On nearly C-compact space

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Abstract : In this paper we have introduced a new type of convex topological space [5] called nearly C-compact space where a convex topological space is a topological space with a convexity [16]. Some fundamental characterizations and various basic properties have been obtained. Its relationship with other types of compact spaces is also investigated.

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1 Introduction

The development of 'abstract convexity' has emanated from different sources in differentways; the first type of development basically banked on generalization of particular problems such as separation of convex sets [3], extremality [4] ; [2], or continuous selection [10]. The second type of development lay before the reader such axiomatizations, which, in every case of design, express a particular point of view of convexity. With the view point of generalized topology which enters into convexity via the closure or hull operator, Schmidt[1953] and Hammer[1955], [1963], [1963b] introduced some axioms to explain abstract convexity. The arising of convexity from algebraic operations and the related property of domain finiteness received attentions in Birkhoff and Frink[1948] Schmidt[1953] and Hammer[1963].

Throughout this paper the axiomatizations as proposed by M. L. J. Van De Vel in his papers in the seventies and finally incorporated in Theory of Convex Structure [15] will be followed.

In [16] the author has discussed 'Topology and Convexity on the same set' and introduced the compatibility of the topology with a convexity on the same underlying set. At the very early stage of this paper we have set aside the concept of compatibility and started just witha triplet (X, τ, C) and called it convex topological space only to bring back 'compatibility' in another way subsequently. With his compatibility, however, Van De Vel has called the triplet (X, τ, C) a topological convex structure.

In this paper, section 2 deals with some early definitions and in section 3, we have discussed about C-regular open sets and mainly nearly C-compact spaces. The last section deals with relationship between nearly C-compact space and almost C-compact space ; different typesof functions are also introduced here.

2 Prerequisites

Definition 2.1 [16] Let X be a nonempty set. A family ✓ of subsets of the set X iscalleda convexity on X if

1. $\emptyset$, X ∈ ✓
2. ✓ is stable for intersection, i. e. if D ⊆ ✓ is nonempty then ∩D ∈ ✓.
3. ✓ is stable for nested unions, i. e. if D ⊆ ✓ is nonempty and totally ordered by set inclusion then ∪D ∈ ✓.

The pair (X, ✓) is called a convex structure. The members of ✓ are called convex sets and their complements are called concave sets.

Definition 2.2 [16] Let ✓ be a convexity on a set X. Let A ⊆ X. The convex hull of A is denoted by co(A) and defined by
co(A) = ∩{C : A ⊆ C ∈ ✓ }.
Note 2.3 [16] Let (X, ✓) be a convex structure and let Y be a subset of X. The family of sets ✓$_Y$ = {C ∩ Y : C ∈ ✓} is a convexity on Y ; it is called the relative convexity of Y
.
Note 2.4 [16] The hull operator co_Y of a subspace (Y, ✓$_Y$) satisfies the following : ∀A ⊆ Y : co_Y(A) = co(A) ∩ Y .
Definition 2.5 [5] Let (X, τ) be a topological space. Let ✓ be a convexity on X. Then the triplet (X, τ, ✓) is called a convex topological space (CTS, in short).
Definition 2.6 [16] Let (X, τ, ✓) be a CTS . Then τ is said to be compatible with the convex structure (X, ✓) if all polytopes of ✓ are closed in τ where a polytope means the convex hull of a finite set. The triplet (X, τ, ✓) is called a topological convex structure.
Note 2.7 [16] Let (X, τ, ✓) be a topological convex structure. Then collection of all closed sets in (X, τ) are subset of C.
Definition 2.8 [16] A function f : (X_1, ✓$_1$) → (X_2, ✓$_2$) between two convex structures issaid to be a convexity preserving function (a CP function) provided for each convex set Cin X_2 ,the set $f^{-1}(C)$ is convex in X_1 i.e. inverse image of convex set is convex.

