S*-Bornological Group

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Abstract: The new class on bornology, which it is called S*- bornological group are studied, to reduce the boundedness condition for product and inverse maps. This is related structure to bornological group. Also, the certain condition for any codomain of S*-bornological group to be bornological group are given. The result important main, that every bornological group is S*-bornological group. Additionally, every left (right) translation is S*-bornological isomorphism.
Key words: Bornological Group, Semi Bounded Set, Semi Bounded Map, Bounded Map.

1.INTRODUCTION

A bornology is a collection β of sub sets satisfy three conditions. First of all, covers X. Also, stable under hereditary , finite union. The effect of bornology is to solve the problem of bounded for any set or space in general way. Later, to solve the problem of bounded for groups they define and study bornological groups. A group bornology is a group with bornology s.t. the product and inverse maps , $f_1: (G, \beta) \times (G, \beta) \rightarrow (G, \beta)$, $f_2: (G, \beta) \rightarrow (G, \beta)$, respectively,are bounded maps (see [2], [3], [4], [5]).

The main goal in this work is to restrictive the condition of boundedness for product and inverse maps.We are going to assume that the group operation to be S*-bounded map. So, the new class of bornological group called here S*-bornological group are defined.

First of all, the idea of lessening the condition of bounded set in to semi bounded set by introducing semi bounded set in bornological set see [1] .

A is a semi _ bounded set ,if there exists a bounded set B s.t. $A \subset B \subset \overline{B}$ where $\overline{B}$ ={all upper and lower bounds}$\cup$ B . Amap f from abornolgy β to $\overline{\beta}$ is bounded if $\forall B \in \beta, f(B) = \overline{\beta}$ .Also, f is S_ bounded map if $\forall B \in \beta, f(B)$ is semi bounded set in $\overline{\beta}$ and the map is S*_ bounded map if $\forall A \in \beta$. Addition identity map is clearly a semi bounded map. Evidently, a bounded map is S*-bounded map.

In this work, new classes of bornological group with respect to S*-bounded map are studied. The main goal in this work is to require less restrictive the condition on the group operations neither of the operation is required to be bounded, the main important results, every bornological group is S*-bornological group. Also , we gave the certain condition for any codomain of S*-bornological group to be S*-bornological group. Additionally, every left (right) translation is S*-bornological isomorphism.

2. S*- BORNOLOGICAL GROUP

Now, affined structures to bornological group which is said to be S*-bornological group , this new class was defined S*- bounded map. The motivation for this work is come from, the idea of lessening the condition of bounded in to semi bounded. For any information on semi bounded set and semi bounded map in bornological set the readers can see [1].

Definition(2.1): A bornological group $(G,.,\beta)$ it is said to be **S*-bornological group** if for every semi bounded sets S_1, S_2 contain $g_1, g_2 \in G$, there is a bounded set B contain $g_1 \cdot g_2^{-1}$ in G. S.t. $S_1 \cdot S_2^{-1} \subset B$.

In discrete bornology on infinite set any semi bounded set is bounded set , the opposite is true.

Proposition (2.2): Let $(G, \cdot, \beta)$ is a bornological group ,then every $(G, \cdot, \beta)$ is a S*-bornological group.

Proof: Let all bounded sets B_1, B_2 contain $g_1, g_2 \in G$. since G is a bornological group. Then, There is a bounded set B contain $g_1 . g_2^{-1}$ in G. S.t. $B_1 . B_2^{-1} \subset B$ see [1]. Since any bounded set is a semi bounded set. Thus, by Definition (2.1). The prove is obvious.

Definition (2.3): The function f from bornological set (X,β) to (Y,β) is called **S*-bornological isomorphism** if f it is bijective, f, f^{-1} are S*-bounded maps.

Example (2.4):

Let $=\{1,2\}$, $Y=\{a,b\}$ with two discrete bornological set, respectively. $\beta_X = \{\emptyset, X, \{1\}, \{2\}\}$, $\beta_Y = \{\emptyset, Y, \{a\}, \{b\}\}$.

We define $f: (X,\beta_X) \to (Y,\beta_Y)$. As follows $f(B) = B'$, $\forall B \in \beta_X$, $B' \in \beta_Y$. As $f(\emptyset) = \emptyset$, $f(1) = a$, $f(2) = b, f(X) = Y$.

It is clear that f is bijective. Here , as $f(\emptyset) = \emptyset$, $f(X) = Y$, $f(\{1\}) = \{a\}$, $f(\{2\}) = \{b\}$. Then f is bounded map. Since every bounded map is S*-bounded map [1]. So f is S*-bounded map. Also, as $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$, $f^{-1}(\{a\}) = \{1\}$, $f^{-1}(\{b\}) = \{2\}$. Then, f^{-1} is S*-bounded map. Thus, f is S*-bornological isomorphism.

