MODELLING THE ELASTOPLASTIC DEFORMATION OF AN INTERNALLY PRESSURIZED TRANSVERSELY ISOTROPIC SHELL UNDER A TEMPERATURE GRADIENT

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Abstract

In this study, the transition theory and generalized strain measure theory were used to model the elastoplastic deformation in a transversely isotropic spherical shell subjected to a thermal gradient and uniform pressure. The main difference in transition's theory from the classical plasticity theory is neglecting ad-hoc assumptions such as the deformation is infinitesimally small, material incompressibility and yield criterion. However, results obtained by transition theory satisfy the yield condition in plastic state, and it is important to determine elastoplastic and fully plastic stresses on the basis of Lebesgue measure. Results are obtained in non-dimensional quantities and are shown graphically and illustrated numerically. The validations of analytical results of particular cases were compared to other works published in the literature and found to be the same. It is concluded that the pressure required for the initial yielding of the thicker shell made of transversely isotropic material is lower than the pressure required for thinner shell at room temperature. Through adding thermal effects, the pressure required for initial yielding of a transversely isotropic shell decreases. The value of fully plastic circumferential stress at the outer surface of the shell decreases with the increase in temperature and pressure. It is seen that the stress distribution through the shell surface, induced by temperature and separately induced by pressure, is opposite.

1. Introduction

A shell is a cylindrical or spherical curved surface which is much smaller in thickness than its other dimensions. The geometric properties of shells, i.e. single or double curvature give such lightweight structures a tremendous advantage. Shell structures find wide applications in many branches of engineering. Examples include aircraft, spacecraft, nuclear reactors, tanks for liquid and gas storage, and pressure vessels. Researchers applying advanced materials in order to reduce the weight, thickness and cost of the structures. This is particularly important for aircraft and launch vehicles. Researchers have also discovered various uses of spherical shell structures in aircraft, industrial, chemical and mechanical ventures, such as rapid centrifugal separators, gas turbines for high control flying machine motors, turning satellite structures, other rotor systems, and pivoting magnetic shields. Many material properties, such as stiffness, strength, thermal expansion, thermal conductivity, and permeability, are used to compose a spherical shell associated with a direction or axis. If the material's properties along any direction are the same as those along a symmetric direction with respect to a plane, then that plane is defined as a plane of material symmetry. An isotropic material comprises an infinite number of

planes of material symmetry, while a transversely isotropic material has three mutually perpendicular planes of material symmetry and one of its principal planes is an isotropic plane [1, 2, 3, 4]. Researchers dedicated to the study of spherical shell based on classical and non-classical treatment to figure out the optimal design of structural components and they led not to confine themselves to the usual elastic regime, but the elastoplastic regime due to the increasing material scarcity and higher costs of the material. Elastoplastic analysis of spherical shells made from transversely isotropic or isotropic materials is critical in solid mechanics. It has attracted a great amount of scientific interest.
Stresses and strains analysis in spherical shell has been extensively performed in classical treatment. Reuss [5] originally gave the solutions corresponding to the plastic state for shells under pressure, which Hill [6] improved and interest in this issue has never stopped ever since. Reissner [7] studied the analysis of stress and displacements of isotropic spherical shells. Timoshinko and Goodier [4] evaluated the elastic stresses in a thick hollow sphere shell under internal pressure, and Hill [6] provided a comprehensive account of the same problem for work-hardening and non work-hardening materials in the elastic-plastic case. Johnson and Derrington [8] studied the onset of initial yield under pressure and uniform heat flow in a spherical shell. All of the researchers mentioned above used the method of superposition and semi-empirical laws to investigate these types of problems. Starting in 1962 [9], researchers used Seth's transition theory and generalized strain measure theory (non-classical treatment) which does not use the superposition method and semi-empirical laws. The generalized principal strain measure has been defined [10] as

$$e_{ii} = \int_0^{e_{ii}^A} \left(1-2e_{ii}^A\right)^{\frac{n}{2}-1} de_{ii}^A = \frac{1}{n}\left(1-\left(1-2e_{ii}^A\right)^{\frac{n}{2}}\right) \qquad (1)$$

where n is strain measure coefficient, e_{ii}^A is Almansi finite strain component and $i=1,2,3$. It gives $n=-2,-1,0,1,2$ respectively to Green, Cauchy, Hencky, Swainger and Almansi measures.

This theory is applied to a more general and wider range of problems [7, 9, 11, 12, 13, 14, 15, 16]. Using his theory, Seth [17] studied elastic-plastic transition stresses and strains in tubes under pressure. Gupta and Dharmani [11] studied creep transition in transversely isotropic shells under uniform pressure. Pankaj *et al.* [18] investigated the problem in creep transition in the rotating spherical shell under the effect of density variable. Pathania and Verma [19] studied temperature and pressure dependent creep stress analysis of spherical shell. However, the studies using the non-classical treatment of spherical shell are very limited as compared to classical treatment, although the empirical assumptions are ignored and the nonlinear behavior of the material is taken into account, as shown in Figure 1 (b).