3 Nearly C-compact space

Definition 3.1 Let (X, τ, ✓) be a CTS. A set P ⊆ X is said to be C-regular open set ifP = int(co(P)).

Result 3.2 Let (X, τ, ✓) be a CTS. Then for any A ⊆ X, the set int(co(A)) is a C-regular open set.
Proof: Let B = int(co(A)). Clearly B is an open set. Now B ⊆ co(B) ⇒ int(B) = B ⊆ int(co(B)). Again int(co(B)) = int(co(int(co(A)))) ⊆ co(int(co(A))) ⊆ co(co(A)) = co(A) ⇒ int(int(co(B))) = int(co(B)) ⊆ int(co(A)) = B. Thus we have B = int(co(B)). Consequently int(co(A)) = B is a C-regular open set.

Note 3.3 It is clear that in a CTS, every C-regular open set is an open set.

Definition 3.4 Let (X, τ, ✓) be a CTS. A set Q ⊆ X is said to be C-regular closed if its complement is a C-regular open set.

Note 3.5 Let (X, τ, ✓) be a CTS. Let A be a C-regular open set. Then A = int(co(A)). Now X \ A = cl[X \ co(A)]. Suppose that B = X \ A. Then B = cl[X \ co(X \ B)]. Since A is a C-regular open set, B is C-regular closed set. Again for any set B ⊆ X, letP = cl[X \ co(X \ B)]. Then X \ P = int(co(X \ B)). Since int(co(X \ B)) is a C-regular open set, P is C-regular closed set.

Note 3.6 In a CTS (X, τ, ✓) a set P ⊆ X is C-regular closed if and only if P = cl[X \ co(X \ P)].

Result 3.7 Let (X, τ, ✓) be a CTS. Then for any Q ⊆ X, cl[X \ co(Q)] is a C-regular closed set

Proof: Let R = cl[X \ co(Q)]. Clearly R is a closed set. We will show that R = cl[X \ co(X \ R)]. Now X \ R ⊆ co(X \ R) ⇒ X \ co(X \ R) ⊆ R ⇒ cl[X \ co(X \ R)] ⊆ cl(R) = R. Again X \ R = X \ cl[X \ co(Q)] = int(co(Q)) ⊆ co(Q) ⇒ co(X \ R) ⊆ co(Q) ⇒ X \ co(Q) ⊆ X \ co(X \ R) ⇒ cl[X \ co(Q)] ⊆ cl[X \ co(X \ R)] ⇒ R ⊆ cl[X \ co(X \ R)]. Hence R = cl[X \ co(X \ R)] and consequently R is a C-regular closed set [by the abovenote].

Note 3.8 It is clear that in a CTS (X, τ, ✓), for any set Q ⊆ X, cl[X \ co(Q)] is a C-regular closed set. In the above proof actually we have used the fact that a set A is C-regular closed set if and only if A = cl[X \ co(X \ A)] because we want to define a C-regular closed set independently in a CTS. Thus in a (X, τ, ✓), a set A ⊆ X is said to be C-regular closed setif A = cl[X \ co(X \ A)].

Theorem 3.9 Let (X, τ, ✓) be a CTS. Then the collection *B* of all C-regular open sets forma basis for some topology which is weaker than τ.

Proof: Let (X, τ, ✓) be a CTS and *B* be the collection of all C-regular open sets. Clearly ∅, X ∈ *B*. Let P, Q ∈ *B*. Then P = int(co(P)) and Q = int(co(P)). Now int(co(P∩ Q)) ⊆ int(co(P)) ∩int(co(Q)) ⇒ int(co(P ∩ Q)) ⊆ P ∩ Q. Since int(co(P ∩ Q)) ∈ *B*, we conclude that *B* forms a base for some topology say τ_1. Again since every C-regular open set is open,we have $\tau_1 \subseteq \tau$.