By the next theorem, we give the certain condition for any codomain of S*-bornological group to be S* -bornological group.

Theorem (2.5): If f is a map from a S*-bornological group $(H,.,\beta)$ in to a bornological group $(L,\circ,\beta')$ be a group homomorphism and S*- bornological isomorphism, then $(L,\circ,\beta')$ is also a S*-bornological group.

Proof: Let $S_1, S_2 \subset L$ be two semi bounded sets contain $g_1, g_2 \in L$. Let $x = f^{-1}(g_1)$, $y = f^{-1}(g_2)$. Since f^{-1} is a S*-bounded map, then $f^{-1}(S_1)$, $f^{-1}(S_2)$ are semi bounded sets contain x , y. Since H is S*-bornological group (By hypothesis).

Then, there exists a bounded set $f^{-1}(B)$ contain $x \cdot y^{-1}$ where B is bounded set contain $g_1 \circ g_2^{-1}$ with $f^{-1}(S_1) \cdot f^{-1}(S_2^{-1}) \subset f^{-1}(B)$.

Since f is S*-bounded map. Then $f(f^{-1}(S_1)) \cdot f^{-1}(S_2^{-1})) \subset B$

Thus, $S_1 \circ S_2^{-1} \subset B$. Then, there is a bounded set B which contain $g_1 \circ g_2^{-1}$. Hence $(L,\circ,\beta')$ is a S* -bornological group.

In result below we give sufficient condition for left or right S*-bornological group to be a S*-bornological isomorphism and it is of the main important results in this chapter.

Proposition (2.6): Let $(G,\cdot,\beta)$ be a S*-bornological group. Then each left (right) translation $l_g: G \to G$, $r_g: G \to G$ is S*-bornological isomorphism.

Proof: Now, we show that the left side is satisfied. Only for left translations can we prove the statement. As we know that left translations is bijective map. We must to prove that for every

fixed arbitrary $g \in G$ the translation l_g is a S*-bounded map. Suppose that g, g_1 be two elements in G and S_1, S_2 be semi bounded sets contain $, g_1$.
By Definition (2.1) there is a bounded set B containing $l_g(g_1) = g \cdot g_1^{-1}$, s.t. $S_1 \cdot S_2^{-1} \subset B$. In particular, we have $g \cdot S_2^{-1} \subset B$. The set S_2^{-1} is semi bounded set contain g_1^{-1}, As a result the last contain says that l_g is a S* -bounded map at g.
Since l_g is S* -bounded on G and $g \in G$ is an element in G.
It's easy to prove that ${l^{-1}}_g = (l_g)^{-1}$ as well, is a S*-bounded map.
Then each left (right) translation is S*-bornological isomorphism.

Definition (2.7): A bornological set y is called **S*-homogenous space** if for all $x_1, x_2 \in y$ there is a S*-bornological isomorphism f of bornological set y onto itself. S.t. $f(x_1) = x_2$.

Proposition (2.8): Every S*-bornological group $(G, \cdot, \beta)$ is a S*-homogenous space.

Proof: Let $a, b \in G$ and suppose, $c = b \cdot a^{-1}$. Since, l_c is a S*- bornological isomorphism of G, by Proposition (2.7). Then

$$l_c(a) = c \cdot a = b \cdot a^{-1} a = b.$$

Theorem (2.9): let $(G, \cdot, \beta_G)$ is a S*-bornological group and let H is a subgroup of S* - bornological group, then every subgroup H is called S*-bornological subgroup.

Proof: For every semi bounded sets $S_1 \subset H$ contain g_1 , $S_2 \subset H$ contains g_2, and we proved to be here a bounded set $B \subset H$ containg $g_1 \cdot g_2^{-1}$. S.t. $S_1 \cdot S_2^{-1} \subset B$. Since , H is bounded in G , S_1, S_2 are semi bounded subsets of G , G is a S*- bornological group. There exists, a bounded set B_G containing $g_1 \cdot g_2^{-1}$. S.t. $S_1 \cdot S_2^{-1} \subset B_G$, the set $B = B_G \cap H$ is bounded subsets of H. Also, $S_1 \cdot S_2^{-1} \subset B \subset B_G$, its meant that, H is a S*- bornological group.

3. CONCLUSION

To reduce the boundedness condition for product and inverse maps,we study new classes of bornological group which is called S*_ bornological group.

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