A NOTE ON Q-INTUITIONISTIC L-FUZZY ℓ-SUBSEMIRING OF A ℓ-SEMIRING

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ABSTRACT: In this paper, we introduce the notion of Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring. We made an attempt to study the algebraic nature of ℓ-semiring. We also made an attempt to study the some properties of Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring and study the main theorem for homomorphism and anti-homomorphism.

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KEY WORDS: fuzzy subset, (Q, L)-fuzzy subset, (Q,L)-fuzzy ℓ-subsemiring, Q-intuitionistic L-fuzzy subset, Q-intuitionistic L-fuzzy ℓ-subsemiring, Q-intuitionistic L-fuzzy relation, Product of Q-intuitionistic L-fuzzy subsets.

INTRODUCTION: After the introduction of fuzzy sets by L.A.Zadeh [31], several researchers explored on the generalization of the concept of fuzzy sets. The concept of lattice was first defined by Dedekind in 1897 and then developed by Birkhofft, G.,[8,9]. Boole introduced Boolean algebra; a special class of lattice was equivalent to Boolean ring with identity. This relation gave a link between lattice theory and modern algebra. The idea of intuitionistic fuzzy subset was presented by K.T.Atanassov [5,6], as a speculation of the thought of fuzzy set. The notion of fuzzy subnearrings and ideals was introduced by Abou Zaid.S [1]. A.Solairaju and R.Nagarajan [26,27] have presented and characterized another mathematical design called Q-fuzzy subgroups. Sampathu.S, Anita Shanthi .S, and Praveen Prakash.A [23] have introduced (Q,L)-fuzzy Subsemiring of a Semiring. In this paper, we introduce the some theorems in Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring and established some results.

1.PRELIMINARIES:

1.1 Definition: Let X be a non–empty set. A **fuzzy subset** A_μ of X is a function $A_\mu: X \to [0, 1]$.

1.2 Definition: Let X be a non-empty set and L= (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A **(Q, L)-fuzzy subset** A_μ of X is a function $A_\mu: X \times Q \to L$.

1.3 Definition: Let R be a ℓ-semiring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a **(Q, L)-fuzzy ℓ-subsemiring (QLFLSSR)** of R if the following conditions are satisfied:

(i) $A(x+y, q) \geq A(x, q) \wedge A(y, q)$,

(ii) $A(xy, q) \geq A(x, q) \wedge A(y, q)$,

(iii) $A(x \vee y, q) \geq A(x, q) \wedge A(y, q)$,

(iv) $A(x \wedge y, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

1.1 Example: Let (Z , + , ●,∨,∧) be a ℓ-semiring and Q={p}, Then the (Q,L)-Fuzzy Set A of Z is defined by

$$A(x,q) = \begin{cases} 1 & if \quad x = 0 \\ 0.33 & if \quad x \epsilon < 2 > -0 \\ 0 & if \quad x \epsilon\, Z - < 2 > \end{cases}$$

Clearly A is an (Q,L)-Fuzzy ℓ-subsemiring of a ℓ-semiring.

1.4 Definition: An **intuitionistic fuzzy subset (IFS)** A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$, where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.5 Definition: Let (L, ≤) be a complete lattice with an involutive order reversing operation $\mathcal{N} : L \rightarrow L$ and Q be a nonempty set. A **Q-intuitionistic L-fuzzy subset (QILFS)** A in X is defined as an object of the form $A = \{< (x, q), \mu_A(x, q), \nu_A(x, q) >/$ x in X and q ε Q }, where $\mu_A : X \times Q \rightarrow L$ and $\nu_A : X \times Q \rightarrow L$ define the degree of membership and the degree of non-membership of the element x ϵ X respectively and for every x ϵ X satisfying $\mu_A(x) \leq \mathcal{N}(\nu_A(x))$.

1.6 Definition: Let A and B be any two Q-intuitionistic L-fuzzy subsets of a set X. We define the following operations:

(i) $A \cap B = \{ \langle x, \mu_A(x,q) \wedge \mu_B(x,q), \nu_A(x,q) \vee \nu_B(x,q) \rangle \}$, for all $x \in X$ and q in Q.

(ii) $A \cup B = \{ \langle x, \mu_A(x,q) \vee \mu_B(x,q), \nu_A(x,q) \wedge \nu_B(x,q) \rangle \}$, for all $x \in X$ and q in Q.

(iii) A И B = $\{ \langle x, 2 (\mu_A(x,q) . \mu_B(x,q)) / (\mu_A(x,q) + \mu_B(x,q)), 2 (\nu_A(x,q) . \nu_B(x,q)) / (\nu_A(x,q) + \nu_B(x,q)) \rangle / x \in X \}$, for all $x \in X$ and q in Q.

(iv) $A \leftrightarrow B = \{ \langle x, \max \{\nu_A(x,q), \mu_B(x,q) \}, \min\{\mu_A(x,q), \nu_B(x,q) \} \rangle / x \in X \}$, for all $x \in X$ and q in Q.

(v) □A = $\{ \langle x, \mu_A(x,q), 1 - \mu_A(x,q) \rangle / x \in X \}$, for all x in X and q in Q.