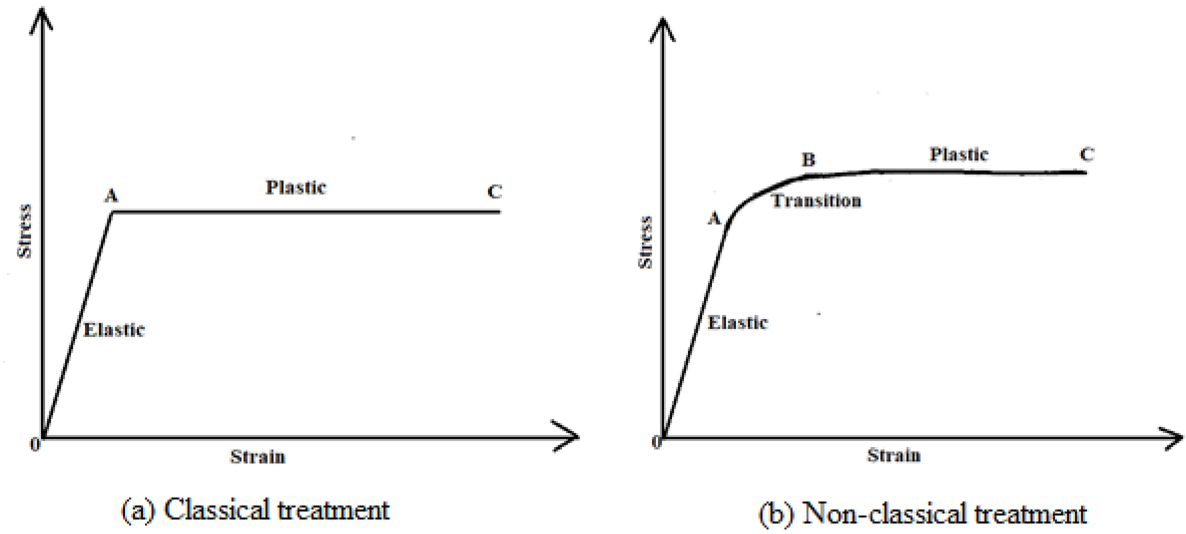


Figure 1: Stress-strain curves

In this paper, elastoplastic stress and fully plastic stress analysis are carried out on an isotropic or a transversely isotropic spherical shell subjected to a thermal gradient and uniform pressure using the transition theory of Seth and generalized strain measure theory. Results are obtained in non-dimensional quantities and are shown graphically and illustrated numerically.

2. Governing Equation

2.1 Mathematical Model

Consider an isotropic or a transversely isotropic spherical shell having internal and external radii a and b $(a < b)$ respectively, subjected to a thermal gradient φ_O and uniform pressure P on the inner surface $r = a$, the cross-section of the shell subjected to these loads is shown in Figure 2.

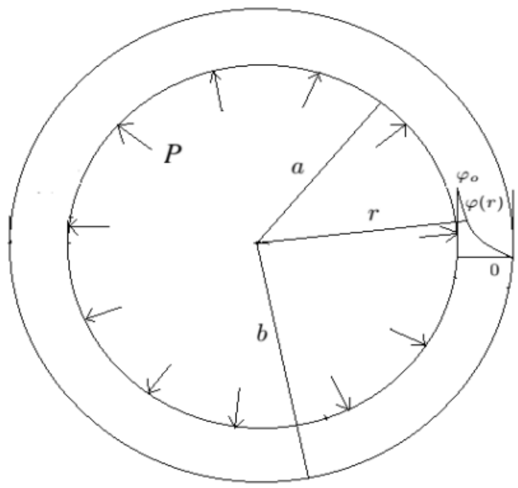


Figure 2: Geometry of spherical shell under a radial temperature gradient and uniform pressure.

2.2 Displacement Coordinates and Strain Measures

Due to the spherical symmetry of the structure, the displacements are purely radial and a function of r only. The components of these displacements in spherical coordinates are given by Seth [17]

$$u = (1-\eta)r \ , \ v = 0 \ , \ w = 0 \ , \qquad (2)$$

where $u, v,$ and w are the physical components of displacement, η is a function of $r=\sqrt{x^2+y^2+z^2}$ only.

For finite deformation, the Almansi strain components in spherical coordinates are:

$$e_{rr}^{A}=\frac{1}{2}\left[1-(\eta+r\eta')^2\right]$$
$$e_{\theta\theta}^{A}=\frac{1}{2}\left[1-\eta^2\right]=e_{\phi\phi}^{A} \qquad (3)$$
$$e_{\phi r}^{A}=e_{r\theta}^{A}=e_{\theta\phi}^{A}=0$$

where $e_{rr}^{A}, e_{\theta\theta}^{A}, e_{\phi\phi}^{A}, e_{r\theta}^{A}, e_{\theta\phi}^{A}, e_{\phi r}^{A}$ are the components of the strain tensor e_{ij}^{A}, the superscripts "A" is Almansi and $\eta'=\frac{d\eta}{dr}$.

Substituting equation (3) into equation (1), the generalized strain components are:

$$e_{rr}=\frac{\left[1-(\eta+\eta' r)^n\right]}{n}$$
$$e_{\theta\theta}=\frac{\left[1-\eta^n\right]}{n}=e_{\phi\phi} \qquad (4)$$
$$e_{\phi r}=e_{r\theta}=e_{\theta\phi}=0$$

2.3 Stress-Strain Relation for Transversely Isotropic Material

The thermo-elastic constitutive equations for transversely isotropic materials are given by [12]

$$\sigma_{11}=C_{11}e_{11}+(C_{11}-2C_{66})e_{22}+C_{13}e_{33}-\beta_1\varphi$$
$$\sigma_{22}=(C_{11}-2C_{66})e_{11}+C_{11}e_{22}+C_{13}e_{33}-\beta_1\varphi \quad (5)$$
$$\sigma_{33}=C_{13}e_{11}+C_{13}e_{22}+C_{33}e_{33}-\beta_2\varphi$$
$$\sigma_{23}=C_{44}(2e_{23}),\ \sigma_{13}=C_{44}(2e_{13}),\ \sigma_{12}=C_{66}(2e_{12})$$

where φ is the temperature change, C_{ij} are the elastic stiffness constants, β_1 and β_2 are the thermal moduli, σ_{ij} and e_{ij} denote respectively the stress and strain tensor.

