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[j,k]- Set Domination in Fuzzy Graphs

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Abstract

Domination is a theoretical concept in graph theory. In this paper, [j, k]- set domination in Fuzzy Graphs is introduced. We study some important properties and derive some results. By [J, K]-set domination, a subset D in a Fuzzy graph G = (V, E) is a [j,k]-set, if every Fuzzy vertex v that is not in the dominating set is adjacent to at least j but not more than k vertices in D. We consider the vertices j and k as small positive integers. The Domination number is denoted by $\gamma_{[j,k]}(G)$, which is the minimum cardinality of a dominating set.

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1. Introduction

Introduction of Graph Theory was introduced by D.B West [17]. Zadeh. L. A introduced the concept of fuzzy relations [18] in his paper. Rosenfield. A [14] introduced the notion of Fuzzy graphs and several Fuzzy analogs. Somasundaram. A and Somasundaram. S[16] discussed domination in Fuzzy graphs using effective edges. Domination in Fuzzy graphs using strong arcs was discussed by Nagoorgani and Chandrasekaran [11]. Fundamentals of domination have been studied by Haynes. T.W et al [5]. The Basic idea of [1,2]-domination was studied by N.Murugasan et al.[8] Musthapha Chellalia et al.[9] are the authors who have given clearer definitions and properties of [1,2]-sets. Domination in Fuzzy graphs was defined by S. Ramya, S. Lavanya, and D. Jayanthi Prasanna [13,4]. A New approach to the domination of Fuzzy graphs was given by S. Ismail Mohideen et al. 3]. Domination in Fuzzy directed graphs was founded by Enrico Enriquez et al. [2]. Application of Fuzzy graphs was concentrated by A.Muneera and Dr R.V.N. Srinivasa Rao [7]. Strong domination in Fuzzy graphs was discovered and applied by O. J. Manjula and M. S. Sunitha [6].

2- Domination in Fuzzy graphs was studied by A.Nagoorgani et al. [12]-Vertex domination in Fuzzy line graphs has been implemented by N. Sarala and T. Kavitha [15]. Fuzzy converting problems of Fuzzy graphs and their applications in the Indian economy are founded by Anushree et al. [1].

In this paper, we have generalised the [j, k] - set domination number of a Fuzzy Graph.

2. Preliminary:

Definition 2.1: Consider the graph G = (V, E). A subset D of a vertex set V is a dominating set of G if, for every vertex$u \in V\setminus D$there exists a vertex v $\in D$ such that uv is an edge of G. The domination number$\gamma\ (G)$of G is the smallest cardinality of a dominating set D of G.

Definition 2.2: A Fuzzy Graph G = (σ, μ) is a set with two functions σ : V → [0, 1] and μ: E → [0,1] such that $\mu(\{u,v\}) \leq\ \sigma(u) \wedge \sigma(v)\ for\ all\ u,v \in V.$

Definition 2.3: if G = (σ, μ) is a Fuzzy Graph on$V\ and u,v \in V, then\ u\ dominates\ v\ in G$

$$if \mu\ (u,v) =\ \sigma(u) \wedge \sigma(v)$$

Definition 2.4: The underlying crisp graph of G = (σ, μ) is denoted by$G^* = (\sigma^*. \mu^*)$ where

$$\sigma^* = \{ u\ \in V\ :\ \sigma(u) > 0\} and$$

$$\mu^* = \{u,v\ ) \in V\ \times V:\ \mu\ (u,v) > 0\}$$

Definition 2.5: The order p and size q of the fuzzy graph G = (σ, μ) are defined by

$$p = \sum_{v \in V} \sigma\ (v)\ \ and\ \ q = \sum_{u,v \in E} \mu(u,v)$$

Definition 2.6: A Fuzzy Graph G is said to be a strong Fuzzy Graph ifμ^* is a strong neighbour and a complete Fuzzy Graph if$\mu\ (u,v) =\ \sigma(u) \wedge \sigma\ (v) \forall\ u,v\ in \sigma^*$

Definition 2.7: The [j, k]-dominating set D of a Fuzzy Graph G = (V, E) is defined as if for every node $v \in V \setminus D$ satisfies $j \leq |N(v) \cap D| \leq k$.

3. [1, 2] Set domination in Fuzzy Generalised Petersen Graph

Theorem 3.1: The number of [1, 2] dominating sets of the generalised Petersen graph is

$$|D_{[1,2]}[F(P(n,2))]| = \begin{cases} k+2 \\ when K = 1,2,3\cdots \end{cases}$$

Proof: Let $v_1, v_2, v_3 \cdots, v_n$ and $u_1, u_2, u_3 \cdots u_n$ are the Fuzzy vertices of the Generalised Petersen Graph.

For each Fuzzy vertex $u, v \in V$ if $\mu(u,v) = \sigma(u) \wedge \sigma(v) =>$ the Fuzzy vertex u dominates the vertex v.

In [1, 1]- dominating set of a Fuzzy Graph G for every vertex $v \in V - D$, there exists $u \in D$ such that u dominates v. In the generalised Petersen graph, every vertex v_i in $V - D$. We have the adjacent vertices u_{i-1} and $u_{i+1}, i = 2,3 \ldots$ such that u_i in D dominaes the Fuzzy vertices v_{i-1} and v_{i+1} in $V - D$.