(vi) ◊A = $\{ \langle x, 1 - \nu_A(x,q), \nu_A(x,q) \rangle / x \in X \}$, for all x in X and q in Q.

1.7 Definition: Let R be a ℓ-semiring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-i**ntuitionistic L-fuzzy** ℓ**-subsemiring (QILFLSSR)** of R if it satisfies the following conditions:

(i) $\mu_A(x+y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$,

(ii) $\mu_A(xy, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$,

(iii) $\mu_A(x \vee y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$,

(iv) $\mu_A(x \wedge y, q) \geq \mu_A(x, q) \wedge \mu_A(y, q)$,

(v) $\nu_A(x+y, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$,

(vi) $\nu_A(xy, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$,

(vii) $\nu_A(x \vee y, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$,

(viii) $\nu_A(x \wedge y, q) \leq \nu_A(x, q) \vee \nu_A(y, q)$, for all x and y $\in$ R and q $\in$ Q.

1.2 Example: Let (Z , + , ●,∨,∧) be a ℓ-semiring and Q={p}, Then Q-intuitionistic L-Fuzzy subset A={< (x, q), $\mu_A(x, q)$, $\nu_A(x, q)$ >/ x in Z and q in Q } of Z is defined by

$$\mu_A(x, q) = \begin{cases} 0.6 \; if \; x \in <2> \\ 0.3 \; otherwise \end{cases}$$

and

$$\nu_A(x, q) = \begin{cases} 0.4 \; if \; x \in <2> \\ 0.7 \; otherwise \end{cases}$$

Clearly A is a Q-intuitionistic L-Fuzzy ℓ-subsemiring of a ℓ-semiring.

1.8 Definition: Let A and B be any two Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring G and H, respectively. The product of A and B, denoted by A×B, is defined as A×B = { 〈 (x,y),q), $\mu_{AxB}((x,y),q)$, $\nu_{AxB}((x,y),q)$ 〉 / for all x in G and y in H and q $\in$ Q }, where $\mu_{AxB}((x,y),q) = \mu_A(x,q) \wedge \mu_B(y,q)$ and $\nu_{AxB}((x, y),q) = \nu_A(x,q) \vee \nu_B(y,q)$.

1.9 Definition: Let A be an Q-intuitionistic L-fuzzy subset in a set S, the strongest Q-intuitionistic L-fuzzy relation on S, that is a Q-intuitionistic L-fuzzy relation on A is V given by $\mu_V((x,y),q) = \mu_A(x,q) \wedge \mu_A(y,q)$ and $\nu_V((x, y),q) = \nu_A(x,q) \vee \nu_A(y,q)$, for all x and y in S and q $\in$ Q.

1.10 Definition: Let R and R^l be any two ℓ-semirings. Let f : R → R^l be any function and A be an Q-intuitionistic L-fuzzy ℓ-subsemiring in R, V be an Q-intuitionistic L-fuzzy ℓ-subsemiring in f(R) = R^l, defined by $\mu_V(y,q) = \sup_{x \in f^{-1}(y)} \mu_A(x,q)$ and $\nu_V(y,q) = \inf_{x \in f^{-1}(y)} \nu_A(x,q)$, for all x in R and y in R^l. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

1.11 Definition: Let A be an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R and a in R. Then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is defined by $((a\mu_A)^p)(x,q) = p(a)\mu_A(x,q)$ and $((a\nu_A)^p)(x,q) = p(a)\nu_A(x,q)$, for every x in R and for some p in P and q $\in$ Q.

1.3 Example: Let (Z , + , ●,∨,∧) be a ℓ-semiring and Q={p}, Then Q-intuitionistic L-Fuzzy subset A={<(x, q), $\mu_A(x, q)$, $\nu_A(x, q)$ >/ x in Z and q $\in$ Q } of Z is defined by

$$\mu_A(x, q) = \begin{cases} 0.6 \; if \; x \in <2> \\ 0.3 \; otherwise \end{cases}$$

and

$$\nu_A(x, q) = \begin{cases} 0.4 \; if \; x \in <2> \\ 0.7 \; otherwise \end{cases}$$

Clearly A is a Q-intuitionistic L-Fuzzy ℓ-subsemiring of a ℓ-semiring.

Now taking p(a) = 0.1 for every a in Z.

Then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is defined by

$$\mu_A(x, q) = \begin{cases} 0.06 \; if \; x \in <2> \\ 0.03 \; otherwise \end{cases}$$

and

$$\nu_A(x, q) = \begin{cases} 0.04 \; if \; x \in <2> \\ 0.07 \; otherwise \end{cases}$$

Clearly $(aA)^p$ is a Q-Intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring.