Definition 3.10 Let (X, τ, ✓) be a CTS. The space (X, τ, ✓) is said to be nearly C- compact if for every open cover ***t*** = {U_a : α ∈ Λ} of X , there exists a finite subfamily {U_a : α ∈ I} where I is a finite set, of *t* such that $\cup_{a\in}$ U_a .

Note 3.11 Let (X, τ, ✓) be a CTS. If (X, τ) is compact, then the space (X, τ, ✓) is nearly C-compact. Observe that if U is an open set, then U ⊆ int(co(U)). The converse is not truein general which follows from the next example.

Example 3.12 Let X be an infinite set. Also let τ be the discrete topology and ✓ = {∅, X}. Then for any nonempty set A ⊆ X, clearly co(A) = X. Thus the CTS (X, τ, ✓) is nearly C-compact but (X, τ) is not compact.

Note 3.13 Nearly compactness is a topological concept where as nearly C-compactness is defined in terms of topology as well as convexity. In a CTS , a topology and a convexity are given on an arbitrary set. So it is natural to ask whether there is a relation between nearly compact space and nearly C-compact space. We shall show that these two concepts are independent which follows from the next examples.

Example 3.14 Consider the above example 3.12. Here (X, τ, ✓) is nearly C-compact but (X, τ) is not nearly compact.

Example 3.15 Let X be an infinite set. Let $p \in X$ and a set $U \subset X$ is defined to be open iff $p \in U$ i.e. the topology τ is given by $\tau = \{U : p \in U\}$. Clearly every open set is dense in X. So (X, τ) is nearly compact. Again let the convexity ✓ on (X, τ) be P(X). Then for any $A \subseteq X$, $co(A) = A$. Let us consider the open cover $U = \{\{p, x\} : x(\neq p) \in X\}$. This cover has no finite subfamily, the interior of the convex hull of whose members covers the space X. Consequently (X, τ, ✓) is not nearly C-compact space.

Theorem 3.16 Let (X, τ, ✓) be a CTS. Then the following conditions are equivalent.

a) (X, τ, ✓) is nearly C-compact.

b) Every basic open cover of X admits of a finite subfamily, the interior of the convex hullof whose members cover the space.

c) Every cover of X by C-regular open sets has a finite sub cover.

d) Every family of C-regular closed sets having the finite intersection property has non void intersection.

e) Every family ↗ of closed sets having the property that for any finite subfamily $\{F_i : i = 1, 2, \ldots\ldots, n\}$, of ↗, $\bigcap_{i=1}^{n} cl[X \setminus co(X \setminus F_i)]$ G ∅, has nonempty intersection .

Proof: a) ⇒ b).Obvious.

b)⇒ c). Let $t = \{U_a : \alpha \in \Lambda\}$ be C-regular open cover of X. Let $x \in X$. Then $x \in U_a$ for some $\alpha \in \Lambda$. Since U_a is open set and $x \in U_a$ there exists a basic open set V_{a_x} such that $x \in V_{a_x} \subseteq U_a$..........(1). Now the family $\{V_{a_x} : x \in X\}$ is a basic open cover X. Hence by b) there exists a finite subfamily $\{V_{a_{x_i}} : i = 1, 2, \ldots\ldots, n\}$ such that $\bigcup_{i=1}^{n}[int(co(V_{a_{x_i}}))] = X$........ (2).

From(1), $int(co(V_{a_x})) \subseteq int(co(U_a)) = U_a$,[since U_a is C-regular open set]. So from (2), we have $\bigcup_{i=1}^{n} U_{a_{x_i}} = X$.

c) ⇒ d). Let ↗ = {F_a : α ∈ Λ} be a family of C-regular closed sets having the finite intersection property. If possible let ∩ {F_a : a ∈ Λ} = ∅. Then {X \ F_a : α ∈ Λ} is an C-regular open cover of X. So by c) there exists a finite subfamily {X \ F_{a_i} : i = 1, 2, , n} such that ⋃{X \ F_{a_i} : i = 1, 2,......., n} = X. This implies that ⋂{F_{a_i} : i = 1, 2, , n} = ∅, which is a contradiction. Hence we infer that ∩ {F_a : a ∈ Λ} G ∅.

