Some Results on Strongly*-2 Divisor Cordial Labeling

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A strongly*-2 divisor cordial labeling of a graph G with the vertex set $V(G)$ is a bijection $f: V(G) \to \{1,2,3,\dots,|V(G)|\}$ such that each edge uv assigned the label 1 if $\lfloor \frac{f(u)+f(v)+f(u)f(v)}{2} \rfloor$ is odd and 0 if $\lfloor \frac{f(u)+f(v)+f(u)f(v)}{2} \rfloor$ is even, then the number of edges labeled with 0 and the number of edges labeled with 1 differs by atmost 1. A graph which admits a Strongly*-2 divisor cordial labeling is called a Srongly*-2 divisor cordial graph. In this paper, we proved that Path, Cycle, Star and comb graph are Strongly*-2 divisor cordial graphs.
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1 Introduction

Graph labeling was introduced in 1960's. We refer Harary [4] for looking over the fundamental terms and notations. We refer [3] and [5] for divisor cordial labeling. In [1] and [2], the strongly*-graphs are studied.

Stimulated by these, we introduce strongly*-2 divisor cordial labeling. In this paper we prove that the path, cycle, star and comb graph are strongly*-2 divisor cordial graphs. We give the basic definitions which are neccessary for our present work.

Definition 1.1 *A graph of $G = (V, E)$ is said to be a strongly*-graph if there exists a bijection $f: V \to \{1,2,\dots,n\}$ in such a way that when an edge, whose vertices are labeled i and j, is labeled with the value $i + j + ij$, all edge labels are dintinct.*

Definition 1.2 *A graph is called cordial if it is possible to label its vertices wih 0^s and 1^s so that when the edges are labeled with the difference of the labels at their end points, the number of vertices(edges) labeled with ones and zeros differs atmost by one.*

Definition 1.3 *Let P_n be a path. Attach a single pendent vertex to every vertex of the path. The resulting graph is a comb graph.*

2 Main Results

Theorem 2.1 *Any path P_n is a strongly*-2 divisor cordial graph.*

Proof. Let P_n be a path with $V(P_n) = \{u_i: 1 \le i \le n\}$ and $E(P_n) = \{u_i u_{i+1}: 1 \le i \le n-1\}$.

Then P_n has n vertices and $n-1$ edges. Define $f: V(P_n) \to \{1,2,\dots,n\}$ by $f(u_i) = i$, $1 \le i \le n$. Let f^* be the induced edge label of f. Then

$$f^*(u_i u_{i+1}) = \lfloor \frac{f(u_i)+f(u_{i+1})+f(u_i)f(u_{i+1})}{2} \rfloor = \lfloor \frac{i+i+1+i(i+1)}{2} \rfloor = \lfloor \frac{i^2+3i+1}{2} \rfloor.$$

Clearly, $\lfloor \frac{i^2+3i+1}{2} \rfloor$ is even if $i \equiv 0,1 (mod\ 4)$ and $\lfloor \frac{i^2+3i+1}{2} \rfloor$ is odd if $i \equiv 2,3 (mod\ 4)$. Therefore the edges $u_i u_{i+1}$, $1 \le i \le n-1$ can be assinged label 0 if $i \equiv 0,1 (mod\ 4)$ and 1 if $i \equiv 2,3 (mod\ 4)$. (1)

Let us find the number of edges with label 0 and 1.

Case 1. $n \equiv 1 (mod\ 4)$

Let $n = 4k+1$. Then there are $4k$ edges. Clearly there are k numbers which are congruent to i modulo 4, $(0 \le i \le 3)$. Hence by (1), there are $2k$ edges with label 0 and $2k$ edges with label 1 and so $|e_f(0) - e_f(1)| = 0$.

Case 2. $n \equiv 2 (mod\ 4)$

Let $n = 4k+2$. Then there are $4k+1$ edges. Clearly there are k numbers which are congruent to i modulo 4, $(i = 0,2,3)$ and $k+1$ numbers which is congruent to 1 modulo 4. Hence by (1), there are $2k+1$ edges with label 0 and $2k$ edges with label 1 and so $|e_f(0) - e_f(1)| = 1$.