2. PROPERTIES OF Q-INTUITIONISTIC L-FUZZY ℓ-SUBSEMIRING OF A ℓ-SEMIRING

2.1 Theorem: Intersection of any two Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let A and B be any two Q-intuitionistic L-fuzzy ℓ-subsemirings of a ℓ-semiring R and x and y in R and q ∈ Q. Let $A=\{(x,q),\mu_A(x,q),\nu_A(x,q))/ x\in R$ and $q \in Q\}$ and $B=\{(x,q),\mu_B(x,q),\nu_B(x,q))/x\in R$ and $q\in Q\}$ and also let $C=A\cap B = \{(x,q),\mu_C(x,q), \nu_C(x,q))/x\in R$ and $q\in Q\}$,where $\mu_A(x,q)\wedge\mu_B(x,q)=\mu_C(x,q)$ and $\nu_A(x,q) \vee\nu_B(x,q)=\nu_C(x,q)$. Now, $\mu_C(x+y,q)=\mu_A(x+y,q)\wedge\mu_B(x+y,q)\geq\{\mu_A(x,q)\wedge\mu_A(y,q)\} \wedge \{\mu_B(x,q)\wedge\mu_B(y,q)\}=\{\mu_A(x,q)\wedge\mu_B(x,q)\}\wedge\{\mu_A(y,q)\wedge\mu_B(y,q)\}=\mu_C(x,q)\wedge\mu_C(y,q)$. Therefore, $\mu_C(x+y,q)\geq\mu_C(x,q)\wedge\mu_C(y,q)$, for all x and $y\in R$ and $q\in Q$. And, $\mu_C(xy,q) =\mu_A(xy,q)\wedge\mu_B(xy,q)\geq\{\mu_A(x,q)\wedge\mu_A(y,q)\}\wedge\{\mu_B(x,q)\wedge\mu_B(y,q)\}=\{\mu_A(x,q)\wedge\mu_B(x,q)\} \wedge \{\mu_A(y,q)\wedge\mu_B(y,q)\}\}=\mu_C(x,q)\wedge\mu_C(y,q)$. Therefore, $\mu_C(xy,q)\geq\mu_C(x,q) \wedge \mu_C(y,q)$, for all x and $y\in R$ and $q \in Q$. Also $\mu_C(x\vee y,q)= \mu_A(x\vee y,q) \wedge \mu_B(x\vee y,q)\geq\{\mu_A(x,q) \wedge\mu_A(y,q)\} \wedge\{\mu_B(x,q)\wedge\mu_B(y,q)\}=\{\mu_A(x,q)\wedge\mu_B(x,q)\}\wedge\{\mu_A(y,q)\wedge\mu_B(y,q)\}=\mu_C(x,q)\wedge\mu_C(y,q)$. Therefore, $\mu_C(x\vee y,q)\geq\mu_C(x,q)\wedge\mu_C(y,q)$, for all x and $y\in R$ and $q\in Q$. And $\mu_C(x\wedge y,q)=\mu_A(x\wedge y,q)\wedge\mu_B(x\wedge y,q)\geq\{\mu_A(x,q)\wedge\mu_A(y,q)\}\wedge\{\mu_B(x,q)\wedge\mu_B(y,q)\}= \{\mu_A(x,q)\wedge\mu_B(x,q)\}\wedge\{\mu_A(y,q)\wedge\mu_B(y,q)\}=\mu_C(x,q)\wedge\mu_C(y,q)$. Therefore, $\mu_C(x\wedge y,q) \geq \mu_C(x,q)\wedge\mu_C(y,q)$, for all x and $y \in R$ and $q \in Q$. Now, $\nu_C(x+y,q)= \nu_A(x+y,q) \vee \nu_B(x+y,q)\leq\{\nu_A(x,q)\vee\nu_A(y,q)\}\vee\{\nu_B(x,q)\vee\nu_B(y,q)\}=\{\nu_A(x,q)\vee\nu_B(x,q)\}\vee\{\nu_A(y,q)\vee \nu_B(y,q)\}=\nu_C(x,q)\vee\nu_C(y,q)$. Therefore, $\nu_C(x+y,q)\leq\nu_C(x,q)\vee \nu_C(y,q)$, for all x and $y \in R$ and $q\in Q$. And, $\nu_C(xy,q)=\nu_A(xy,q)\vee\nu_B(xy,q)\leq\{\nu_A(x,q)\vee\nu_A(y,q)\}\vee\{\nu_B(x,q)\vee \nu_B(y,q)\}\}=\{\nu_A(x,q)\vee\nu_B(x,q)\}\vee\{ \nu_A(y,q) \vee \nu_B(y,q)\}\}=\nu_C(x,q)\vee\nu_C(y,q)$ .Therefore, $\nu_C(xy,q) \leq\nu_C(x,q) \vee \nu_C(y,q)$, for all x and y in R and $q \in Q$. Also, $\nu_C(x\vee y,q) = \nu_A(x\vee y,q)\vee\nu_B(x\vee y,q)\leq\{\nu_A(x,q) \vee\nu_A(y,q)\}\vee\{\nu_B(x,q)\vee\nu_B(y,q)\} = \{\nu_A(x,q) \vee\nu_B(x,q)\} \vee\{\nu_A(y,q)\vee\nu_B(y,q)\}=\nu_C(x,q)\vee\nu_C(y,q)$. Therefore, $\nu_C(x\vee y,q)\leq\nu_C(x,q)\vee\nu_C(y,q)$, for all x and $y\in R$ and $q \in Q$. And, $\nu_C(x\wedge y,q) = \nu_A(x\wedge y,q) \vee \nu_B(x\wedge y,q)\leq\{\nu_A(x,q) \vee\nu_A(y,q)\} \vee\{\nu_B(x,q) \vee\nu_B(y,q)\}=\{\nu_A(x,q) \vee \nu_B(x,q)\} \vee\{\nu_A(y,q) \vee \nu_B(y,q)\} =\nu_C(x,q) \vee\nu_C(y,q)$. Therefore, $\nu_C(x\wedge y,q)\leq \nu_C(x,q) \vee \nu_C(y,q)$, for all x and $y \in R$ and $q \in Q$. Therefore C is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R. Hence the intersection of any two Q-intuitionistic L-fuzzy ℓ-subsemirings of a ℓ-semiring R is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

