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Research Article

Fuzzy Dynamic Programming to Determine the Shortest Route Problem

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Abstract: The purpose of this paper is to develop a new paradigm for a fuzzy dynamic programming for solving the shortest route problem. Dynamic programming is a mathematical technique used for obtaining the solution for a sequence of interrelated decisions. In real life, situations are not crisp enough moreover, they are fuzzy in nature. Therefore, in order to deal with the real situations a new method is being proposed here to find the more realistic answers to the dynamic problems. For this purpose the MATLAB software is used where the decision parameters are trapezoidal fuzzy numbers.

Keywords: Dynamic programming, fuzzy dynamic programming, shortest path problem

1. Introduction

Dynamic programming is a mathematical technique used to solve the optimal sub-divisional problems. The process involves the disintegration of original complex problem into simpler sub-problems and save the answer for the future, thereby avoiding the work of re-computing the answers every time problem is solved. . It is applied in many fields such as production, scheduling, inventory, salesman allocation, advertising media, probability decision problem etc.

The optimisation of problems involving large number of decision variables along with inequality constraints cannot be handled adequately by classical mathematics, therefore dynamic programming technique is used as it provides the solution in orderly manner from one stage to another until the final stage is reached. Optimisation of problem is done using deterministic formulation but very often in real world, problems include unknown parameters.

In real life, situations are not so crisp and deterministic. Therefore adopting fuzzy approach to represent parametric uncertainty can be helpful. Fuzzy was introduced by scientist **Lotfi A. Zadeh** of University of California, Berkeley (1965). Fuzzy significantly deals with the ambiguity in real world problem. The process of applying fuzzy set theory in the field of dynamic programming is known as fuzzy dynamic programming.

2. Significance Of The Study

Shortest routes are of great importance in network and optimization problems. Shortest route problem is used to determine an optimal shortest path between two nodes analysing the large network. To travel from one place to another can be done through different paths, but to determine the most effective path helps us to save the overall travelling time of the journey and to reach the destination in the least time possible.

3. Review Of Related Studies

The scientist **Sepideh (2003) and Rehmat et. al(2007)** have discussed their ideas on the fuzzy multi-objective linear programming to determine the solution of fuzzy travelling salesman problems. **Dhanasekar et. al(2013)** also applied classical travelling salesman problem based approach for solving fuzzy travelling salesman problems using ranking technique. **Majumdar and Bhunia(2011)** dealt with asymmetric fuzzy travelling salesman problem. **Singhal Abha and Pandy P.(2016)** used dynamic programming algorithm for optimal solution of fuzzy travelling salesman problem where all the parameters are in matrix form.

Scientist **Kacprzyk(1997)** has stated the applications and developments in the field of fuzzy dynamic programming. **Hussien et. al(1995)** have approached a new method using fuzzy dynamic programming for solving multiple criteria resources allocation problem. **Baldwin et. al(1982)** has worked on dynamic programming with fuzzy system and fuzzy environment for multi stage problem and solved by a procedure of fuzzy interpolation. It is an extention of **Bellman R.E. (1952)** model. **Nagalakshmi et. al(2018)** have proposed a paper in solving a fuzzy optimal sub-divisional problem using fuzzy least cost route problem by using generalized trapezoidal number. **Hosseinzadeh et. al** have also approached a new technique for resolving fully fuzzy linear programming by using the lexicography way.

4. Objectives Of The Study

The objective is to determine the optimal solution for the fuzzy least cost route problem with vague parameters using dynamic programming approach.

5.Definition and Preliminaries

5.1.Fuzzy Set

Let X be a non empty set, then a fuzzy set $\tilde{A}$ of this set X is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$

Where; $\mu_{\tilde{A}}: X \rightarrow [0, 1]$ is known as membership function of $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is the degree of membership of $x \in X$ in $\tilde{A}$.

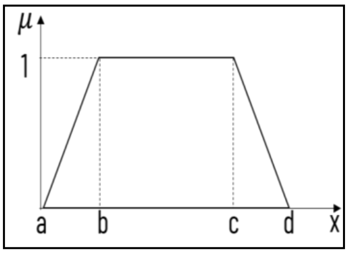
5.2.Membership Function

Membership function is a graphical representation that is used to characterize fuzziness, i.e., whether the elements in a fuzzy set are discrete or continuous. It represents the degree of truth in fuzzy logic. A fuzzy set is described by a membership function $\mu_{\tilde{A}}(x)$ of $\tilde{A}$.