Case 3. $n \equiv 3 (mod\ 4)$

Let $n = 4k+3$. Then there are $4k+2$ edges. Clearly there are $k+1$ numbers which are congruent to i modulo 4, $(i = 1,2)$ and k numbers which are congruent to i modulo 4, $(i = 0,3)$. Hence by (1), there are $2k+1$ edges with label 0 and $2k+1$ edges with label 1 and so $|e_f(0) - e_f(1)| = 0$.

Case 4. $n \equiv 0 (mod\ 4)$

Let $n = 4k$. Then there are $4k-1$ edges. Clearly there are k numbers which are congruent to i modulo 4, $(i = 1,2,3)$ and $k-1$ numbers which is cngruent to 0 modulo 4. Hence by (1), there are $2k-1$ edges with label 0 and $2k$ edges with label 1 and so $|e_f(0) - e_f(1)| = 1$.

Thus $|e_f(0) - e_f(1)| \le 1$ and hence any path P_n is a strongly*-2 divisor cordial graph.

Example 2.2 *A strongly**-2 divisor cordial labeling of the graph* P_{10} *is given in Figure 1.*

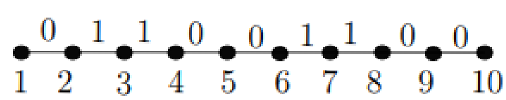

Figure 1.

Theorem 2.3 *Any cycle* C_n *is a strongly**-2 divisor cordial graph.*

Proof. Let $C_n: u_1 u_2 \dots u_n$ be the cycle with $V(C_n) = \{u_i : 1 \le i \le n\}$ and $E(C_n) = \{u_i u_{i+1}, u_1 u_n : 1 \le i \le n-1\}$.

Case 1. n is odd

Define $f: V(C_n) \to \{1,2,\dots,n\}$ by $f(u_i) = i$, $1 \le i \le n$. Let f^* be the induced edge label of f. Consider the edges $u_i u_{i+1}, 1 \le i \le n-1$. Now $f^*(u_i u_{i+1}) = \lfloor \frac{f(u_i)+f(u_{i+1})+f(u_i)f(u_{i+1})}{2} \rfloor = \lfloor \frac{i+i+1+i(i+1)}{2} \rfloor = \lfloor \frac{i^2+3i+1}{2} \rfloor$ which is even if $i \equiv 0,1 (mod\ 4)$ and odd $if\ \ i \equiv 2,3(mod\ 4)$. Therefore, the edges $u_i u_{i+1}, 1 \le i \le n-1$ can be assinged label 0 if $i \equiv 0,1(mod\ 4)$ and 1 if $i \equiv 2,3(mod\ 4)$ (1)

For the edge u_1u_n, $f^*(u_1u_n)= \lfloor\frac{1+n+n}{2}\rfloor = \lfloor\frac{2n+1}{2}\rfloor = n$ which is odd since n is odd and so the edge u_1u_n gets the label 1. (2)

Let us find the number of edges with label 0 and 1.

Subcase 1.1. $n \equiv 1(mod\ 4)$

Let $n = 4k+1$. Then there are $4k+1$ edges. Clearly there are k numbers which are congruent to i modulo 4, $(0 \le i \le 3)$. By (1), there are $2k$ edges with label 0 and $2k$ edges with label 1 and the edge u_1u_n with label 1. Hence there are $2k$ edges with label 0 and $2k+1$ edges with label 1 and so $|e_f(0) - e_f(1)| = 1$.

Subcase 1.2. $n \equiv 3(mod\ 4)$

Let $n = 4k+3$. Then there are $4k+3$ edges. Clearly there are $k+1$ numbers which are congruent to i modulo 4, $(i = 1,2)$ and k numbers which are cngruent to i modulo 4, $(i = 0,3)$. By (1), there are $2k+1$ edges with label 0 and $2k+1$ edges with label 1 and the edge u_1u_n wth label 1. Hence there are $2k+1$ edges with label 0 and $2k+2$ edges with label 1 and so $|e_f(0) - e_f(1)| = 1$.

