An Efficient Algorithm for Solving Multi- Objective Problem

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Abstract
In this paper, we propose an algorithm (CLLE) to find efficient" solutions for multi-objective sequencing problem on one machine .The Objectives are total completion time(∑Cj),total lateness(∑Lj), maximum Lateness(Lmax) and maximum earliness (Emax). A collection of dependent jobs(tasks) has to work out sequenced on one tool ,jobs j (j=1,2,3,….,n)required processing time p_j and due history d_j. Conclusions is formulated in the from the (CLLE) algorithm based on results of computational experiments.
Keywords: Multi-objective Sequencing, one machine, Solution, Efficient, Pareto optimal.

1.Introdution

Sequencing indicates the division of limited resources to tasks through time. It is a decision-action process that has a aim the optimization" of one or extra objectives [1].
The essential sequencing problem enable be presented you finding for all of the mission which are alse called jobs a runuing while on one from the machines that are capable to execute them it that all part bonds are met , that should be well- done in such a direction that the obtained solution who is called sequencing is better possible that minimize the offerend objective function [2].

In that paper, the one machine case is involved the job j (j=1,2.3,..,n) request processing times (P_j) debt date (d_j),know completion times ($C_j=\sum_{i=1}^{j} p_i$) for particular sequence of job .the lateness criterion $L_j=C_j-d_j$ for and the earliness criterion is $E_j=\max\{d_j-C_j,0\}$.

In they simultaneous multi-criteria problems approach, two or extra criteria (objectives) are considered as simultaneously. This way usually generates all effective sequences and select the odd that gives the better objective function amount of that criteria(objective) . Most multi-criteria sequencing issue are NP- herd [3]. [4] gave me a heuristic method from multi-criteria sequencing problem for sequencing dependent order time for lessen the weighted total of total completion time, maximum retard "and maximum earliness means of integer programming model.

Manufacturing finesse are dynamic , complex and randomly systems. . Form the start of organized and top earliness by manufacturing, factor, supervisors, engineers and leader have developed many feasible methods for controlling output activities[5].Much manufacturing organizations beget and update product "Ion sequences which are schema that state when sure controllable activities(e.g., operating of jobs(tasks) by purse) should pick place. Production sequences coordinate activities for increasing output and minimize operating assess. A production sequence can determine resources conflicts, rule the release from (tasks) jobs to they shop, include that required explicit materials are sequenced in period, determine whether allocation promises can occur met and identify time periods obtainable for deterrent maintenance[6].[7] Solved multi-criteria problem in a hierarchical method.[8] presents a p custom- built Simulator designed to particular ort solution approaches for sequencing case in complex assembly cross found in made-up environments. Industrial environments. In modern years a powerful optimization too [9],[10] Evolutionary Algorithms (EAs) reign been presented to solve they sequencing Problem.[11] presented an effective algorithm to locate the set of all "efficient sol for the multi-objective problem $1//f(\sum C_j, \sum T_j, L_{max})$.[12] Proposed an algorithm to find effective solutions for Multi-objective problem $1//F(\sum C_j , V_{max} ,L_{max})$.

2.Materlals and Methods

2.1.Definition (1.1) [2]: A feasible sol (sequence) σ are efficient (Pareto optimum or non –dominated) with regard to the performance standard f and g if there is no suitable solution (sequence) π it that both $f(\pi) \le f(\sigma)$ and $g(\pi) \le g(\sigma)$ where on least single of the inequalities is strict. Lawler's Algorithm (LA) which settle the 1 /pres/f_{max} problem or $1//f_{max} \in (C_{max}, L_{max}, T_{max}, V_{max})$(1) to discovery minimum f_{max} Lawler's Algorithm (LA) is described by the" next steps (algorithm 1):

2.2 .Algorithm (1.2); Lawler's algorithm

Step(1): let N={1,2,….,n} F is the set of all tasks (jobs) with no successor and $\pi=\phi$

Step(2):let J* be a job such that $f_{j*}(\sum_{j\in N} Pi)=\min_{i\in f}\{f_j (\sum_{i\in N} Pi)\}$

Step(3):Set N=N-{J*} and sequence job J* in last position of π, i.e.,π =(π,J*)

Step(4):Modify F with respect to the new set of sequencing jobs

Step(5): LF N=ϕ stop, otherwise go to step (2)

3 . Formulation of the simultaneons multi objective problem

The simltaneous multi-objective sequencing (P) problem of overall Completion time ,total lateness maximum lateness and extreme Earliness is formulated as follows

$$\begin{aligned}
&\text{Min}\{\sum C_j, \sum L_j, L_{max}, E_{max}\}\\
&\text{s.t}\\
&C_j=\sum\nolimits^{j}_{i=1}P_i \quad j=1,2,...,n. \qquad \ldots(P)\\
&L_j=C_j-d_j \ , \quad j=1,2,...n.\\
&E_j=\max\{d_j-c_j,0\} \ , j=1,2,..,n.
\end{aligned}$$