5.3.Trapezoidal Membership Function

Trapezoidal membership function consists of four parameters a, b, c and d. Here b to c span represents the highest membership value (i.e. 1). At a and d the membership value is least (i.e. 0) and the value between (a, b) and (c, d) is in between 0 and 1.

$$\mu_A(a, b, c, d; x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \\ 0, & d \le x \end{cases} = max\left(min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$



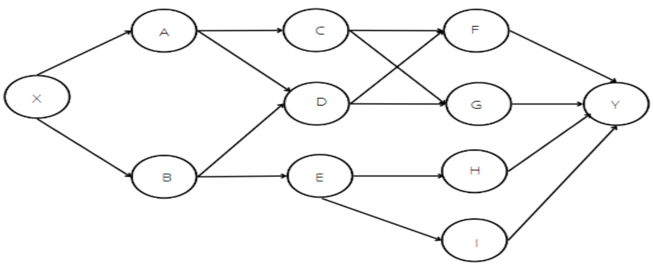
5.4.Arctitecture Of Fuzzy Expert System

6.Fuzzy Shortest Route Problem

Here the auto-parts shop owner wants to send his loading vehicle from Mhow to Maheshwar in such a way that the time required to reach the destination is least. On studying the various connectivities we have obtained the routes in following manner:

Table.1. Names of the cities

X	MHOW
A	MANPUR
B	BARGONDA
C	GUJRI
D	AASHAPURA
E	GULAWAT
F	DHAMNOD
G	MATMOR
H	CHOLI
I	KARHI
Y	MAHESHWAR



On dividing the problem into sub-stages we obtain four stages for the given problem. At each stage a decision is to be taken to select the optimal path out of the given alternatives. This is to be solved using fuzzy dynamic programming.

The time taken by the vehicle to reach from one city to another city is dependent on various factors such as distance, load in vehicle etc. Using fuzzy approach to the dynamic programming helps to take the optimum decision in advance by considering various factors that can affect the time taken during the journey.

Therefore, in order to use fuzzy approach two input parameters (distance, weight) are taken which affects the output parameter (time). Then fuzzy rules are given to fuzzy inference system to determine the optimum values for time required to reach from one city to another. This method is implemented in MATLAB using Fuzzy toolbox.

Figure.1. The design structure

6.1. Proposed Method

STEP1: Collect the information about the input variables from the shop owner.
STEP 2: Design the expert system using fuzzy inference system.
STEP 3: Construct the rule base.
STEP 4: Give the input parameters in the fuzzy system to determine the output.
STEP 5: Use the defuzzified output values to determine the result of the problem using the dynamic programming approach.

6.2.Data Analysis

The membership functions of the inputs and output are given as follows:

Table.2. Membership Functions Of Distance

MINIMUM	MEDIUM	MAXIMUM
$\mu_{(min)}(x) = \begin{cases} 1, & 9 \le x \le 11 \\ \frac{x-5}{4}, & 5 \le x \le 9 \\ \frac{15-x}{4}, & 11 \le x \le 15 \\ 0, & \le 5 \text{ and } \ge 15 \end{cases}$	$\mu_{(medium)}(x) = \begin{cases} 1, & 18 \le x \le 24 \\ \frac{x-12}{6}, & 12 \le x \le 18 \\ \frac{30-x}{6}, & 24 \le x \le 30 \\ 0, & \le 12 \text{ and } \ge 30 \end{cases}$	$\mu_{(max)}(x) = \begin{cases} 1, & 35 \le x \le 42 \\ \frac{x-25}{10}, & 25 \le x \le 35 \\ \frac{50-x}{8}, & 42 \le x \le 50 \\ 0, & \le 25 \text{ and } \ge 50 \end{cases}$

