

DOUBLE DISPERSION RELIES ON COMBINED CONVECTION IN A FLUID WITH VARIABLE PROPERTIES

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ABSTRACT:

The present study is to investigate the influence of the variable properties and double dispersion on mixed convection flow over a vertical plate embedded in a power-law fluid saturated porous medium. The variable viscosity is assumed to vary as a inverse linear function of temperature and thermal conductivity as a linear function of temperature. Similarity transformations of the governing partial differential equations are obtained using Lie scaling group transformations. These similarity equations are solved numerically by using the Shooting technique. The numerical results for the non-dimensional velocity, temperature and concentration are displayed graphically for different values of variable viscosity, thermal conductivity, thermal dispersion and solutal dispersion. Local heat and mass transfer are shown in a tabular form. The present results are compared with previously published work and are found to be in good agreement.

Keywords: Dispersion; viscosity; Thermal conductivity; Power-law ; fluid Mixed convection

1. INTRODUCTION

The mixed convection boundary layer flow along a vertical surface embedded in porous media has received considerable theoretical and practical interest. The mixed convection flow occurs in several industrial and technical applications such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. A number of studies have been reported in the literature focusing on the problem of mixed convection about different surface geometries in porous media. A review of convective heat transfer in porous medium is presented in the book by Nield and Bejan. The majority of these studies dealt with the traditional Newtonian fluids. It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. Due to the important applications of non-Newtonian fluids in biology, physiology, technology, and industry, considerable efforts have been directed towards the analysis and understanding of such fluids. A number of mathematical models have been proposed to explain the rheological behavior of non Newtonian fluids. Among these, a model which has been most widely used for non-Newtonian fluids, and is frequently encountered in chemical engineering processes, is the power-law model. Although this model is merely an empirical relationship between the stress and velocity gradients, it has been successfully applied to non Newtonian fluids experimentally.

Stratification of fluid arises due to temperature variations, concentration differences, or the presence of different fluids. In practical situations where the heat and mass transfer mechanisms run parallel, it is interesting to analyze the effect of double stratification (stratification of medium with respect to thermal and concentration fields) on the convective transport in power-law fluid. The analysis of natural convection in a doubly stratified medium is a fundamentally interesting and important problem because of its broad range of engineering applications. The applications include heat rejection into the environment such as lakes, rivers and the seas; thermal energy storage systems such as solar ponds and heat transfer from thermal sources such as the condensers of power plants. Although the effect of stratification of the medium on the heat removal process in a fluid is important, very little work has been reported in the literature. Jumah and Mujumdar studied the free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical flat plate in saturated porous media. Murthy et al. discussed the effect of double stratification on free convection heat and mass transfer in a Darcian fluid saturated porous medium using the similarity solution technique for the case of uniform wall heat and mass flux conditions. Lakshmi Narayana and Murthy [13] analyzed the free convection heat and mass transfer from a vertical flat plate in a doubly stratified non-Darcy porous medium using series solution technique. Cheng [14] discussed the combined heat and mass transfer in natural convection flow from a vertical wavy surface in a power-law fluid saturated porous medium with thermal and mass stratification. Recently, Postelnicu et al. analyzed the free convection heat and mass transfer in a doubly stratified porous medium saturated with a power-law fluid. Motivated by the investigations mentioned above, the purpose of the present work is to investigate the thermal and solutal stratification effects on mixed convection heat and mass transfer from vertical plate in Darcy porous media saturated with power-law fluid with variable surface temperature and concentration conditions.

2. MATHEMATICAL FORMULATION

Consider a steady, laminar, viscous incompressible, mixed convection heat and mass transfer boundary layer flow over the vertical plate in a non-Newtonian power-law fluid saturated Darcy porous medium. Choose the two dimensional coordinate system such that the \bar{x} -axis is along the vertical plate and \bar{y} -axis normal to the plate. The wall is maintained at constant temperature and concentration T_w and C_w respectively, and these values are assumed to be greater than the ambient temperature and concentration T_∞ and C_∞ respectively. By Boussinesq approximation and boundary layer approximation, the governing equations, namely, the equations of continuity, momentum, energy and concentration for the non-Newtonian power-law fluid are given by

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

$$n\bar{u}^{n-1} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left[\frac{gK\rho_\infty}{\mu} (\beta_T(T - T_\infty) + \beta_C(C - C_\infty)) \right]$$

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left[(\alpha + \gamma d\bar{u}) \frac{\partial T}{\partial \bar{y}} \right]$$

$$\bar{u} \frac{\partial C}{\partial \bar{x}} + \bar{v} \frac{\partial C}{\partial \bar{y}} = \frac{\partial}{\partial \bar{y}} \left[(D + \zeta d\bar{u}) \frac{\partial C}{\partial \bar{y}} \right]$$

where \bar{x} and \bar{y} are the cartesian coordinates, \bar{u} and \bar{v} are the velocity components in \bar{x} and \bar{y} directions respectively, T is the temperature, C is the concentration, β_T and β_C are the thermal and concentration expansion coefficients respectively, ν is the kinematic viscosity of the fluid, K is the permeability, K_T is the thermal diffusion ratio, α is the thermal conductivity, D is the molecular diffusivities, γ is the mechanical thermal dispersion, ζ is the mechanical solutal dispersion and d is the pore diameter, n is the index in the power-law variation of viscosity, $n < 1$ for the pseudo-plastic fluids (shear-thinning fluid), $n > 1$ for the dilatant fluids (shear-thickening fluids) and $n = 1$ for the Newtonian fluids.