3.1. Notation and some fundamental concepts of multi-objective sequencing

N= Set of jobs

n=Number of jobs

P_j= processing time for jobs j

d_j=Due date for jobs j

C_J= completion time for jobs j

L_j= lateness for jobs j

L_{max}=Maximum lateness
E_{max} = Maximum earliness
$\sum C_{j=}$ total completion time
$\sum L_j$= total lateness

3.2. An algorithm(CLLE) to find efficient solution of the (P) problem
$1//F(\sum C_j, \sum L_j, L_{max}, E_{max})$ (P)
Step(1): Let $\Delta=\sum p_i$, $\sigma=(\phi)$.
Step(2): Let $N=\{1,2,\ldots,n\}$ $k=n$, $t=\sum p_i$.
Step(3): Calculate L_i by Lawler 's algorithm ($\forall_i \in N$).
Step(4): Find $j\in N \ni L_j \le \Delta$, $P_j \ge P_i$, $\forall$ j, i $\in$ N & $L_i \le \Delta$, assign j in position k of σ, If no job j with $L_j \le \Delta$, set $E_{max}(\sigma) = E_{max}(SPT)$ go to j step (8).
Step(5): Set $t=t-p_j$, $N=N-\{j\}$, $k=k-1$, if $k>1$ go to step (3) .
Step(6): For the resulting sequence job $\sigma = (\sigma(1),\ldots,\sigma(n))$, calculate $(\sum C_i(\sigma), \sum L_i(\sigma), L_{max}(\sigma), E_{max}(\sigma))$.
Step(7): Put $\Delta=L_{max}(\sigma)-1$, go to step (3).
Step(8):Put $\Delta=E_{max}(\sigma)-1$ $N=\{1,\ldots,n\}$, $k=1$, $t=\sum p_i$ and $\sigma=(\phi)$ if $\Delta \le E_{max}$ (MST) go To step (12).
Step(9): Calculate $r_i = \max\{S_i-\Delta,0\}$ $\forall_i \in N$.
Step(10): Find a job $j\in N$ with $r_j \le C_{k-1}$ and $P_j \le P_i$ $\forall$ j, i $\in$ N , $C_0 = 0$ (break tie with min S_j) assign j in position k of σ .
Step(11): Set $N=N-\{j\}$, $k=k+1$, if $k \le n$ go to step (10) for the resulting Sequence $\sigma = (\sigma(1),\ldots,\sigma(n))$ calculate $(\sum C_i(\sigma), \sum L_i(\sigma), L_{max}(\sigma), E_{max}(\sigma))$ And go to step (8).
Step(12): Stop with a set of efficient solutions .

Example(3.2.1)
consider the problem (P) with the following data:

i	1	2	3	4
P_i	2	3	5	8
d_i	11	3	7	8

Set$\Delta= \sum P_i$=18

I		1	2	3	4	j
T	k	L_i				
18	4	7	15	11	10	4
10	3	-1	7	3	*	3
5	2	-6	2	*	*	2
2	1	-9	*	*	*	1

$t=\sum P_i=18$

J	1	2	3	4
C_j	2	5	10	18
L_j	-9	2	3	10
E_j	9	0	0	0

The first efficient sequence is the SPT sequence (1,2,3,4) which gives **($\sum C_j$, $\sum L_j$, L_{max}, E_{max})** =(35,6,10,9)

Put $\Delta=L_{max}$ -1=10-1=9

I		1	2	3	4	j
T	K	L_i				
18	4	7	15	11	10	1
16	3	*	13	9	8	4
8	2	*	5	1	*	3
3	1	*	0	*	*	2

J	2	3	4	1
C_j	3	8	16	18
L_j	0	1	8	7
E_j	0	0	0	0

the second efficient sequence (2,3,4,1) gives **($\sum C_j$, $\sum L_j$, L_{max}, E_{max})**=(45,16,8,0) repeating the process, the first part of the algorithm is stopped.

Now let $\Delta=E_{max}$(SPT)-1= 9-1=8

i	1	2	3	4
P_i	2	3	5	8
d_i	11	3	7	8
S_i	9	0	2	0

i	2	4	3	1
C_i	3	11	16	18
E_i	0	0	0	**0**

E_{max}(MST)=0 $\Delta \nleq E_{max}$(MST)

$r_i=\max\{s_i - \Delta , 0\}$

K	1	2	3	4	j
1	1	0	0	0	2
2	*	*	0	0	1
3	*	*	*	0	3
4	*	*	*	*	4

The third efficient sequence

J	2	1	3	4
C_j	3	5	10	18
L_j	0	-6	3	10
E_j	0	6	0	0

($\sum C_j$, $\sum L_j$, L_{max}, E_{max})=(36,7,10,6)
$\Delta=E_{max}(\sigma\text{-}1)=6-1=5$