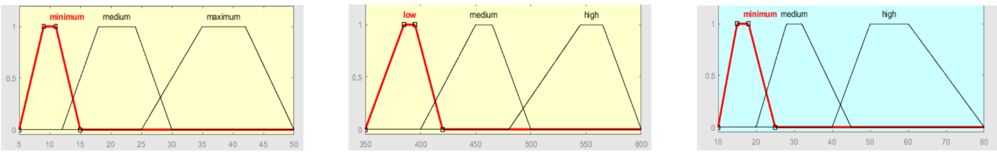
Table.3. Membership Functions Of Weight

LOW	MEDIUM	HIGH
$\mu_{(low)}(x) = \begin{cases} 1, & 385 \le x \le 395 \\ \frac{x-350}{35}, & 350 \le x \le 385 \\ \frac{420-x}{25}, & 395 \le x \le 420 \\ 0, & \le 350 \text{ and } \ge 420 \end{cases}$	$\mu_{(medium)}(x) = \begin{cases} 1, & 450 \le x \le 465 \\ \frac{x-400}{50}, & 400 \le x \le 450 \\ \frac{500-x}{35}, & 465 \le x \le 500 \\ 0, & \le 400 \text{ and } \ge 500 \end{cases}$	$\mu_{(high)}(x) = \begin{cases} 1, & 545 \le x \le 565 \\ \frac{x-480}{65}, & 480 \le x \le 545 \\ \frac{600-x}{35}, & 565 \le x \le 600 \\ 0, & \le 480 \text{ and } \ge 600 \end{cases}$

Table.4. Membership Functions Of Time

MINIMUM	MEDIUM	MAXIMUM
$\mu_{(min)}(x) = \begin{cases} 1, & 15 \le x \le 18 \\ \frac{x-10}{5}, & 10 \le x \le 15 \\ \frac{25-x}{7}, & 18 \le x \le 25 \\ 0, & \le 10 \text{ and } \ge 25 \end{cases}$	$\mu_{(medium)}(x) = \begin{cases} 1, & 28 \le x \le 32 \\ \frac{x-20}{8}, & 20 \le x \le 28 \\ \frac{45-x}{12}, & 32 \le x \le 45 \\ 0, & \le 20 \text{ and } \ge 45 \end{cases}$	$\mu_{(medium)}(x) = \begin{cases} 1, & 28 \le x \le 32 \\ \frac{x-20}{8}, & 20 \le x \le 28 \\ \frac{45-x}{12}, & 32 \le x \le 45 \\ 0, & \le 20 \text{ and } \ge 45 \end{cases}$

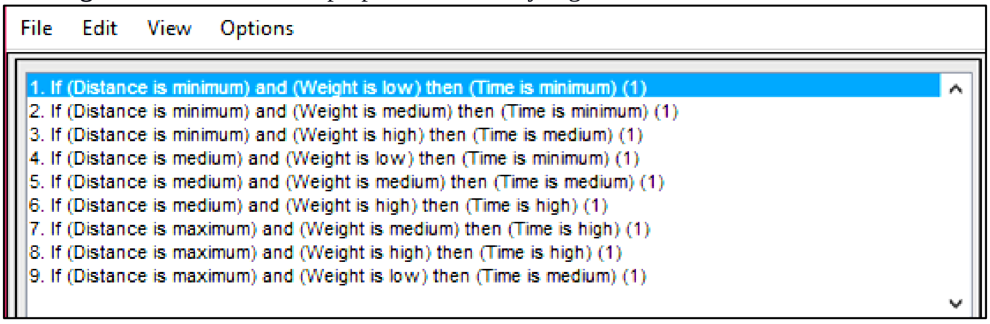
Figure.2. Membership Functions (a) Distance (b) Weight (c) Time