The stress components can be calculated if the distribution of temperature is given. Consider the case of steady flow and we take the temperature

$$\varphi=\begin{cases}\varphi_o, & \text{for } r=a,\\ 0, & \text{for } r=b\end{cases}$$

where φ_o is constant, then the temperature at any distance r from the center is given by [4]

$$\varphi=\bar{\varphi}_o\left(\frac{b-r}{r}\right) \qquad (6)$$

where $\bar{\varphi}_o=\frac{\varphi_o a}{b-a}$

Substituting equation (4) in equation (5), we get

$$\sigma_{rr}=C_{33}\frac{\left[1-(\eta+r\eta')^n\right]}{n}+2C_{13}\frac{\left[1-\eta^n\right]}{n}-\beta_2\varphi$$

$$\sigma_{\theta\theta} = 2C_{13}\frac{\left[1-(\eta+r\eta')^n\right]}{n} + \left(2C_{11}-2C_{66}\right)\frac{\left[1-\eta^n\right]}{n} - \beta_1\varphi \qquad (7)$$
$$\sigma_{\phi\phi} = \frac{C_{13}}{n}\left[1-(\eta+r\eta')^n\right] + \frac{(2C_{11}-2C_{66})}{n}\left[1-\eta^n\right] - \beta_1\varphi$$
$$\sigma_{\phi r} = 0 = \sigma_{r\theta} = \sigma_{\theta\phi}$$

The equilibrium stress-equations are given by [3]

$$\frac{\partial(\sigma_{rr})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sigma_{r\phi})}{\partial\phi} + \frac{1}{r}\frac{\partial(\sigma_{r\theta})}{\partial\theta} + \frac{2\sigma_{rr}-\sigma_{\phi\phi}-\sigma_{\theta\theta}+\sigma_{r\theta}\cot\theta}{r} = 0$$
$$\frac{\partial(\sigma_{r\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sigma_{\theta\phi})}{\partial\phi} + \frac{1}{r}\frac{\partial(\sigma_{\theta\theta})}{\partial\theta} + \frac{3\sigma_{r\theta}+(\sigma_{\theta\theta}-\sigma_{\phi\phi})\cot\theta}{r} = 0 \qquad (8)$$
$$\frac{\partial(\sigma_{r\phi})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sigma_{\phi\phi})}{\partial\phi} + \frac{1}{r}\frac{\partial(\sigma_{\phi\theta})}{\partial\theta} + \frac{3\sigma_{r\phi}+2\sigma_{\theta\phi}\cot\theta}{r} = 0$$

Substituting the values of stresses from equation (7) in equation (8), we see that the equations of equilibrium are all satisfied except,

$$\frac{\partial(\sigma_{rr})}{\partial r} + \frac{2}{r}(\sigma_{rr}-\sigma_{\theta\theta}) = 0 \qquad (9)$$

Using equations (6) and (7) in equation (9), we obtain a non-linear differential equation governing transitions in η as:

$$\eta p(1+p)^{n-1}\frac{dp}{d\eta} = -p(1+p)^n + 2(\alpha_3-1)p + \frac{2\alpha_3}{n\eta^n}\left(1-\eta^n(1+p)^n\right) + \frac{2\alpha_3(\alpha_2-1)}{n\eta^n}\left(1-\eta^n\right)$$
$$+\frac{\beta_2\bar{\varphi}_o}{C_{33}\eta^n}\left(\frac{b}{r}\right) + \frac{2(\beta_1-\beta_2)\bar{\varphi}_o}{C_{33}\eta^n}\left(\frac{b}{r}-1\right) \qquad (10)$$

where $r\eta' = p\eta$ and $\alpha_3 = 1-\frac{C_{13}}{C_{33}}$. In equation (10) the transitional points of η are $p\to-1$ and $p\to\pm\infty$

3. Boundary Conditions

The problem's boundary conditions are set by:

$$\sigma_{rr}|_{r=a} = -P \qquad (11)$$
$$\sigma_{rr}|_{r=b} = 0 \qquad (12)$$

where P is a pressure on the inner surface of the spherical shell.

4. Analytical Solution

To evaluate the plastic stresses, the transition function Ψ at the transition point $p\to\pm\infty$ [11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23] is taken through the principal stresses. We define the transition function Ψ as

$$\Psi = \eta^n\left[(1+p)^n + 2(1-\alpha_3)\right] \qquad (13)$$

Then,

$$\frac{d\ln\Psi}{d\ln r}=\frac{np\left[\left(1+p\right)^{n}+2(1-\alpha_3)\right]+np\eta\left(1+p\right)^{n-1}\frac{dp}{d\eta}}{\left(1+p\right)^{n}+2(1-\alpha_3)} \tag{14}$$

From equation (10) substitute the value of $\frac{dp}{d\eta}$ in equation (14), we get

$$\frac{d\ln\Psi}{d\ln r}=\frac{2\alpha_3\left[\frac{1}{\eta^n}-\left(1+p\right)^{n}\right]+\frac{2\alpha_3(1-\alpha_2)}{\eta^n}(\eta^n-1)+\frac{n\beta_2\bar{\varphi}_o}{C_{33}\eta^n}\left(\frac{b}{r}\right)+\frac{2n(\beta_1-\beta_2)\bar{\varphi}_o}{C_{33}\eta^n}\left(\frac{b}{r}-1\right)}{\left(1+p\right)^{n}+2(1-\alpha_3)} \tag{15}$$

where $\frac{C_{11}+C_{12}-2C_{13}}{C_{33}}=\alpha_3(1-\alpha_2)$.