I	1	2	3	4
P_i	2	3	5	8
d_i	11	3	7	8
s_i	9	0	2	0

i	2	4	3	1
C_i	3	11	16	18
E_i	0	0	0	0

Stop if $\Delta \leq E_{max}$(MST) if not go to step (9)
$r_i=\max(s_i - \Delta, 0)$

K	1	2	3	4	j
1	4	0	0	0	2
2	4	*	0	0	3
3	4	*	*	0	1
4	*	*	*	0	4

J	2	3	4	1
C_j	3	8	10	18
L_j	0	1	-1	10
E_j	0	0	1	0

($\sum C_j$, $\sum L_j$, L_{max}, E_{max})=(39,10,10,1)

Table(3.2.1): the result of efficient sequence solutions for example(3.1) by CEM and algorithm CLLE.

Efficient solutions for problem(P)					
CEM	AIg(CLLE)	$\sum C_j$	$\sum L_j$	L_{max}	E_{max}
(1,2,3,4)	(1,2,3,4)	35	6	10	9
(2,3,4,1)	(2,3,4,1)	45	16	8	0
(2,1,3.4)	(2,1,3,4)	36	7	10	6
(2,3,1,4)	(2,3,1,4)	39	10	10	1

In this example we find all efficient sequence .

3.3. Test problems with computational experiments

The (CLLE) algorithm and complete enumeration method are tested for (P) problem for generating efficient solution means of coding it in MATLAB 2019 and run on a personal computer HP for Ram 2.50 GB test case are generated as following :for all job j an integer processing once P_j generated From the discrete regular distribution(3) .Also ,for each task j an integer due history d_j is generated of the discrete orderly distribution [P(1-TF-RDD/2) ,P(1-TF+RDD/2)] where, $P=\sum^{p}_{i=1}p_l$ depending on the relative extent of due date (RDD) and in the average lag factor (TF). For together parameters, the values 0.2,0.4, 0.6,0.8,1.0” are consider. For each chosen value of n , two problems are produced for each of the five figure of parameters making 10 problems form each account of n where the number of jobs $n \leq 1000$ for the (P) problem .A rate computation time in seconds and average size of efficient" points for complete enumeration method (CEM) and algorithm for n=3,4,5,6,7,8 are compared. And given in table (1).

TABLE(1): A comparison of average account time in seconds and average number of efficient points for (CEM)and (CLLE) algorithm .

No of jobs(n)	**Average computation time for CEM**	**Average no. of efficient points for CEM**	**Average computation time for CLLE**	**Average no. of efficient points for CLLE**
3	0.0026594	1.4	0.0038085	1.3
4	0.0005489	2.6	0.0012247	2.6
5	0.0023876	4.1	0.0021648	3.6
6	0.0245867	3.7	0.00075272	3.1
7	0.5358357	5.6	0.0132736	5
8	5.9417348688	9.7	0.0138298	4.8

3.4. Analysis of number efficient solutions

As our aim is to identify They collection of efficient solutions , we should try to hold the entire set. It is clear that if them objectives ability be optimized individually, we deduce that the set of efficient solution has no additional elements only unit with extreme account of the individual objective service .

It should be noted that the SPT sequences is one of the efficient. Solutions for the problem (P) From the results we can conclude they the average number rom efficient points is very small when contrast to the size of permutation sequences and the average computation times rapidly increase with the problem size $n \geq 8$. the objective of the experimental work reported here was to obtain some idea of the computation performance of the (CLLE) algorithm . Also ,we solved the problem (P) by complete enumeration method (CEM) to find exact efficient solutions set and programmed in MATLAB and implemented on

The same above personal computer and we get the same results with(CLLE) algorithm with size number n=3-8 jobs. but this method is not practically , since , the sequencing problem is defined on finite set of candidate sequences .

3.5. Conclusion

In this study, an algorithm (CLLE) is presented to multi-objective optimization and investigated its performance on a specific single machine multi-objective sequencing problem(P). Since, we are using (CLLE)algorithm , we can be sure that a solution for problem(P)is truly an efficient solution. As a result of our experiments, we conclude that the(CLLE)algorithm performs quite well for the multi objective problem (P)of size n=3,4,..8. the research presented, here, contributes to the multi-objective sequencing literature by adapting (CLLE) algorithm to multi objective problems. For future research, we recommend the topic that would use experimentation with the following machine sequencing problem .

1. $1//F(\sum C_j, \sum E_j ,\sum L_j, L_{max})$
2. $1//F(\sum C_j ,\sum L_j, \sum T_j ,E_{max})$
3. $1//F(\sum C_j, \sum E_j, \sum T_j, \sum L_j, L_{max}, E_{max})$.

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