(a) **(b)** **(c)**

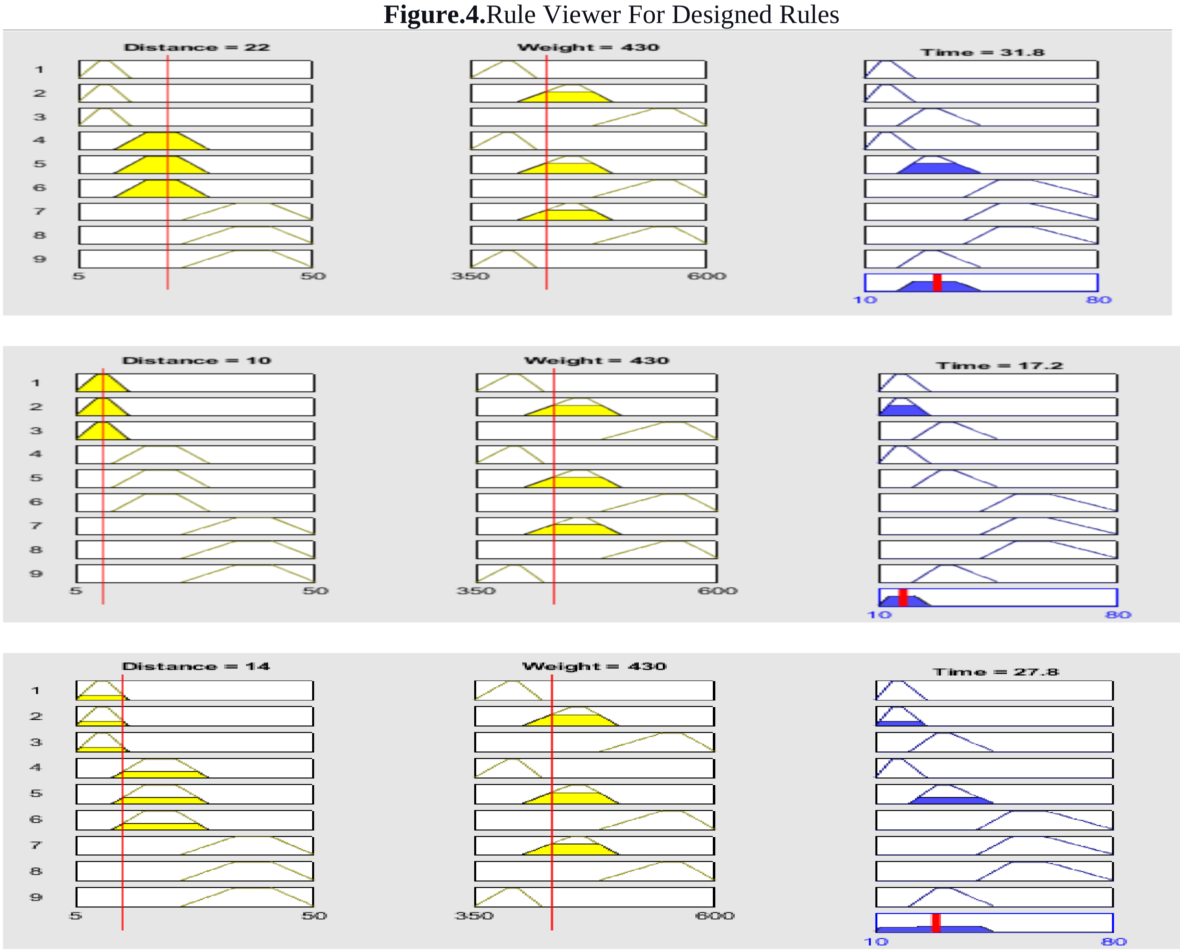
Here Mamdani method is used as the inference method and then IF-THEN rules are constructed.

Figure.3.IF-THEN rules prepared with fuzzy logic toolbox in MATLAB software



Based on the above rules we have determined the values of time taken by the vehicle to reach from one city to another depending on various input factors and noted down in a tabular form.

Figure.4. Rule Viewer For Designed Rules



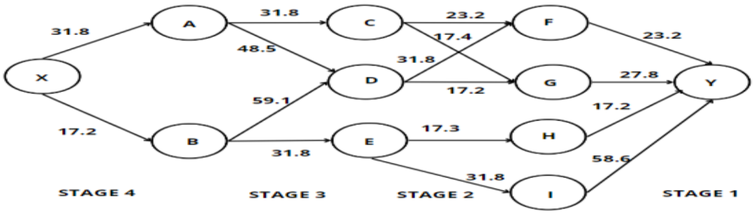
Here the distances of various cities along with weight kept in vehicle are taken while travelling from Mhow to Maheshwar (the weight may be different for different day). Putting these values in the system we have obtained the optimized values of time to reach each city.

Table.5. Observed Values

CITIES TO BE TRAVELLED	**DISTANCE TO BE TRAVELLED**	**WEIGHT IN THE VEHICLE**	**TIME**
X→A	22	430	31.8
X→B	11	430	17.2
A→C	19	430	31.8
A→D	28	430	48.5
B→D	47	430	59.1
B→E	22	430	31.8
C→F	13	430	23.2
C→G	6	430	17.4
D→F	18	430	31.8
D→G	8	430	17.2
E→H	7	430	17.3
E→I	18	430	31.8
F→Y	13	430	23.2
G→Y	14	430	27.8
H→Y	10	430	17.2
I→Y	31	430	58.6

7.Result

Now using these defuzzified values of time variable we solve the shop owner's problem using the dynamic programming approach.