2.2 Theorem: The intersection of a family of Q-intuitionistic L-fuzzy ℓ-subsemirings of ℓ-semiring R is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let $\{V_i: i\in I\}$ be a family of Q-intuitionistic L-fuzzy ℓ-subsemirings of a ℓ-semiring R and let $A = \underset{i \in I}{\cap V_i}$. Let x and $y\in R$ and $q\in Q$. Then, $\mu_A(x+y,q)=\inf_{i\in I} \mu_{V_i}$

$(x+y,q)\geq \inf_{i\in I}\{\mu_{Vi}(x,q)\wedge\mu_{Vi}(y,q)\}=\inf_{i\in I}\mu_{Vi}(x,q)\wedge\inf_{i\in I}\mu_{Vi}(y,q)=\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, $\mu_A(x+y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and $y\in R$ and $q\in Q$. And, $\mu_A(xy,q)=\inf_{i\in I}\mu_{Vi}(xy,q)\geq\inf_{i\in I}\{\mu_{Vi}(x,q)\wedge\mu_{Vi}(y,q)\}=\inf_{i\in I}\mu_{Vi}(x,q)\wedge\inf_{i\in I}\mu_{Vi}(y,q)=\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, $\mu_A(xy,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and $y\in R$ and $q\in Q$.Also, $\mu_A(x\vee y,q)=\inf_{i\in I}\mu_{Vi}(x\vee y,q)\geq\inf_{i\in I}\{\mu_{Vi}(x,q)\wedge\mu_{Vi}(y,q)\}=\inf_{i\in I}\mu_{Vi}(x,q)\wedge\inf_{i\in I}\mu_{Vi}(y,q)=\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, $\mu_A(x\vee y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and $y\in R$ and $q\in Q$. And, $\mu_A(x\wedge y,q)=\inf_{i\in I}\mu_{Vi}(x\wedge y,q)\geq\inf_{i\in I}\{\mu_{Vi}(x,q)\wedge\mu_{Vi}(y,q)\}=\inf_{i\in I}\mu_{Vi}(x,q)\wedge\inf_{i\in I}\mu_{Vi}(y,q)=\mu_A(x,q)\wedge\mu_A(y,q)$. Therefore, $\mu_A(x\wedge y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)$, for all x and $y\in R$ and $q\in Q$. Now, $\nu_A(x+y,q)=\sup_{i\in I}\nu_{Vi}(x+y,q)\leq\sup_{i\in I}\{\nu_{Vi}(x,q)\vee\nu_{Vi}(y,q)\}=\sup_{i\in I}\nu_{Vi}(x,q)\vee\sup_{i\in I}\nu_{Vi}(y,q)=\nu_A(x,q)\vee\nu_A(y,q)$. Therefore, $\nu_A(x+y,q)\leq\nu_A(x,q)\vee\nu_A(y,q)$, for all x and $y\in R$ and $q\in Q$. And, $\nu_A(xy,q)=\sup_{i\in I}\nu_{Vi}(xy,q)\leq\sup_{i\in I}\{\nu_{Vi}(x,q)\vee\nu_{Vi}(y,q)\}=\sup_{i\in I}\nu_{Vi}(x,q)\vee\sup_{i\in I}\nu_{Vi}(y,q)=\nu_A(x,q)\vee\nu_A(y,q)$. Therefore, $\nu_A(xy,q)\leq\nu_A(x,q)\vee\nu_A(y,q)$, for all x and $y\in R$ and $q\in Q$. Also, $\nu_A(x\vee y,q)=\sup_{i\in I}\nu_{Vi}(x\vee y,q)\leq\sup_{i\in I}\{\nu_{Vi}(x,q)\vee\nu_{Vi}(y,q)\}=\sup_{i\in I}\nu_{Vi}(x,q)\vee\sup_{i\in I}\nu_{Vi}(y,q)=\nu_A(x,q)\vee\nu_A(y,q)$. Therefore, $\nu_A(x\vee y,q)\leq\nu_A(x,q)\vee\nu_A(y,q)$, for all x and $y\in R$ and $q\in Q$.And,$\nu_A(x\wedge y,q)=\sup_{i\in I}\nu_{Vi}(x\wedge y,q)\leq\sup_{i\in I}\{\nu_{Vi}(x,q)\vee\nu_{Vi}(y,q)\}=\sup_{i\in I}\nu_{Vi}(x,q)\vee\sup_{i\in I}\nu_{Vi}(y,q)=\nu_A(x,q)\vee\nu_A(y,q)$.Therefore, $\nu_A(x\wedge y,q)\leq\nu_A(x,q)\vee\nu_A(y,q)$, for all x and $y\in R$ and $q\in Q$. That is, A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. Hence, the intersection of a family of Q-intuitionistic L-fuzzy ℓ-subsemirings of R is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

2.3 Theorem: If A and B are any two Q-intuitionistic L-fuzzy ℓ-subsemirings of the ℓ-semirings R_1 and R_2 respectively, then A×B is a Q-intuitionistic L-fuzzy ℓ-subsemiring of $R_1\times R_2$.