By taking asymptotic value of equation (15) as $p\to\pm\infty$, we get

$$\frac{d\ln\Psi}{d\ln r}=-2\alpha_3 \tag{16}$$

Then integration of equation (16) yields,

$\Psi=Ar^{-2\alpha_3}$, where A is an integration constant. (17)

Using equation (17) in equation (13), we get

$$\sigma_{rr}=\frac{C_{33}}{n}\left[3-2\alpha_3-Ar^{-2\alpha_3}\right]-\beta_2\bar{\varphi}_o\left(\frac{b}{r}-1\right) \tag{18}$$

The relationship between yielding stress in tension and material constants at the transition range [13] is given by

$$Y=\frac{C_{33}\alpha_3(3-\alpha_2-2\alpha_3)}{n(2-\alpha_3-\alpha_2\alpha_3)} \tag{19}$$

where Y is the yield stress in tension. Using equation (19) in equation (18), we get

$$\sigma_{rr}=\frac{Y(2-\alpha_3-\alpha_2\alpha_3)}{(3-\alpha_2-2\alpha_3)\alpha_3}\left[3-2\alpha_3-Ar^{-2\alpha_3}\right]-\beta_2\bar{\varphi}_o\left(\frac{b}{r}-1\right) \tag{20}$$

Using the boundary condition (12) in equation (20), one gets

$$A=(3-2\alpha_3)b^{2\alpha_3}$$

Thus by substituting the value of the constant A, equation (20) becomes

$$\sigma_{rr}=\frac{Y(2-\alpha_3-\alpha_2\alpha_3)(3-2\alpha_3)}{(3-\alpha_2-2\alpha_3)\alpha_3}\left[1-\left(\frac{b}{r}\right)^{2\alpha_3}\right]-\beta_2\bar{\varphi}_o\left(\frac{b}{r}-1\right) \tag{21}$$

Using the boundary condition (11) in equation (21), we get

$$P=\frac{Y(2-\alpha_3-\alpha_2\alpha_3)(3-2\alpha_3)}{(3-\alpha_2-2\alpha_3)\alpha_3}\left[\left(\frac{b}{a}\right)^{2\alpha_3}-1\right]+\beta_2\bar{\varphi}_o\left(\frac{b}{a}-1\right) \tag{22}$$

Substituting equation (21) in equation (9), we get

$$\sigma_{\theta\theta}=\sigma_{\phi\phi}=\sigma_{rr}+\frac{Y(2-\alpha_3-\alpha_2\alpha_3)(3-2\alpha_3)}{(3-\alpha_2-2\alpha_3)}\left(\frac{b}{r}\right)^{2\alpha_3}+\frac{\beta_2\bar{\varphi}_o}{2}\left(\frac{b}{r}\right) \tag{23}$$

4.1 Initial Yielding

We note that the value of $\left|\sigma_{\theta\theta}-\sigma_{rr}\right|$ is maximum at $r=a$. Hence yielding of the shell will take place at the inner surface. Thus,

$$\left|\sigma_{rr}-\sigma_{\theta\theta}\right|_{r=a}=\frac{Y(2-\alpha_3-\alpha_2\alpha_3)(3-2\alpha_3)}{(3-\alpha_2-2\alpha_3)}\left(\frac{b}{a}\right)^{2\alpha_3}+\frac{\beta_2\bar{\varphi}_o}{2}\left(\frac{b}{a}\right)\cong Y_1(say) \qquad (24)$$

Substituting the value of Y in terms of Y_1 in equations (21), (22) and (23), one gets the transitional stress as

$$\sigma_{rr}=\frac{1}{\alpha_3\left(\frac{b}{a}\right)^{2\alpha_3}}\left[1-\left(\frac{b}{r}\right)^{2\alpha_3}\right]\left[Y_1-\frac{\beta_2\bar{\varphi}_o}{2}\left(\frac{b}{a}\right)\right]-\beta_2\bar{\varphi}_o\left(\frac{b}{r}-1\right) \qquad (25)$$

$$\sigma_{\theta\theta}=\sigma_{rr}+\frac{1}{\left(\frac{b}{a}\right)^{2\alpha_3}}\left(\frac{b}{r}\right)^{2\alpha_3}\left[Y_1-\frac{\beta_2\bar{\varphi}_o}{2}\left(\frac{b}{a}\right)\right]+\frac{\beta_2\bar{\varphi}_o}{2}\left(\frac{b}{r}\right) \qquad (26)$$

and a relation between pressure and temperature for initial yielding at the internal surface as

$$P=\frac{1}{\alpha_3\left(\frac{b}{a}\right)^{2\alpha_3}}\left[\left(\frac{b}{a}\right)^{2\alpha_3}-1\right]\left[Y_1-\frac{\beta_2\bar{\varphi}_o}{2}\left(\frac{b}{a}\right)\right]-\beta_2\bar{\varphi}_o\left(\frac{b}{a}-1\right) \qquad (27)$$

We introduce the following non-dimensional quantities

$$R=\frac{r}{b},\quad R_o=\frac{a}{b},\quad \sigma_{rt}=\frac{\sigma_{rr}}{Y_1},$$

$$\sigma_{\theta t}=\frac{\sigma_{\theta\theta}}{Y_1},\ \beta_o=\frac{\beta_2\bar{\varphi}_o}{Y_1},\ P_t=\frac{P}{Y_1}$$

Thus equations (25), (26) and (27) in non-dimensional quantities become

$$\sigma_{rt}=\frac{R_o^{2\alpha_3}}{\alpha_3}\left[1-R^{-2\alpha_3}\right]\left[1-\frac{\beta_o}{2R_o}\right]-\beta_o\left(\frac{1}{R}-1\right) \qquad (28)$$

$$\sigma_{\theta t}=\sigma_{rt}+\left(\frac{Ro}{R}\right)^{2\alpha_3}\left[1-\frac{\beta_o}{2R_o}\right]+\frac{\beta_o}{2R} \qquad (29)$$

$$P_t=\frac{R_o^{2\alpha_3}}{\alpha_3}\left[R_o^{-2\alpha_3}-1\right]\left[1-\frac{\beta_o}{2R_o}\right]+\beta_o\left(\frac{1}{R_o}-1\right) \qquad (30)$$