Case 2. n is even

Subcase 2.1. $n \equiv 0(mod\ 4)$

Define $f: V(C_n) \to \{1,2,\dots,n\}$ by $f(u_i) = i$, $1 \le i \le n$. Let f^* be the induced edge label of f. As in case 1, $f^*(u_iu_{i+1})= \lfloor\frac{i^2+3i+1}{2}\rfloor$ which is even $if\ \ i \equiv 0,1(mod\ 4)$ and odd $if\ \ i \equiv 2,3(mod\ 4)$. For the edge u_1u_n, $f^*(u_1u_n)=\lfloor\frac{1+n+n}{2}\rfloor = \lfloor\frac{2n+1}{2}\rfloor= \lfloor\frac{2n+1}{2}\rfloor = n$ which is even and so the edge u_1u_n gets label 0.

Let us find the number of edges with label 0 and 1. Now $n = 4k$. Then there are $4k$ edges. Clearly there are k numbers which are congruent to i modulo 4, $(i = 1,2,3)$ and $k-1$ numbers which is cngruent to 0 modulo 4. By (1), there are $2k-1$ edges with label 0 and $2k$ edges with label 1 and the edge u_1u_n with label 0. Hence there are $2k$ edges with label 0 and $2k$ edges with label 1 and so $|e_f(0) - e_f(1)| = 0$.

Subcase 2.2. $n \equiv 2(mod\ 4)$

Define $f: V(C_n) \to \{1,2,\dots,n\}$ by

$f(u_i)$ = { 1 i = 1 i+1 if i $iseven$ i-1 if i $isoddand$ $i \neq 1$.

Consider the edges u_iu_{i+1}, $1 \le i \le n-1$.

For $i = 1$, $f^*(u_iu_{i+1}) = f^*(u_1u_2) = \lfloor\frac{f(u_1)+f(u_2)+f(u_1)f(u_2)}{2}\rfloor = \lfloor\frac{1+3+1(3)}{2}\rfloor= \lfloor\frac{7}{2}\rfloor = 3$.

For odd i and $i \neq 1$, $f^*(u_iu_{i+1}) = \lfloor\frac{i-1+i+2+(i-1)(i+2)}{2}\rfloor = \lfloor\frac{i^2+3i-1}{2}\rfloor$ which is even $if\ \ i \equiv 3(mod\ 4)$ and odd $if\ \ i \equiv 1(mod\ 4)$.

For even i, $f^*(u_iu_{i+1})=\lfloor\frac{i+i+i(i+1)}{2}\rfloor = \lfloor\frac{i^2+3i+1}{2}\rfloor$ which is even $if\ \ i \equiv 0(mod\ 4)$ and odd $if\ \ i \equiv 2(mod\ 4)$. Therefore the edges u_iu_{i+1}, $1 \le i \le n-1$ can be assinged label 0 if $i \equiv 0,3(mod\ 4)$ and 1 if $i \equiv 1,2(mod\ 4)$. (3)

For the edge u_1u_n, $f^*(u_1u_n)=\lfloor\frac{1+n+n}{2}\rfloor = \lfloor\frac{2n+1}{2}\rfloor= n$ which is even and so the edge u_1u_n gets label 0. Let us find the number of edges with label 0 and 1. Here $n = 4k+2$. Then there are $4k+2$ edges. Clearly there are k numbers which are congruent to i modulo 4, $(i = 0,2,3)$ and $k+1$ numbers which is congruent to 1 modulo 4. By (3), there are $2k$ edges with label 0 and $2k+1$ edges with label 1 and the edge u_1u_n label with 0. Hence there are $2k+1$ edges with label 0 and $2k+1$ edges with label 1 and so $|e_f(0) - e_f(1)| = 0$.

Thus $|e_f(0) - e_f(1)| \leq 1$ and hence any cycle C_n is a strongly*-2 divisor cordial graph.

Example 2.4 *A strongly*-2 divisor cordial labeling of the graph C_{10} is given in Figure 2.*

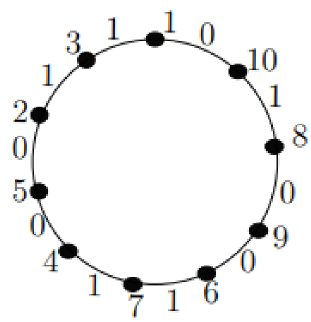

Figure 2.