Proof: Let A and B be two Q-intuitionistic L-fuzzy ℓ-subsemirings of the ℓ-semirings R_1 and R_2 respectively. Let x_1 and $x_2\in R_1$, y_1 and $y_2\in R_2$ and $q\in Q$. Then (x_1,y_1) and (x_2,y_2) are in $R_1\times R_2$. Now, $\mu_{A\times B}[((x_1,y_1)+(x_2,y_2)),q]=\mu_{A\times B}((x_1+x_2,y_1+y_2),q)=\mu_A((x_1+x_2),q)\wedge\mu_B((y_1+y_2),q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_B(y_1,q)\wedge\mu_B(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_B(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_B(y_2,q)\}\}=\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$. Therefore, $\mu_{A\times B}[((x_1,y_1)+(x_2,y_2),q)]\geq\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$.And, $\mu_{A\times B}[((x_1,y_1)(x_2,y_2)),q)]=\mu_{A\times B}((x_1x_2,y_1y_2),q)=\mu_A(x_1x_2,q)\wedge\mu_B(y_1y_2,q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_B(y_1,q)\wedge\mu_B(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_B(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_B(y_2,q)\}\}=\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$. Therefore, $\mu_{A\times B}[((x_1,y_1)(x_2,y_2)),q]\geq\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$. Also, $\mu_{A\times B}[((x_1,y_1)\vee(x_2,y_2)),q]=\mu_{A\times B}((x_1\vee x_2,y_1\vee y_2),q)=\mu_A((x_1\vee x_2),q)\wedge\mu_B((y_1\vee y_2),q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_B(y_1,q)\wedge\mu_B(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_B(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_B(y_2,q)\}\}=\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$. Therefore, $\mu_{A\times B}[((x_1,y_1)\vee(x_2,y_2),q)]\geq$

$\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$. And, $\mu_{A\times B}[((x_1,y_1)\wedge(x_2,y_2)),q]=\mu_{A\times B}((x_1\wedge x_2,y_1\wedge y_2), q)=\mu_A((x_1\wedge x_2),q)\wedge\mu_B((y_1\wedge y_2),q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_B(y_1,q)\wedge\mu_B(y_2,q)\}\}= \{\{\mu_A(x_1,q)\wedge\mu_B(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_B(y_2,q)\}\}=\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$. Therefore, $\mu_{A\times B}[((x_1,y_1)\wedge(x_2,y_2),q)]\geq\mu_{A\times B}((x_1,y_1),q)\wedge\mu_{A\times B}((x_2,y_2),q)$. Now, $\nu_{A\times B}[((x_1,y_1)+(x_2,y_2),q)]=\nu_{A\times B}((x_1+x_2,y_1+y_2),q)=\nu_A((x_1+x_2),q)\vee\nu_B((y_1+y_2),q)\leq\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_B(y_1,q)\vee\nu_B(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_B(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_B(y_2,q)\}\}= \nu_{A\times B}((x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)$. Therefore, $\nu_{A\times B}[((x_1,y_1)+(x_2,y_2),q)]\leq\nu_{A\times B}[(x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)]$. And, $\nu_{A\times B}[(x_1,y_1)(x_2,y_2),q]=\nu_{A\times B}((x_1x_2,q)(y_1y_2,q))=\nu_A(x_1x_2,q)\vee\nu_B(y_1y_2,q)\leq\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_B(y_1,q)\vee\nu_B(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_B(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_B(y_2,q)\}=\nu_{A\times B}((x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)$.Therefore, $\nu_{A\times B}[((x_1,y_1)(x_2,y_2)), q]\leq\nu_{A\times B}((x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)$. Also, $\nu_{A\times B}[((x_1,y_1)\vee(x_2,y_2),q)]=\nu_{A\times B}((x_1\vee x_2, y_1\vee y_2),q)=\nu_A((x_1\vee x_2),q)\vee\nu_B((y_1\vee y_2),q)\leq\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_B(y_1,q)\vee\nu_B(y_2,q)\}\}= \{\{\nu_A(x_1,q)\vee\nu_B(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_B(y_2,q)\}\}=\nu_{A\times B}((x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)$. Therefore, $\nu_{A\times B}[((x_1,y_1)\vee(x_2,y_2),q)]\leq\nu_{A\times B}[(x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)]$. And, $\nu_{A\times B}[((x_1, y_1)\wedge(x_2,y_2),q)]=\nu_{A\times B}((x_1\wedge x_2,y_1\wedge y_2),q)=\nu_A((x_1\wedge x_2),q)\vee\nu_B((y_1\wedge y_2),q)\leq\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_B(y_1,q)\vee\nu_B(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_B(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_B(y_2,q)\}\} = \nu_{A\times B}((x_1,y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)$. Therefore, $\nu_{A\times B}[((x_1,y_1)\wedge(x_2,y_2),q)] \leq \nu_{A\times B}[(x_1, y_1),q)\vee\nu_{A\times B}((x_2,y_2),q)]$. Hence A×B is a Q-intuitionistic L-fuzzy ℓ-subsemiring of ℓ-semiring of $R_1\times R_2$.