Application of scaling group of transformation Finding the similarity solutions is equivalent to determining the invariant solutions of these equations under a particular continuous one parameter group one of the methods is to search for a transformation group from the elementary set of one parameter scaling transformation, defined by the following group (Γ).

Here $\epsilon \neq 0$ is the parameter of the group and α 's are arbitrary real numbers whose interrelationship will be determined by our analysis. Transformation in may be treated as a point transformation, transforming the coordinates $(x, y, \psi, \theta, \phi) = (x^*, y^*, \psi^*, \theta^*, \phi^*)$. Using the transformations and the property that group transformations keeps the system invariant, we get the values of the parameters as $\alpha_1 = 2\alpha_3$, $\alpha_2 = \alpha_3$, $\alpha_4 = \alpha_5 = 0$.

Heat and Mass transfer coefficients:

The non dimensional heat and mass transfer coefficients in terms of local Nusselt number Nu and the local Sherwood number Sh respectively given by

$$q_w = -k_e \left[\frac{\partial T}{\partial \bar{y}} \right]_{\bar{y}=0} = -(k + k_d) \left[\frac{\partial T}{\partial \bar{y}} \right]_{\bar{y}=0} \quad \text{and} \quad q_m = -D_e \left[\frac{\partial C}{\partial \bar{y}} \right]_{\bar{y}=0} = -(D + D_d) \left[\frac{\partial C}{\partial \bar{y}} \right]_{\bar{y}=0}$$

3. RESULTS AND DISCUSSION

The non-dimensional heat and mass transfer coefficients (Nux and Shx) is plotted against power-law index (n) for different values of thermal and solutal stratification parameters in with $N = 1$, $Le = 1$. It is observed that the increasing the power-law index (n), increases the Nusselt number (Nux), but increasing the value of thermal stratification parameter (ϵ_1), Nusselt number is decreased. It is noted from that the increasing the power-law index (n), increases the Sherwood number (Shx), but increasing the value of thermal stratification parameter decreases Shx . It is noted from Fig.4 that the Nusselt number (Nux) is increased with increasing the power-law index (n), but increasing the value of solutal stratification parameter (ϵ_2) decreases Nux demonstrates that increasing the power-law index (n), increases the Sherwood number (Shx), but increasing the value of solutal stratification parameter Sherwood number is decreased. Figure-6 illustrates the variation of heat transfer coefficient (Nusselt number, Nux) with Lewis number (Le) for different values of power-law index (n) and $N = 1$, $\epsilon_1 = 0.5$ and $\epsilon_2 = 0.5$. It is observed from the figure that increasing Lewis number decreases the Nusselt number, but increasing the values of power law index increases Nusselt number (Nux). depict the variation of mass transfer coefficient (Sherwood number, Shx) with Lewis number (Le) for different values of power-law index (n) and $N =$

1, $\varepsilon_1=0.5$ and $\varepsilon_2=0.5$. It is observed from the figure that increasing Lewis number increases the Sherwood number.

The effect of thermal dispersion and the power-law index n for fixed values of the parameters on the non dimensional velocity $f(\eta)$, temperature $\theta(\eta)$ and concentration $\varphi(\eta)$ profiles respectively. Enhancing the thermal dispersion increases the non dimensional velocity $f(\eta)$. that increases in g values of thermal dispersion, increases the non dimensional temperature $\theta(\eta)$. i.e thermal dispersion enhances the transport of heat along the normal direction to the wall as compared with the case where dispersion is neglected (i.e., $Pe \gamma=0$). It displays that concentration boundary layer $\varphi(\eta)$ reduces with increasing the value of thermal dispersion. It presents the non dimensional velocity $f(\eta)$, temperature $\theta(\eta)$ and concentration $\varphi(\eta)$ profiles for different values of power-law fluid index n and solutal dispersion. It is observed from the that non dimensional velocity $f(\eta)$ profile increases with an increase in the solutal dispersion. Also an increase in the value of the solutal dispersion slightly decreases the thermal boundary layer thickness as shown in depicts that the concentration $\varphi(\eta)$ profile continuously increases with increasing the value of solutal dispersion. Hence the concentration boundary layer thickness enhances with an enhance in the solutal dispersion parameter.

CONCLUSIONS

In this paper, mixed convection heat and mass transfer along a vertical plate embedded in a power-law fluid saturated Darcy porous medium in presence thermal and solutal stratification has been considered. The wall is maintained at variable temperature and concentration x_T (w and x_C) (w respectively). It can be concluded from the present analysis that the increasing the of thermal stratification parameter decreases the Nusselt number and Sherwood number. The same trend is observed in case of solutal stratification parameter increases the Nusselt and Sherwood numbers decreases. An increase in the values of the power-law index parameter Nusselt and Sherwood numbers increased. Also, the higher value of Lewis number Nusselt number decreases, but Sherwood number increases. It is also observed that as Lewis number increases, that is, the thermal boundary layer thickness increases and the concentration boundary layer thickness decreases rapidly. The Lewis number has a more pronounced effect on the concentration field than on the temperature field.

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