In [1, 2]-dominating set for each Fuzzy vertex v_i in $V - D$, we have two adjacent vertices u_{i-1} and u_{i+1}, such that u_{i-1} and u_{i+1} dominate the vertex v_i in $V - D$, $i = 2,3\cdots$

The Fuzzy dominating vertices are given below.

$$u_1, u_3, u_4, u_5 \cdots u_{n-1}$$

$$u_2, u_4, u_5, u_6 \cdots u_n$$

$$u_3, u_5, u_6, u_7 \cdots u_{n+1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$u_n, u_2, u_3, \cdots u_{n-2}$$

Therefore, the Generalised [1, 2]-Fuzzy dominating set of the Generalised Petersen Graph is

$$D_{[1,2]}[F(P(n,2)] = \begin{cases} u_{2k1+s-2}, \quad u_{k2+s+1} \quad if\, 2k1+s-2 \leq n \; if\, k2+s+1 \leq n \; where\, k1 = 1,2 \\ k2 = 1,2,3\cdots \\ 2k1+s-2 = pn+q, \\ if\, 2k1+s-2 > n k2+s+1 = pn+q, \\ if\, k2+s+1 > n \\ for any fixer integer p \\ k1 = 1,2,\cdots \\ k2 = 1,2,3\cdots \\ where s = 1,2,3\cdots \end{cases}$$

$$Hence |D_{[1,2]}[F(P(n,2)] = k+2$$

$$and\; \gamma_{[1,2]}\, F[P(n,2)] = k+2$$

Theorem 3.2: If $1 \leq |N(v) \wedge D| \leq 2$, then the Fuzzy vertex v belongs to every [1,2]- dominating set.

Proof: Let G be a fuzzy graph, so the graph must satisfy the condition $\mu(n,v) \leq \sigma(u) \wedge \sigma(v)$, i.e., the length of u and v must be the min {σ (u), σ(v)}. Suppose if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for a vertex $u \in V$ then u has a strong neighbour v in $V - D$. i.e., (u,v) is the strong arc.

For [1, 1]-domination, we could find only one strong neighbour for [1, 2]- domination.

We could find two strong neighbours v_{i-1} and v_{i+1}, that is, for each $v \in V - D$ we can find u in D such that N (u) = v_{i-1} and N (u) =v_{i+1}.

Suppose let us assume that u has no strong neighbour in D. Then we have $N_s(u) = \Phi$. So, the Fuzzy vertex v is not dominated by u_{i-1} and u_{i+1}. So you belong to either [1, 1]-dominating set or [1, 2] - dominating set.

Theorem 3.3: [1, 2]-A dominating set of a Fuzzy Graph G is a dominating set.

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Proof: Let $v_1, v_2, v_3 \cdots v_n$ be vertices of the fuzzy graph. $\sigma(v_1), \sigma(v_2) \cdots \sigma(v_n)$ are the weights of the fuzzy vertices.

$\mu(u_1, v_1), \mu(u_1, v_2) \cdots\cdots \mu(u_{n-1}, v_n)$ are the weights of the lengths $v_1v_2, v_2v_3 \cdots v_{n-1}v_n$ for the Fuzzy Graph $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for each fuzzy vertices

$v \in$ V-D is adjacent to u in D, and when $\mu(u,v) = \sigma(u) \wedge \sigma(v)$, that is, for strong domination the arc $(u,v), u\ strongly\ dominates\ v.$

Suppose every vertex $v\ in\ V - D$ is adjacent to u in D. The Fuzzy vertex u dominates the Fuzzy vertex v so it is called a [1, 1]-dominating set.

If the vertex $v_i \in V - D$ is adjacent to $u_{i-1} and u_{i+1}$ where i = 2, 3, 4.... Therefore v is dominated by $u_{i-1} and u_{i+1}$ it comes under [1, 2]-domination.

Remark 1: Strong domination is symmetric.

Remark 2: $\sigma(u) = 1 \Rightarrow u\ is called the strongest vertex for u \in V$

Remark 3: $\sigma(u) \in (0.7,1) \Rightarrow u\ is called the first grade vertex for u \in V$

Remark 4: $\sigma(u) \in (0.5,0.7) \Rightarrow u\ is called the second grade vertex for u \in V$

Remark 5: $\sigma(u) \in (0,0.5) \Rightarrow u\ is called the third grade vertex for u \in V$

Application: To handle real–life uncertainties, fuzzy mathematics is the best of all fields of mathematics. The Fuzzy graph is one of the strong tools to model the relationship between various features with imprecision. Fuzzy logic has been used in numerous applications such as facial pattern recognition, air conditioners, washing machines, vacuum cleaners, antiskid braking systems, transmission systems, control of subway systems, and unmanned helicopters.

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Conclusion:

In this paper, we have generalised the [1, 1] and [1, 2]-dominating number of a fuzzy graph and its application. Similarly, we can study [j, k]-domination number of special graphs.

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