d) ⇒ e). Let ↗ = {F_a : α ∈ Λ} be a family of closed sets with the stated property. Now ↗$_1$ = {cl[X \ co(X \ F_a)] : a ∈ Λ} is a family of C-regular closed sets and it has finite intersection property. So by d) ⋂{cl[X \ co(X \ F_a)]: a ∈ Λ} G ∅ (1). Again X \ F_a ⊆ co(X \ F_a) ⇒ cl[X \ co(X \ F_a)] ⊆ cl(F_a) = F_a [since F_a is closed set]. Hence from 1), it follows that ⋂{F_a : a ∈ Λ} G ∅.

e) ⇒ a). Let t = {U_a : a ∈ Λ} be an open cover of X. If possible suppose that for every finite subfamily {U_a : a ∈ I} where I is a finite set, of t, ⋃{int(co(U_a)): a ∈ I} G X. Then this implies that X \ ⋃{int(co(U_a)): a ∈ I} G ∅ ⇒ ⋂{X \ int(co(U_a)): a ∈ I} G ∅ ⇒ ⋂{cl[X \ co(U_a)]: a ∈ I} G ∅———(1). Let V_a = X \ U_a. Then V_a is a closed set and from (1) we have ⋂{cl[X \ co(X \ V_a)]: a ∈ I} G ∅. Now the family {V_a : a ∈ Λ} of closed sets satisfies the condition e). So by e) we get, ⋂{V_a : a ∈ Λ} G ∅. ⇒ ⋂{X \ U_a : a ∈ Λ} G ∅ ⇒ ⋃{U_a : a ∈ Λ} G X, which is a contradiction. Hence (X, τ, ✓) is nearly C-compact.

Definition 3.17 A CTS (X, τ, ✓) is said to be semi C-regular space if every point in X has fundamental system of C-regular open neighbourhood.

Theorem 3.18 Let (X, τ, ✓) be semi C-regular space. Then (X, τ, ✓) is nearly C- compactif and only if it is compact.

Proof: We know that compact space is nearly C-compact space. So one part is obvious. On the other hand let (X, τ, ✓) be a semi C-regular space which is nearly C-compact. Also let t = {U_a : a ∈ Λ} be an open cover of X. Let x ∈ X. Then ∈ U_{a_x} for some a ∈ Λ. Since X is semi C-regular space, there exists a C-regular open set V_{a_x} such that x ∈ V_{a_x} ⊆ U_{a_x} .

Now {V_{a_x} : x ∈ X} is a C-regular open cover of X and therefore it has a finite sub cover

{$V_{a_{x_i}}$: i = 1, 2, ..., n}. Consequently {$U_{a_{x_i}}$: i = 1, 2,. , n} is a finite sub cover of t. This shows that (X, τ, ✓) is compact.

Corollary 3.19 Let (X, τ, ✓) be a semi C-regular space. If (X, τ, ✓) is nearly C-compact then it is nearly compact.

Proof: Observe that compactness implies nearly compactness.

Note 3.20 We see that in a semi C-regular space, nearly C-compactness implies nearly compactness. In the next theorem we shall show that there exists a special type of convex topological space in which nearly C-compactness implies nearly compactness also.

Theorem 3.21 Let (X, τ, ✓) be a CTS where τ is compatible with the convex structure (X, ✓) i.e. (X, τ, ✓) be a topological convex structure. If (X, τ, ✓) is nearly C-compact then it is nearly compact.