2.4 Theorem: Let A be a Q-intuitionistic L-fuzzy subset of a ℓ-semiring R and V be the strongest Q-intuitionistic L-fuzzy relation of R. Then A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R if and only if V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R×R.

Proof: Suppose that A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. Then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R and $q\in Q$. We have, $\mu_V((x+y),q)=\mu_V[((x_1,x_2)+(y_1,y_2)),q)]=\mu_V((x_1+y_1,x_2+y_2),q)=\mu_A((x_1+y_1),q)\wedge\mu_A((x_2+y_2), q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_A(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q)\wedge\mu_A(y_2,q)\}\}=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)=\mu_V(x,q)\wedge\mu_V(y,q)$. Therefore, $\mu_V((x+y),q)\geq\mu_V(x,q)\wedge\mu_V(y,q)$, for all x and y in R×R and $q\in Q$. And, $\mu_V(xy,q) = \mu_V[((x_1,x_2)(y_1,y_2),q)]=\mu_V((x_1y_1,x_2y_2),q)=\mu_A(x_1y_1,q)\wedge\mu_A(x_2y_2,q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_A(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q)\wedge\mu_A(y_2,q)\}\}=\{\mu_V((x_1,x_2), q)\wedge\mu_V((y_1,y_2),q)\}=\{\mu_V(x,q)\wedge\mu_V(y,q)\}$.Therefore, $\mu_V(xy,q)\geq\mu_V(x,q)\wedge\mu_V(y,q)$, for all x and y in R×R and $q\in Q$. Also, $\mu_V((x\vee y),q)=\mu_V[((x_1,x_2)\vee(y_1,y_2)),q)]=\mu_V((x_1\vee y_1, x_2\vee y_2),q)=\mu_A((x_1\vee y_1),q)\wedge\mu_A((x_2\vee y_2),q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_A(y_2,q)\}\} =\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q)\wedge\mu_A(y_2,q)\}\}=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q) = \mu_V(x,q)\wedge\mu_V(y,q)$. Therefore, $\mu_V((x\vee y),q)\geq\mu_V(x,q)\wedge\mu_V(y,q)$, for all x and y in R×R and $q\in Q$. And, $\mu_V((x\wedge y),q)=\mu_V[((x_1,x_2)\wedge(y_1,y_2)),q)]=\mu_V((x_1\wedge y_1,x_2\wedge y_2),q)=\mu_A((x_1\wedge y_1),q)\wedge\mu_A((x_2\wedge y_2),q)\geq\{\{\mu_A(x_1,q)\wedge\mu_A(y_1,q)\}\wedge\{\mu_A(x_2,q)\wedge\mu_A(y_2,q)\}\}=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q)\wedge\mu_A(y_2,q)\}\}=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)=\mu_V(x,q)\wedge\mu_V(y,q)$. Therefore, $\mu_V((x\wedge y),q)\geq\mu_V(x,q)\wedge\mu_V(y,q)$, for all x and y in R×R and $q\in Q$. We have, $\nu_V((x+y),q)=\nu_V[((x_1,x_2)+(y_1,y_2),q)]=\nu_V((x_1+y_1,x_2+y_2),q)=\nu_A((x_1+y_1),q)\vee\nu_A((x_2+y_2),q)\}\leq\{\{\nu_A(x_1,q)\vee\nu_A(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_A(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}=\{\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)\}=\nu_V(x,q)\vee\nu_V(y,q)$.Therefore, $\nu_V((x+y),q)\leq\nu_V(x,q)\vee\nu_V(y,q)$, for all x and y in R×R and $q\in Q$. And, $\nu_V(xy,q)=\nu_V[((x_1,x_2)(y_1,y_2),q)]=\nu_V((x_1y_1,x_2y_2),q)=\nu_A(x_1y_1,q)\vee\nu_A(x_2y_2,q)\leq\{\{\nu_A(x_1,q)\vee\nu_A(y_1,q)\}\vee\{\nu_A(x_2,q),\nu_A(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}=\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)=\nu_V(x,q)\vee\nu_V(y,q)$. Therefore, $\nu_V(xy,q)\leq\nu_V(x,q)\vee\nu_V(y,q)$, for all x and y in R×R and $q\in Q$. Also, $\nu_V((x\vee y),q)=\nu_V[((x_1,x_2)\vee(y_1,y_2),q)]=\nu_V((x_1\vee y_1,x_2\vee y_2),q)=\nu_A((x_1\vee y_1),q)\vee\nu_A((x_2\vee y_2),q)\}\leq\{\{\nu_A(x_1,q)\vee\nu_A(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_A(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}=\{\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)\}=\nu_V(x,q)\vee\nu_V(y,q)$. Therefore, $\nu_V((x\vee y),q)\leq\nu_V(x,q)\vee\nu_V(y,q)$,for all x and y in R×R and $q\in Q$. And, $\nu_V((x\wedge y),q)=\nu_V[((x_1,x_2)\wedge(y_1,y_2),q)]=\nu_V((x_1\wedge y_1,x_2\wedge y_2),q)=\nu_A((x_1\wedge y_1),q)\vee\nu_A((x_2\wedge y_2),q)\}\leq\{\{\nu_A(x_1,q)\vee\nu_A(y_1,q)\}\vee\{\nu_A(x_2,q)\vee\nu_A(y_2,q)\}\}=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}=\{\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)\}=\nu_V(x,q)\vee\nu_V(y,q)$. Therefore, $\nu_V((x\wedge y),q)\leq\nu_V(x,q)\vee\nu_V(y,q)$, for all x and y in R×R and $q\in Q$. This proves that V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R×R.