Theorem 2.5 *Any star $K_{1,n}$ is a strongly*-2 divisor cordial graph.*

Proof. Let $V(K_{1,n}) = \{u, v_i : 1 \leq i \leq n\}$ and $E(K_{1,n}) = \{uv_i : 1 \leq i \leq n\}$. Then $|V(K_{1,n})| = n+1$ and $|E(K_{1,n})| = n$. Define $f: V(K_{1,n}) \to \{1,2,\ldots,n+1\}$ by $f(u) = 1$ and $f(v_i) = i+1$, $1 \leq i \leq n$. Let f^* be the induced edge label of f. Then $f^*(uv_i) = \lfloor\frac{1+i+1+i+1}{2}\rfloor = \lfloor\frac{2i+3}{2}\rfloor = i+1$ which is odd if i is even and even if i is odd. (1)

Case 1. n is even

Let $n = 2k$. Then there are $2k$ edges. Clearly there are k odd and k even numbers. Hence by (1), k edges with label 1 and k edges with label 0 and so $|e_f(0) - e_f(1)| = 0$.

Case 2. n is odd

Let $n = 2k+1$. Then there are $2k+1$ edges. Then there are k even numbers and $k+1$ odd numbers. Hence by (1), there are k edges with label 1 and $k+1$ edges with label 0 and so $|e_f(0) - e_f(1)| = 1$.

Thus $|e_f(0) - e_f(1)| \leq 1$ and hence any star $K_{1,n}$ is a strongly*-2 divisor cordial graph.

Example 2.6 *A strongly*-2 divisor cordial labeling of the graph $K_{1,6}$ is shown in Figure 3.*

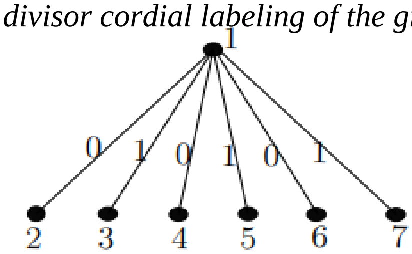

Figure 3.

Theorem 2.7 *The comb* $P_n \odot K_1$ *is a strongly*-2 divisor cordial graph.*

Proof. Let $V(P_n \odot K_1) = \{u_i, v_i: 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{u_i u_{i+1}, u_j v_j: 1 \leq i \leq n-1, 1 \leq j \leq n\}$. Then the graph $P_n \odot K_1$ has $2n$ vertices and $2n-1$ edges.

Case 1. n is odd

Define $f: V(P_n \odot K_1) \to \{1,2,3,\dots,2n\}$ by $f(u_i) = i$, $1 \leq i \leq n$ and $f(v_i) = n+i$, $1 \leq i \leq n$. Let f^* be the induced edge label of f. Consider the edges $u_i u_{i+1}$, $1 \leq i \leq n-1$.

Now $f^*(u_i u_{i+1})=\lfloor\frac{i+i+1+i(i+1)}{2}\rfloor = \lfloor\frac{i^2+3i+1}{2}\rfloor$ which is even if $i \equiv 0,1(mod\ 4)$ and odd if $i \equiv 2,3(mod\ 4)$. Therefore, the edges $u_i u_{i+1}, 1 \leq i \leq n-1$ can be assinged label 0 if $i \equiv 0,1(mod\ 4)$ and 1 if $i \equiv 2,3(mod\ 4)$ (1)

Subcase 1.1. $n \equiv 1(mod\ 4)$

In this case, $n = 4k+1$. For the edges $u_j v_j, 1 \leq j \leq n$,

$f^*(u_j v_j) = \lfloor\frac{j+n+j+j(n+j)}{2}\rfloor = \lfloor\frac{(n+2)j+n+j^2}{2}\rfloor = \lfloor\frac{(4k+3)j+4k+1+j^2}{2}\rfloor = \lfloor\frac{4kj+3j+4k+1+j^2}{2}\rfloor = 2k(1+j) + \lfloor\frac{j^2+3j+1}{2}\rfloor$.