Proof: Let S be the collection of all closed sets in (X, τ, ✓). Since (X, τ, ✓) is a topological convex structure, S ⊆ ✓. Thus for any set A ⊆ X we have co(A) ⊆ cl(A). Let t = $\{U_a : a \in \Lambda\}$ be an open cover of X .Since X is nearly C-compact, t has finite subfamily $\{U_{a_i}: i = 1,2,\ldots,n\}$ such that $\cup\{int(co(U_{a_i})) : i = 1,2,\ldots,n\} = X \Rightarrow \cup\{int(cl(U_{a_i})) : i = 1,2,\ldots,n\} = X$. This shows that X is nearly compact.

4 Comparison

Definition 4.1 Let (X, τ, ✓) be a CTS. The space is said to be almost C-compact if for every open cover $\{U_a : a \in \Lambda\}$, there exists a finite subfamily $\{U_{a_i} : i = 1,2,\ldots\ldots,n\}$ such that $\cup\{co(U_{a_i}) : i = 1,2,\ldots,n\} = X$.

Note 4.2 If a CTS (X, τ, ✓) is nearly C-compact then it is also almost C-compact.

Definition 4.3 A CTS (X, τ, ✓) is said to be almost C-regular if for each x ∈ X andeachopen neighbourhood U of x, there exists an open neighbourhood V of x such that x ∈ V ⊆ co(V) ⊆ int(co(U)).

Note 4.4 Every locally convex space is almost C-regular space.

If x ∈ X and U is an open neighbourhood of x then ∃ a convex open neighbourhood V of x such that x ∈ V ⊆ U . This shows that V = co(V) = int(V) ⊆ int(co(U)).

Theorem 4.5 An almost C-regular space (X, τ, ✓) is almost C-compact if and only if it is nearly C- compact.

Proof: Nearly C-compactness implies almost C-compactness, so one part is obvious.

To prove the converse, let (X, τ, ✓) be an almost C-regular space which is almost C-compact. Let $t = \{U_a : a \in \Lambda\}$ be any C-regular open cover of X. Let x ∈ X. Then ∈ U_{a_x} , for some $a \in \Lambda$. Since X is almost C-regular, there exists an open neighbourhood V_{a_x} of x such that x $\in V_{a_x} \subseteq co(V_{a_x}) \subseteq int(co(U_{a_x})) = U_{a_x}$ —-(1)[Note that U_{a_x} is a C-regular open set].

Now $\{V_{a_x} : x \in X\}$ is an open cover of X which is almost C-compact. So it admits a finite subfamily $\{V_{a_{x_i}} : i = 1, 2, .., n\}$ such that $\cup\{co(V_{a_{x_i}}) : i = 1, 2, \ldots, n\} = X$. Hence from (1) we have $\cup\{U_{a_{x_i}}: i = 1, 2, \ldots, n\} = X$. Consequently (X, τ, ✓) is nearly C-compact.

Theorem 4.6 Let (X, τ, ✓) be a nearly C-compact space. Then every open cover of a C-regular closed subset of Y of X admits a finite subfamily, the interior of the convex hull of whose members cover the set Y .

Proof: Let Y be a C-regular closed set in (X, τ, ✓). Also let $t = \{U_a : a \in \Lambda\}$ be any open cover of Y . Clearly X \ Y is C-regular open set and so it is open in X. Thus $t_1 = \{U_a : a \in \Lambda\} \cup \{(X \setminus Y)\}$ is an open cover of X which is nearly C-compact. Thus t_1 has a finite subfamily $\{V_{a_i} : i = 1,2\ldots,n\}$ such that $\cup\{int(co(V_{a_i})) : i = 1,2,\ldots\ldots,n\} = X$. Let there is no $i \in \{1,2,\ldots\ldots,n\}$ such that $V_{a_i} = X \setminus Y$. So we have $\cup\{int(co(V_{a_i})) : i = 1,2,,n\} = Y$.

Again if $V_{a_i} = X \setminus Y$ for some $i \in \{1, 2, \ldots\ldots, n\}$ and since int(co(X \ Y)) = X \ Y we infer that $t_1 \setminus \{X \setminus Y\}$ serves the required purpose.