4.2 Fully Plastic State

When fully plastic state is reached $\alpha_3\to 0$, equations (28), (29) and (30) become

$$\sigma_{rf} = 2\ln R\left[1-\frac{\beta_o}{2R_o}\right]-\beta_o\left(\frac{1}{R}-1\right) \qquad (31)$$

$$\sigma_{\theta f} = \sigma_{rf} + \left[1-\frac{\beta_o}{2R_o}\right]+\frac{\beta_o}{2R} \qquad (32)$$

$$P_f = -2\ln Ro\left[1-\frac{\beta_o}{2R_o}\right]+\beta_o\left(\frac{1}{R_o}-1\right) \qquad (33)$$

Particular Cases

(a) Spherical shell under uniform internal pressure only ($\varphi_o = 0$).

Transitional stresses and the pressure required for initial yielding of equations (28), (29) and (30) become

$$\sigma_{rt} = \frac{R_o^{2\alpha_3}}{\alpha_3}\left[1-R^{-2\alpha_3}\right] \qquad (34)$$

$$\sigma_{\theta t} = \sigma_{rt} + \left(\frac{Ro}{R}\right)^{2\alpha_3} \qquad (35)$$

$$P_t = \frac{R_o^{2\alpha_3}}{\alpha_3}\left[R_o^{-2\alpha_3}-1\right] \qquad (36)$$

These results are the same as obtained by Seth [17].

Fully plastic stresses and the pressure required for fully plastic state of equations (31), (32) and (33) become

$$\sigma_{rf} = 2\ln R \qquad (37)$$

$$\sigma_{\theta f} = \sigma_{rf} + 1 \qquad (38)$$

$$P_f = -2\ln Ro \qquad (39)$$

These results are the same as obtained by Johnson and Mellor [24] and Hill [6].

(b) Spherical shell under a temperature gradient only ($P = 0$).

Using the boundary condition $\sigma_{rr}\,|_{r=a} = 0$ in equations (21), (22) and (23), the transitional stresses and the pressure required for initial yielding of equations in non-dimensional quantities become

$$\sigma_{rt} = \beta_o\left[\frac{1-R^{-2\alpha_3}}{1-R_o^{-2\alpha_3}}\left(\frac{1}{R_o}-1\right)-\left(\frac{1}{R}-1\right)\right] \qquad (40)$$

$$\sigma_{\theta t} = \sigma_{rt} + \beta_o\left[\frac{\alpha_3 R^{-2\alpha_3}}{1-R_o^{-2\alpha_3}}\left(\frac{1}{R_o}-1\right)+\frac{1}{2R}\right] \qquad (41)$$

$$\beta_o = \frac{2R_o\left[1-R_o^{-2\alpha_3}\right]}{2R_o\left(\frac{1}{R_o}-1\right)\alpha_3 R_o^{-2\alpha_3}+1-R_o^{-2\alpha_3}} \qquad (42)$$

Fully plastic stresses and the pressure required for fully plastic state of equations (31), (32) and (33) become

$$\sigma_{rf} = \beta_o \left[\frac{\ln R}{\ln R_o} \left(\frac{1}{R_o} - 1 \right) - \left(\frac{1}{R} - 1 \right) \right] \qquad (43)$$

$$\sigma_{\theta f} = \sigma_{rf} + \beta_o \left[\frac{1}{2 \ln R_o} \left(\frac{1}{R_o} - 1 \right) + \frac{1}{2R} \right] \qquad (44)$$

$$\beta_o = \frac{2R_o \ln R_o}{1 - R_o + \ln R_o} \qquad (45)$$

5. Numerical Illustration and Discussion

As a numerical illustration, the C_{ij}, elastic stiffness constants for transversely isotropic material (Titanium) [14] and isotropic material (Steel) [20] are given in Table 1.

Table 1: Elastic stiffness constants C_{ij} used in units of $10^{10} N/m^2$

Materials	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}
Transversely isotropic (α_3=0.618, Titanium)	16.24	9.20	6.90	18.07	4.67
Isotropic (α_3=0.563, Steel)	2.908	1.27	1.27	2.908	0.819

It is seen from Figure 3 that the pressure required for the initial yielding of the thicker shell made of titanium and steel material is lower than the pressure required for thinner shell made of titanium and steel material at no temperature. Through adding thermal effects, the pressure required for initial yielding of the shell made of titanium and steel material decreases. At higher temperature titanium shell requires low pressure compared to steel shell but at lower temperature reverse is the case. Compared to thinner shells, thicker shell requires high pressure to become fully plastic with temperature rise. Despite uniform internal pressure, thicker shell requires high temperature compared to thinner shell to become fully plastic.

The pressure required for transition and fully plastic state along with the radii ratio is illustrated in Figure 4. It is depicted in the figure that the thicker shell requires high pressure for initial yielding and fully plastic as compared to thinner shell when the shell is subjected to pressure only. Figure 5 shows that thinner shell requires high temperature for initial yielding and fully plastic compared to thicker shell when the shell is subjected to temperature only. From both Figure 4 and Figure 5, shell made of titanium material requires high temperature and high pressure as compared to steel material for initial yielding when the shell is subjected to temperature only and pressure only respectively. Pressure required for fully plastic state is much lower than the pressure required for initial yielding, while reverse results are obtained for the temperature required.

It is observed in Figure 6 that circumferential stress on the outer surface is maximum for titanium and steel shells. The value of the transitional circumferential stress for steel and titanium shells increases at the shell's outer surface with the rise in temperature and pressure. The transitional circumferential stress value for steel shell at lower temperature is higher than the titanium shell on outer surface. It is also observed that fully plastic circumferential stress on the outer surface is maximum. With the rise in temperature and pressure, the value of fully plastic circumferential stress decreases at the shell's outer surface.