For $j \equiv 0,1(mod\ 4)$, $\lfloor\frac{j^2+3j+1}{2}\rfloor$ is even and so $f^*(u_j v_j)$ is even and for $j \equiv 2,3(mod\ 4)$, $\lfloor\frac{j^2+3j+1}{2}\rfloor$ is odd and so $f^*(u_j v_j)$ is odd. Therefore, for $1 \leq j \leq n$, the edges $u_j v_j$ can be assinged label 0 if $j \equiv 0,1(mod\ 4)$ and 1 if $j \equiv 2,3(mod\ 4)$. (2)

Let us find the number of edges with label 0 and 1. Now $n = 4k+1$ implies that there are $8k+1$ edges. There are $n-1 = 4k$ edges of the form $u_i u_{i+1}$, $1 \leq i \leq n-1$ and so there are k numbers which are congruent to i modulo 4, $(0 \leq i \leq 3)$. By (1), there are $2k$ edges with label 0 and $2k$ edges with label 1. Also there are $n = 4k+1$ edges of the form $u_j v_j, 1 \leq j \leq n$ and so there are k numbers which are congruent to j modulo 4, $(j = 0,2,3)$ and $k+1$ numbers congruent to 1 modulo 4. By (2), there are $2k+1$ edges with label 0 and $2k$ edges with label 1. Hence there are $4k+1$ edges with label 0 and $4k$ edges with label 1 and so $\left|e_f(0) - e_f(1)\right| = 1$.

Subcase 1.2. $n \equiv 3(mod\ 4)$

Here $n = 4k+3$. For the edges $u_j v_j, 1 \leq j \leq n$, $f^*(u_j v_j) = \lfloor\frac{j+n+j+j(n+j)}{2}\rfloor = \lfloor\frac{(n+2)j+n+j^2}{2}\rfloor = \lfloor\frac{(4k+5)j+4k+3+j^2}{2}\rfloor = \lfloor\frac{4kj+3j+4k+3+j^2}{2}\rfloor = 2k(1+j) + \lfloor\frac{j^2+5j+3}{2}\rfloor$.

For $j \equiv 1,2(mod\ 4)$, $\lfloor\frac{j^2+5j+3}{2}\rfloor$ is even and so $f^*(u_j v_j)$ is even and for $j \equiv 0,3(mod\ 4)$, $\lfloor\frac{j^2+5j+3}{2}\rfloor$ is odd and so $f^*(u_j v_j)$ is odd. Threrfore, for $1 \leq j \leq n$, the edges $u_j v_j$ can be assinged label 0 if $j \equiv 1,2(mod\ 4)$ and 1 if $j \equiv 0,3(mod\ 4)$. (3)

Let us find the number of edges with label 0 and 1. Now $n = 4k+3$ implies that there are $8k+5$ edges. Then there are $n-1 = 4k+2$ edges of the form $u_i u_{i+1}$, $1 \leq i \leq n-1$ and so there are $k+1$ numbers which are congruent to i modulo 4, $(i = 1,2)$ and k numbers which are congruent to i modulo 4, $(i = 0,3)$. By (1), there are $2k+1$ edges with label 0 and $2k+1$ edges with label 1. Also there are $n = 4k+3$ edges of the form $u_j v_j, 1 \leq j \leq n$. Then there are $k+1$ numbers which are congruent to j modulo 4, $(j = 1,2,3)$ and k numbers congruent to 0 modulo 4. By (3), there are $2k+2$ edges with label 0 and $2k+1$ edges with label 1. Hence there are $4k+3$ edges with label 0 and $4k+2$ edges with label 1 and so

$\left|e_f(0) - e_f(1)\right| = 1$.

Case 2: n is even

Define $f: V(P_n \odot K_1) \to \{1,2,3,\ldots,2n\}$ by $f(u_i) = i$, $1 \le i \le n$ and $f(v_i) = 2n + 1 - i$, $1 \le i \le n$. Let f^* be the induced edge label of f. As in case 1, $f^*(u_i u_{i+1}) = \lfloor\frac{i^2+3i+1}{2}\rfloor$ which is even $if\ i \equiv 0,1(mod\ 4)$ and odd $if\ i \equiv 2,3(mod\ 4), 1 \le i \le n-1$. (4)

Subcase 2.1. $n \equiv 0(mod\ 4)$

In this case, $n = 4k$. For the edges $u_j v_j, 1 \le j \le n$,

$f^*(u_j v_j) = \lfloor\frac{j+2n+1-j+j(2n+1-j)}{2}\rfloor = \lfloor\frac{2n+1+2nj+j-j^2}{2}\rfloor = \lfloor\frac{2(4k)+1+2(4k)j+j-j^2}{2}\rfloor = \lfloor\frac{8k+8kj+1+j-j^2}{2}\rfloor = 4k(1+j) + \lfloor\frac{1+j-j^2}{2}\rfloor$.