Definition 4.7 A function f : (X, τ, ✓1) → (Y, σ, ✓2) is said to be

1) almost C-continuous if the inverse image of every C-regular open set is open and
2) almost C-open if image of each C-regular open set is open.

Note 4.8 Since every C-regular open set open, we infer that in a CTS every continuous function is almost C-continuous and every open function is almost C-open.

Theorem 4.9 Let f : (X, τ, ✓1) → (Y, σ, ✓2) be a CP function from X onto Y and f is almost C-continuous and almost C-open. If X is nearly C-compact then Y is also nearly C-compact.

Proof: Let f: (X, τ, ✓1) → (Y, σ, ✓2) be a CP, almost C-continuous and almost C-open mapping from a nearly C-compact space X onto Y. Suppose $t = \{U_a : a \in \Lambda\}$ be a C-regular open cover of Y. Then $t_1 = \{f^{-1}(U_a) : a \in \Lambda\}$ is an open cover of X. Then t_1 admits a finite subfamily $\{f^{-1}(U_{a_i}) : i = 1,2,\ldots,n\}$ such that $\cup\{int\ (co\ (f^{-1}(U_{a_i}))) : i = 1,2,\ldots,n\} = X$. Since $int\ (co\ (f^{-1}(U_{a_i})))$ is a regular C-open set in X and f is almost C-open map, $f(int\ (co\ (f^{-1}(U_{a_i}))))$ is an open set in Y. So $f(int\ (co\ (f^{-1}(U_{a_i})))) \subseteq int\ (f(co\ (f^{-1}(U_{a_i}))))$ ……….(1). Again, since f is a CP mapping, $co\ (f^{-1}(U_{a_i})) \subseteq f^{-1}(co(U_{a_i}))$……….(2). Now $Y = f(X) = f(\cup\{int\ (co\ (f^{-1}(U_{a_i}))) : i = 1,2,\ldots,n\}) = \cup\{f(int\ (co\ (f^{-1}(U_{a_i})))) : i = 1,2,\ldots,n\} \subseteq \cup\{int\ (f(co\ (f^{-1}(U_{a_i})))) : i = 1,2,\ldots,n\}$ (from (1)) $\subseteq \cup\{int\ (f(f^{-1}(co(U_{a_i})))) : i = 1,2,\ldots,n\}$ (from (2)) $= \cup\{int\ (co(U_{a_i})) : i = 1,2,\ldots,n\}$ (since f is onto) . Therefore,
$Y \subseteq \cup\{U_{a_i} : i = 1,2,\ldots n\}$. This shows that Y is nearly C- compact space.

Note 4.10 Let (X, τ, ✓1)be a compact CTS and let f : (X, τ, ✓1) → (Y, σ, ✓2) be a continuous onto map. Then it is clear that Y is nearly C-compact space. We now prove something more in the next theorem.

Theorem 4.11 Let f : (X, τ, ✓1) → (Y, σ, ✓2) be an almost C-continuous mapping from a compact space X onto Y . Then Y is nearly C-compact.

Proof: Suppose $t = \{U_a : a \in \Lambda\}$ be a C-regular open cover of Y. Then $t_1 = \{f^{-1}(U_a): a \in \Lambda\}$ is an open cover of X, which is compact. Then t_1 admits a finite subfamily $\{f^{-1}(U_{a_i}): i = 1,2,\dots,n\}$ such that $\cup\{f^{-1}(U_{a_i}): i = 1,2,\dots,n\} = X$. Now $Y = f(X) = f(\cup\{f^{-1}(U_{a_i}): i = 1,2,\dots,n\}) = \cup\{f(f^{-1}(U_{a_i})): i = 1,2,\dots,n\} = \cup\{U_{a_i}: i = 1,2,\dots,n\}$(since f is onto) . This shows that Y is nearly C- compact space.