In Figure 7, stress distribution in a shell along with the radii ratio R separately due to uniform internal pressure and temperature are drawn. It is seen from the figure that the stress distribution through the shell surface, induced by temperature and separately induced by pressure, is opposite.

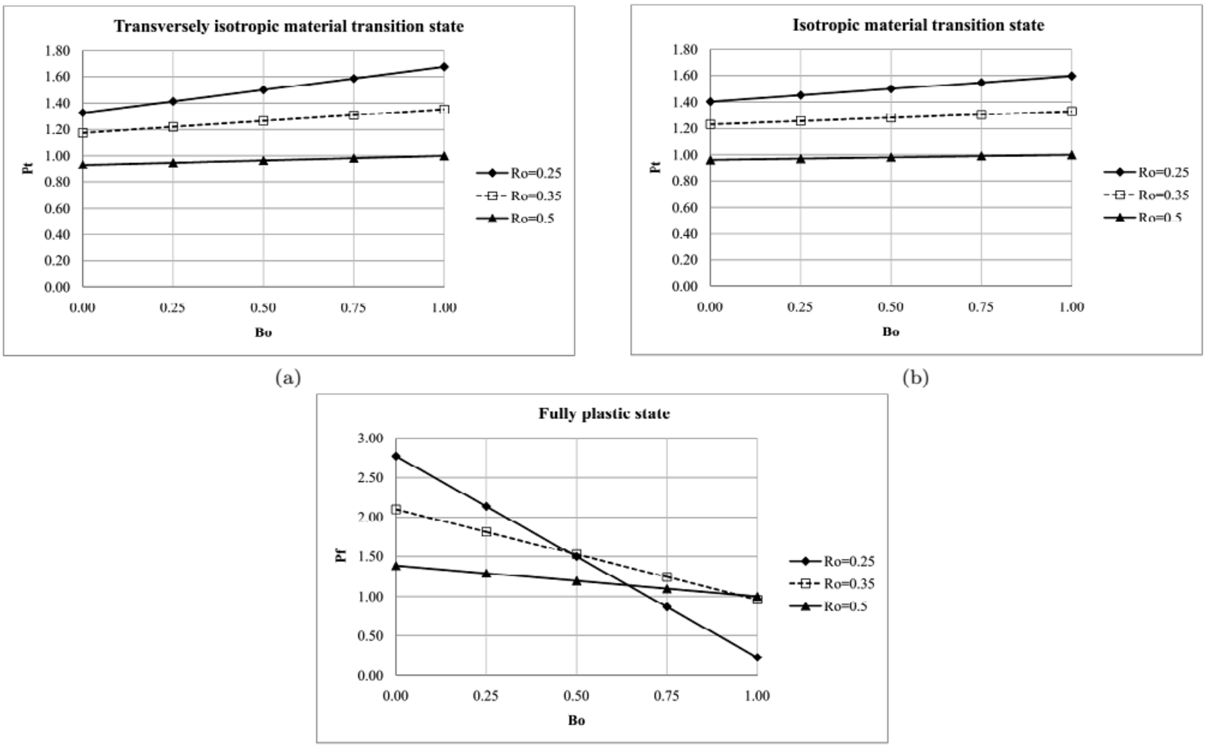


Figure 3: The relationship between β_o and P/Y_1 for initial yielding and fully plastic state at different radii ratio.

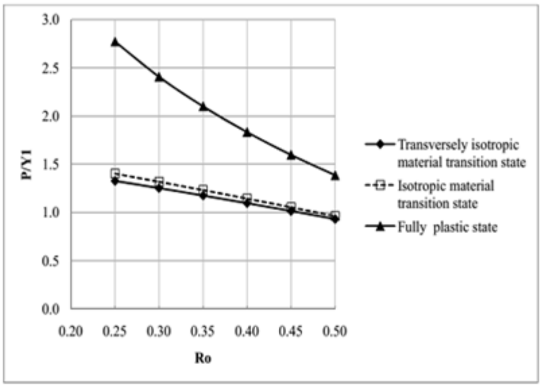


Figure 4: Graph between pressure required for initial and fully plastic state versus $R_o = a/b$.