Conversely assume that V is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R×R, then for any $x=(x_1,x_2)$ and $y=(y_1,y_2)$ are in R×R and $q\in Q$, we have $\mu_A((x_1+y_1),q)\wedge\mu_A((x_2+y_2),q)=\mu_V((x_1+y_1,x_2+y_2),q)=\mu_V[((x_1,x_2),q)+((y_1,y_2),q)]=\mu_V((x+y),q)\geq\mu_V(x,q)\wedge\mu_V(y,q)=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q),\mu_A(y_2,q)\}\}$. If $\mu_A((x_1+y_1),q)\leq\mu_A((x_2+y_2),q)$, $\mu_A(x_1,q)\leq\mu_A(x_2,q)$, $\mu_A(y_1,q)\leq\mu_A(y_2,q)$, we get, $\mu_A((x_1+y_1),q)\geq\mu_A(x_1,q)\wedge\mu_A(y_1,q)$, for all x_1 and y_1 in R and q in Q. And, $\mu_A(x_1y_1),q)\wedge\mu_A(x_2y_2),q)\}=\mu_V((x_1y_1,x_2y_2),q)=\mu_V[((x_1,x_2)(y_1,y_2),q)]=\mu_V(x\ y,q)\geq\mu_V(x,q)\wedge\mu_V(y,q)=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)\}=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q)\wedge\mu_A(y_2,q)\}\}$. If $\mu_A(x_1y_1,q)\leq\mu_A(x_2y_2,q)$, $\mu_A(x_1,q)\leq\mu_A(x_2,q)$, $\mu_A(y_1,q)\leq\mu_A(y_2,q)$, we get $\mu_A(x_1y_1,q)\geq\mu_A(x_1,q)\wedge\mu_A(y_1,q)$, for all x_1 and y_1 in R and $q\in Q$. Also, $\mu_A((x_1\vee y_1),q)\wedge\mu_A((x_2\vee y_2),q)=\mu_V((x_1\vee y_1,x_2\vee y_2),q)=\mu_V[((x_1,x_2),q)\vee((y_1,y_2),q)]=\mu_V((x\vee y),q)\geq\mu_V(x,q)\wedge\mu_V(y,q)=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q),\mu_A(y_2,q)\}\}$. If $\mu_A((x_1\vee y_1),q)\leq\mu_A((x_2\vee y_2),q)$, $\mu_A(x_1,q)\leq\mu_A(x_2,q)$, $\mu_A(y_1,q)\leq\mu_A(y_2,q)$, we get, $\mu_A((x_1\vee y_1),q)\geq\mu_A(x_1,q)\wedge\mu_A(y_1,q)$, for all x_1 and y_1 in R and $q\in Q$. And, $\mu_A((x_1\wedge y_1),q)\wedge\mu_A((x_2\wedge y_2),q)=\mu_V((x_1\wedge y_1,x_2\wedge y_2),q)=\mu_V[((x_1,x_2),q)\wedge((y_1,y_2),q)]=$