For $j \equiv 2,3(mod\ 4)$, $\lfloor\frac{1+j-j^2}{2}\rfloor$ is odd and so $f^*(u_j v_j)$ is odd and for $j \equiv 0,1(mod\ 4)$, $\lfloor\frac{1+j-j^2}{2}\rfloor$ is even and so $f^*(u_j v_j)$ is even. Threrfore, for $1 \le j \le n$, the edges $u_j v_j$ can be assinged label 1 if $j \equiv 2,3(mod\ 4)$ and 0 if $j \equiv 0,1(mod\ 4)$. (5)

Let us find the number of edges with label 0 and 1. Now $n = 4k$ implies that there are $8k-1$ edges. Then there are $n-1 = 4k-1$ edges are of the form $u_i u_{i+1}$, $1 \le i \le n-1$ and so there are k numbers which are congruent to i modulo 4, $(i = 1,2,3)$ and $k-1$ numbers which are congruent to 0 modulo 4. By (4), there are $2k-1$ edges with label 0 and $2k$ edges with label 1. Also there are $n = 4k$ edges of the form $u_j v_j, 1 \le j \le n$ and so there are k numbers which are congruent to j modulo 4, $(0 \le i \le 3)$. By (5), there are $2k$ edges with label 1 and $2k$ edges with label 0. Hence there are $4k-1$ edges with label 0 and $4k$ edges with label 1 and so $\left|e_f(0) - e_f(1)\right| = 1$.

Subcase 2.2. $n \equiv 2(mod\ 4)$

In this case, $n = 4k+2$. For the edges $u_j v_j, 1 \le j \le n$, $f^*(u_j v_j) = \lfloor\frac{j+2n+1-j+j(2n+1-j)}{2}\rfloor = \lfloor\frac{2n+1+2nj+j-j^2}{2}\rfloor = \lfloor\frac{2(4k+2)+1+2(4k+2)j+j-j^2}{2}\rfloor =$

$\lfloor\frac{8k+8kj+4+4j+1+j-j^2}{2}\rfloor = \lfloor\frac{4(1+j)(2k+1)+1+j-j^2}{2}\rfloor = 2(1+j)(2k+1) + \lfloor\frac{1+j-j^2}{2}\rfloor$.

For $j \equiv 2,3(mod\ 4)$, $\lfloor\frac{1+j-j^2}{2}\rfloor$ is odd and so $f^*(u_j v_j)$ is odd and for $j \equiv 0,1(mod\ 4)$, $\lfloor\frac{1+j-j^2}{2}\rfloor$ is even and so $f^*(u_j v_j)$ is even. Threrfore, for $1 \le j \le n$, the edges $u_j v_j$ can be assinged label 1 if $j \equiv 2,3(mod\ 4)$ and 0 if $j \equiv 0,1(mod\ 4)$. (6)

Let us find the number of edges with label 0 and 1. Now $n = 4k+2$ implies that there are $8k+3$ edges. Then there are $n-1 = 4k+1$ edges are of the form $u_i u_{i+1}$, $1 \le i \le n-1$ and so there are k numbers which are congruent to i modulo 4, $(i = 0,2,3)$ and $k+1$ numbers which are congruent to 1 modulo 4. By (4), there are $2k+1$ edges with label 0 and $2k$ edges with label 1. Also there are $n = 4k+2$ edges of the form $u_j v_j, 1 \le j \le n$, then there are k numbers which are congruent to j modulo 4, $(i = 0,3)$ and $k+1$ numbers are congruent to j modulo 4, $(i = 1,2)$. By (6), there are $2k+1$ edges with label 1 and $2k+1$ edges with label 0. Hence there are $4k+2$ edges with label 0 and $4k+1$ edges with label 1 and so $\left|e_f(0) - e_f(1)\right| = 1$.

Thus $\left|e_f(0) - e_f(1)\right| \le 1$ and hence the comb $P_n \odot K_1$ is a strongly*-2 divisor cordial graph.

Example 2.8 *A strongly*-2 divisor cordial labeling of the graph* $P_6 \odot K_1$ *is given in Figure 4*

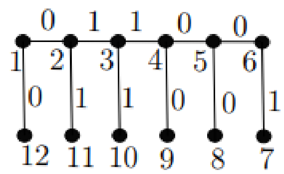

Figure 4.

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