Using the stage coach problem to determine the shortest path:

f(Y) = 0

f(F) =23.2

f(G) = 27.8

f(H) = 17.2

f(I) = 58.6

f(C) = min{23.2+f(F), 17.4+ f(G)} = min {46.4, 45.2} =45.2

f(D) = min{31.8+f(F), 17.2+ f(G)} = min {55, 45} = 45

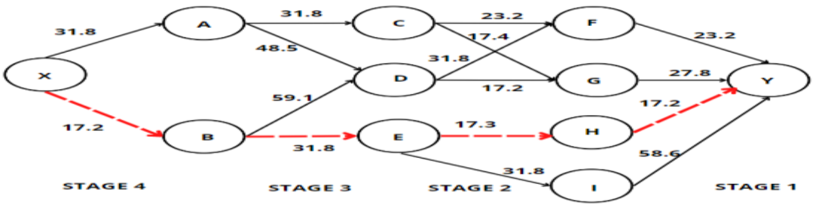
f(E) = min{17.3+f(H), 31.8+ f(I)} = min {34.5, 90.4} = 34.5

f(A) = min{31.8+f(C), 48.5+ f(D)} = min {77, 93.5} = 77

f(B) = min{59.1+f(D), 31.8+ f(E)} = min {104.1, 66.3} = 66.3

f(X) = min{31.8+f(A), 17.2+ f(B)} = min {108.8, 83.5} = 83.5

Hence, the optimal path obtained for given problem is X-B-E-H-Y and the minimum time taken to reach from source (Mhow) to destination (Maheshwar) is 17.2+31.8+17.3+17.2 = 83.5 minutes. This is the most optimal path obtained according to the given input parameters.



8.Conclusion

A fuzzy dynamic programming technique is proposed for the solution of shortest route problem. Here a fuzzy logic expert system is created using MATLAB in order to determine the optimum values of time taken to travel from one city to another based on various input factors affecting the travelling time. The input and output factors are classified into linguistic form. Then the trapezoidal membership function are constructed which are then provided with the IF-THEN rules. On applying the given conditions of input variables provided by the shop owner the values of time are noted in tabular form. Then these defuzzified values of time are used to solve the problem by applying the dynamic programming method. Hence we get the most optimum results of our real life problem going through fuzzy inference system.

References (APA)

Baldwin J F and Pilswoth B W (1982). Dynamic Programming for Fuzzy Systems with Fuzzy Environment. Journal of Mathematics analysis and application, 85(1), 1-23.

Bellman R. E. (1952). The Theory of Dynamic Programming, An Address Delivered before the Summer Meeting of the Society in Laramie.

Dhanasekar S., Hariharan S., and Sekar P. (2013). Classical travelling salesman problem (TSP) based approach to solve fuzzy TSP using Yager's ranking. International journal of Computer Applications (IJCA), 74 (13), 1-4.

Hosseinzadeh A., Edalatpanah S. A. (2016). A New Approach for Solving Fully Fuzzy Linear Programming by using the Lexicography Method, Advances in Fuzzy Systems, 1-6.

Hussien M L, Abo-sinna M A (1995). A Fuzzy Dynamic Approach to the Multi-Criteria Resource Allocation Problem, Fuzzy sets and systems, 69, 115-124.

Kacprzy J (1997). Multistage Fuzzy Control A Model-Based Approach to Fuzzy Control and Decision Making, Wiley New York USA and Engineering, 4(8).

Majumdar J, Bhunia AK. (2011). Solving a multi-objective interval crew-scheduling problem via Genetic Algorithm, Operational research society of India, 48(3), 197-216.

Mohanaselvi S., Mondal S.S. (2019). "A fuzzy dynamic programming approach to fuzzy least cost route problem", Journal of Physics: Conference Series 1377012042.

Nagalakshmi T. And Uthra G. (2017). "An approach of finding an optimal solution for a fuzzy least cost route problem by using generalized trapezoidal fuzzy numbers", Advances in fuzzy mathematics, ISSN 0973-533X Volume 12, Number 3.

Nagalakshmi T and Uthra G (2018). On a Solving Fuzzy Optimal Subdivision Problem using Fuzzy Dynamic Programming, International journal of mathematics trends and technology, IJMTT, 56244-251.

Rehmat A., Saeed H., and Cheema M.S. (2007), Fuzzy multi-objective linear programming approach, Pakistan Journal of Statistics Operation Research, 3(2), 87-98.

Sepideh F. (2003). Solving travelling salesman problem by using a fuzzy multi-objective linear programming, African Journal of Mathematics and Computer Science Research, 4 (11), 339-349.

Singhal Abha and Pandy P. (2016). Travelling salesman problems by dynamic programming algorithm, International Journal of Scientific Engineering and Applied Science, 2, 263-267.