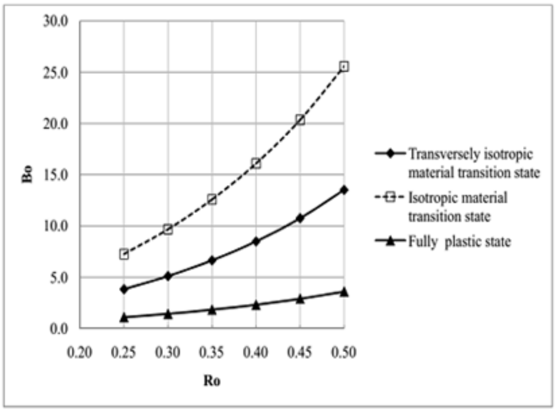
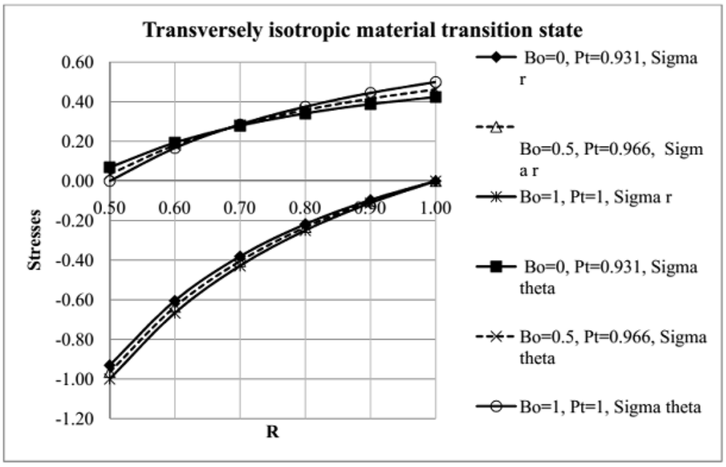
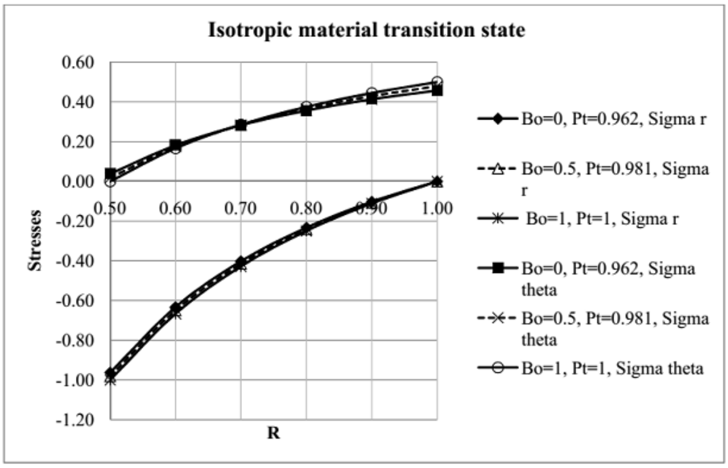


Figure 5: Graph between temperature versus $R_o = a/b$.



(a)



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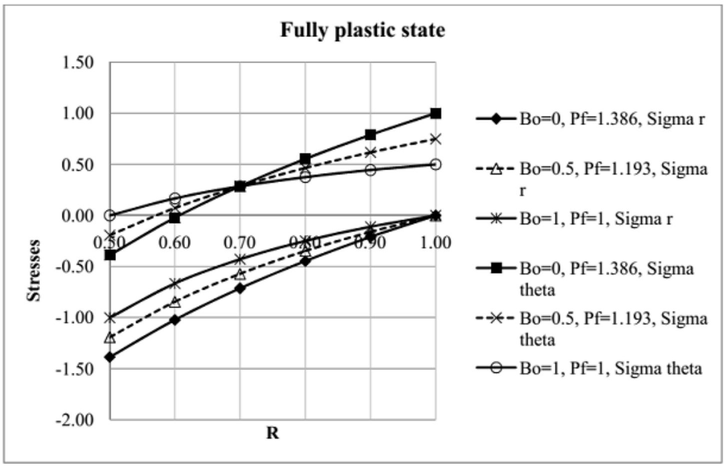
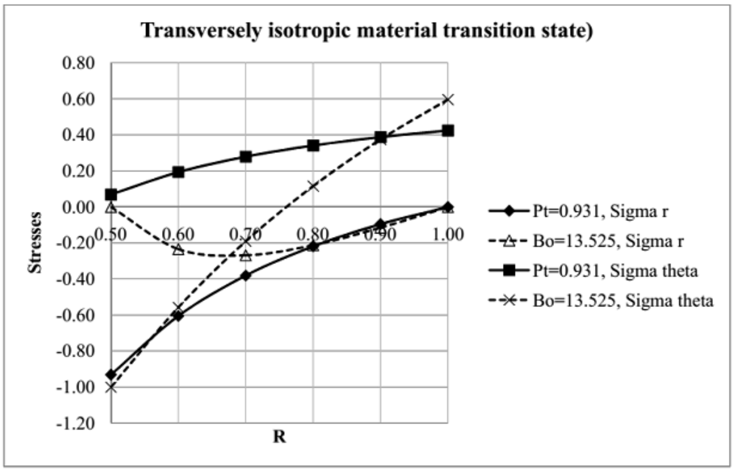
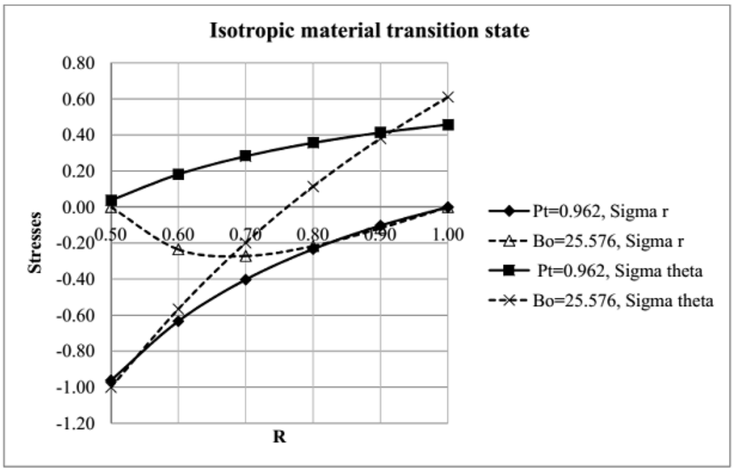


(c)

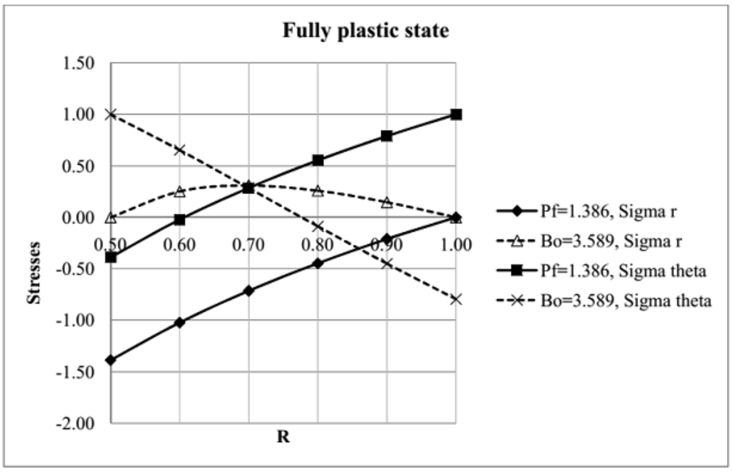
Figure 6: Stress distribution in a shell along with the radii ratio R under internal pressure and temperature gradient.