Theorem 4.12 A CTS (X, τ, ✓) is almost C-compact if and only if every C-regular open cover has a finite subfamily, the convex hull of whose members cover the space.
Proof: Since every C-regular open set is open, it is clear that, if X is almost C-compact then it has the above property.
On the other hand, let $t = \{U_a: a \in \Lambda\}$ be an open cover of X. Since $U_a \subseteq int(co(U_a))$, the family $t_1 = \{int(co(U_a)): a \in \Lambda\}$ is a C-regular open cover of X. From the given condition, t_1 has a finite subfamily $\{int(co(U_{a_i})): i = 1,2,\dots,n\}$ such that $\cup\{co(int(co(U_{a_i}))): i = 1,2,\dots,n\} = X$………(1) Now $int(co(U_{a_i})) \subseteq co(U_{a_i}) \Rightarrow co(int(co(U_{a_i}))) \subseteq co(co(U_{a_i})) = co(U_{a_i}) \Rightarrow$ (from (1)), $\cup\{co(U_{a_i}): i = 1,2,\dots,n\} = X$. Consequently, X is almost C-compact space.

Theorem 4.13 Let f : (X, τ, ✓1) → (Y, σ, ✓2) be a CP and almost C-continuous mapping from X onto Y . If X is almost C-compact then so is Y.
Proof: Suppose $t = \{U_a: a \in \Lambda\}$ be a C-regular open cover of Y. Then $t_1 = \{f^{-1}(U_a): a \in \Lambda\}$ is an open cover of X. Since X is almost C-compact, t_1 admits a finite subfamily $\{f^{-1}(U_{a_i}): i = 1,2,\dots,n\}$ such that $\cup\{co(f^{-1}(U_{a_i})): i = 1,2,\dots,n\} = X$. Now $Y = f(X) = f(\cup\{co(f^{-1}(U_{a_i})): i = 1,2,\dots,n\}) = \cup\{f(co(f^{-1}(U_{a_i}))): i = 1,2,\dots,n\} \subseteq \cup\{f(f^{-1}(co(U_{a_i}))): i = 1,2,\dots,n\} = \cup\{co(U_{a_i}): i = 1,2,\dots,n\}$(since f is onto) . This shows that Y is nearly C- compact space.

Corollary 4.14 Let f : (X, τ, ✓1) → (Y, σ, ✓2) be a CP and almost C-continuous mapping from X onto Y . If X is compact then Y is almost C-compact.

Corollary 4.15 Let f : (X, τ, ✓1) → (Y, σ, ✓2) be a CP and almost C-continuous mapping from X onto Y . If X is nearly C-compact then Y is almost C-compact.

Definition 4.16 A function f : (X, τ, ✓1) → (Y, σ, ✓2) is said to be strongly convex ifforeach $A \subseteq X$, $f(A) = f(co(A))$.
Theorem 4.17 Let f : (X, τ, ✓1) → (Y, σ, ✓2) be continuous and strongly convex mapping from X onto Y . If X is almost C-compact then Y is compact.
Proof: Let $t = \{U_a : a \in \Lambda\}$ be an open cover of Y . Then $t_1 = \{f^{-1}(U_a): a \in \Lambda\}$ isan open cover of X which is almost C-compact. So t_1 has a finite subfamily $\{f^{-1}(U_{a_i}): i = 1,2,\dots,n\}$ such that $\cup\{co(f^{-1}(U_{a_i})) : i = 1,2,\dots\dots,n\} = X$ Now $Y = f(X) = f[\cup\{co(f^{-1}(U_{a_i})) : i = 1,2,\dots\dots,n\}]$

$= \cup\{f\ (co\ (f^{-1}(U_{a_i}))) : i = 1,2,\&,n\} = \cup\{f\ (f^{-1}(U_{a_i})) : i = 1,2,\&,n\}$

$= \cup\{U_{a_i} : i = 1,2,\ldots\ldots,n\}$. Hence Y is compact.

Corollary 4.18 The image of nearly C-compact space under continuous and strongly convex mapping is nearly C-compact.

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