(a)



(b)



(c)

Figure 7: Stress distribution in a shell along with the radii ratio R separately under internal pressure and temperature.

6. **Conclusion**

It is inferred from the numerical discussion that the pressure required for the initial yielding of the thicker shell made of transversely isotropic material is lower than the pressure required for thinner shell at no temperature. Through adding thermal effects, the pressure required for initial yielding of the shell made of transversely isotropic material decreases. Compared to thinner shells, thicker shell requires high pressure to become fully plastic with temperature rise. The thicker shell requires high pressure for initial yielding and fully plastic as compared to thinner shell when the shell is subjected to pressure only, whereas thinner shell requires high temperature for initial yielding and fully plastic compared to thicker shell when the shell is subjected to temperature only. Shell made of titanium material requires high temperature as compared to steel material for initial yielding when the shell is subjected to temperature only. The transitional circumferential stress value for steel shell at lower temperature is higher than the titanium shell on outer surface. With the rise in temperature and pressure, the value of fully plastic circumferential stress decreases at the shell's outer surface. It is seen that the stress distribution through the shell surface, induced by temperature and separately induced by pressure, is opposite.

References

1. Chakrabarty, J., (2002), Applied Plasticity, Springer-Verlag, New York.
2. Fung, Y.C., 1965, Foundations of Solid Mechanics, Engle wood Cliffs, N.J.Prentice-Hall.
3. Sokolinikoff, I.S., (1956), Mathematical Theory of Elasticity (2nd edition), McGraw - Hill Book Co., New York.
4. Timoshenko, S.P., Goodier, J.N, (1951), Theory of Elasticity (3rd edition), McGraw-Hall Book Co. New York.
5. Reuss, E., (1930), Reruecksichtigung der elastischen Formaenderungen in der Plastizitaets theorie, Z. Angew. Math. Mech., **10**:266 – 274.
6. Hill, R., (1950), Mathematical Theory of Plasticity, Clarendon Press, Oxford.
7. Reissner, E., (1947), Stress and Small Displacements of Shallow Spherical Shells, J. Math. Phys., **25**(3):279-
300.
8. Johnson, W. and Derrington, M.G., (1958), The onset of yield in a thick-spherical shell subject to internal
pressure and uniform heat flow, Applied Science Research Sec. A, **7**:408-420.
9. Seth, B.R., (1962), Transition Theory of Elastic-plastic Deformation, Creep and Relaxation, Nature, **195**:896-897.
10. Seth, B.R., (1966), Measure Concept in Mechanics, Int. J. Non-linear Mech., **I** (2):35 – 40.
11. Gupta, S.K., Dharmani R.L., (1976), Creep transition in transversely isotropic shells under uniform pressure, H.P.
University, Simla 171005, **8**(9).
12. Parkus, H., (1976), Thermo-Elasticity, Springer-Verlag Wien, New York, USA.
13. Seth, B.R., (1970), Transition condition: the yield condition, Int J Non-linear Mech, **5**: 279-285.
14. Temesgen, A.G., Singh, S.B., Pankaj, T., (2021/1), Elastoplastic analysis in functionally graded thick-walled rotating transversely isotropic cylinder under a radial temperature gradient and uniform pressure, Math. Mech .Solids, USA, **26**(1):5-17 doi:10.1177/ 1081286520934041.

15. Temesgen, A.G., Singh, S.B., Pankaj, T.,(2020), Modeling of creep deformation of a transversely isotropic rotating disc with a shaft having variable density and subjected to a thermal gradient, Thermal Science and Engineering Progress, **20**:100745.
16. Temesgen, A.G., Singh, S.B., Pankaj, T., (2020), Modeling of elastoplastic deformation of transversely isotropic rotating disc of variable density with shaft under a radial temperature gradient, Structural Integrity and Life, **20**(2):113–121.
17. Seth, B.R., (1963), Elastic-plastic transition in shells and tubes under pressure, ZAMM, **43**:345-51.
18. Pankaj, T., Verma, G., Puneet, R., Pathania, D.S., (2017), Creep transition in the rotating spherical shell under the effect of density variable by Seth's transition theory, **AIP Conference Proceedings**.
19. Pathania, D.S., Verma G., (2019), Temperature and Pressure Dependent creep stress analysis of spherical shell, Int.J. of Applied Mechanics and Engineering, **24**(1):105-115.
20. Sethi, M., Thakur, P., (2020), Elasto-plastic deformation in isotropic material disk with shaft subjected to load and variable density, J Rubber Res, **23**(2): 69–78.
21. Thakur, P., Sethi, M., Gupta, N., Gupta, K., (2020), Effect of density parameter in a disk made of orthotropic material and rubber, J Rubber Res, **25**(3):193-201.
22. Thakur, P., Sethi, M., Gupta, N., Gupta, K., (2021), Thermal effects in rectangular plate made of rubber, copper and glass materials. J. Rubber Res, **24**(1):147-155.
23. Thakur, P., Kumar, N., Sethi, M., (2021), Elastic-plastic stresses in a rotating disc of transversely isotropic material fitted with a shaft and subjected to thermal gradient, Meccanica, **56**(5):1165-1175.
24. Johnson, W., Mellor, P.B., (1962), Plasticity for mechanical engineers, Von-Nostrand Reinhold company, London.