$\mu_V((x\wedge y),q)\geq\mu_V(x,q)\wedge\mu_V(y,q)=\mu_V((x_1,x_2),q)\wedge\mu_V((y_1,y_2),q)=\{\{\mu_A(x_1,q)\wedge\mu_A(x_2,q)\}\wedge\{\mu_A(y_1,q),\mu_A(y_2,q)\}\}$. If $\mu_A((x_1\wedge y_1),q)\leq\mu_A((x_2\wedge y_2),q)$, $\mu_A(x_1,q)\leq\mu_A(x_2,q)$, $\mu_A(y_1,q)\leq\mu_A(y_2,q)$, we get, $\mu_A((x_1\wedge y_1),q)\geq\mu_A(x_1,q)\wedge\mu_A(y_1,q)$, for all x_1 and y_1 in R and $q\in Q$. We have $\nu_A((x_1+y_1),q)\vee\nu_A((x_2+y_2),q)=\nu_V((x_1+y_1,x_2+y_2),q)=\nu_V[((x_1,x_2)+(y_1,y_2),q)]=\nu_V(x+y,q)\leq\nu_V(x,q)\vee\nu_V(y,q)=\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}$. If $\nu_A(x_1+y_1,q)\geq\nu_A(x_2+y_2,q),\nu_A(x_1,q)\geq\nu_A(x_2,q),\nu_A(y_1,q)\geq\nu_A(y_2,q)$, we get, $\nu_A(x_1+y_1,q)\leq\nu_A(x_1,q)\vee\nu_A(y_1,q)$, for all x_1 and y_1 in R and $q\in Q$. And, $\nu_A(x_1y_1,q)\vee\nu_A(x_2y_2,q)=\nu_V((x_1y_1,x_2y_2),q)=\nu_V[((x_1,x_2),(y_1,y_2)),q)]=\nu_V(xy,q)\leq\nu_V(x,q)\vee\nu_V(y,q)=\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q),\nu_A(y_2,q)\}\}$. If $\nu_A(x_1y_1,q)\geq\nu_A(x_2y_2,q),\nu_A(x_1,q)\geq\nu_A(x_2,q),\nu_A(y_1,q)\geq\nu_A(y_2,q)$, we get $\nu_A(x_1y_1,q)\leq\nu_A(x_1,q)\vee\nu_A(y_1,q)$, for all x_1 and y_1 in R and $q\in Q$. Also, $\nu_A((x_1\vee y_1),q)\vee\nu_A((x_2\vee y_2),q)=\nu_V((x_1\vee y_1,x_2\vee y_2),q)=\nu_V[((x_1,x_2)\vee(y_1,y_2),q)]=\nu_V(x\vee y,q)\leq\nu_V(x,q)\vee\nu_V(y,q)=\nu_V((x_1,x_2),q\vee\nu_V((y_1,y_2),q)=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}$. If $\nu_A(x_1\vee y_1,q)\geq\nu_A(x_2\vee y_2,q),\nu_A(x_1,q)\geq\nu_A(x_2,q),\nu_A(y_1,q)\geq\nu_A(y_2,q)$, we get, $\nu_A(x_1\vee y_1,q)\leq\nu_A(x_1,q)\vee\nu_A(y_1,q)$, for all x_1 and $y_1\in R$ and $q\in Q$. And, $\nu_A((x_1\wedge y_1),q)\vee\nu_A((x_2\wedge y_2),q)=\nu_V((x_1\wedge y_1,x_2\wedge y_2),q)=\nu_V[((x_1,x_2)\wedge(y_1,y_2),q)]=\nu_V(x\wedge y,q)\leq\nu_V(x,q)\vee\nu_V(y,q)=\nu_V((x_1,x_2),q)\vee\nu_V((y_1,y_2),q)=\{\{\nu_A(x_1,q)\vee\nu_A(x_2,q)\}\vee\{\nu_A(y_1,q)\vee\nu_A(y_2,q)\}\}$. If $\nu_A(x_1\wedge y_1,q)\geq\nu_A(x_2\wedge y_2,q),\nu_A(x_1,q)\geq\nu_A(x_2,q),\nu_A(y_1,q)\geq\nu_A(y_2,q)$, we get, $\nu_A(x_1\wedge y_1,q)\leq\nu_A(x_1,q)\vee\nu_A(y_1,q)$, for all x_1 and y_1 in R and $q\in Q$. Therefore A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

2.5 Theorem: If A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R, then $H=\{x/x\in R: \mu_A(x,q)=1,\nu_A(x,q)=0\}$ is either empty or is a ℓ-subsemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x and $y\in H$ and $q\in Q$, then $\mu_A(x+y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)=1\wedge1=1$. Therefore, $\mu_A(x+y,q)=1$. And $\mu_A(xy,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)=1\wedge1=1$. Therefore, $\mu_A(xy,q)=1$. Also, $\mu_A(x\vee y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)=1\wedge1=1$. Therefore, $\mu_A(x\vee y,q)=1$. And, $\mu_A(x\wedge y,q)\geq\mu_A(x,q)\wedge\mu_A(y,q)=1\wedge1=1$. Therefore, $\mu_A(x\wedge y,q)=1$.Now, $\nu_A(x+y,q)\leq\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(x+y,q)=0$. And $\nu_A(xy,q)\leq\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(xy,q)=0$. Also, $\nu_A(x\wedge y,q)\leq\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(x\wedge y,q)=0$. And, $\nu_A(x\wedge y,q)\leq\nu_A(x,q)\vee\nu_A(y,q)=0\vee0=0$. Therefore, $\nu_A(x\wedge y,q)=0$.We get $x+y$, xy, $x\vee y$, $x\wedge y$ in H. Therefore, H is a ℓ-subsemiring of R. Hence H is either empty or is a ℓ-subsemiring of R.

2.6 Theorem: If A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R, then □A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. Consider $A=\{\langle(x,q),\mu_A(x,q),\nu_A(x,q)\rangle\}$, for all x in R and $q\in Q$, we take $\Box A=B=\{\langle(x,q),\mu_B(x,q),\nu_B(x,q)\rangle\}$, where $\mu_B(x,q)=\mu_A(x,q),\nu_B(x,q)=1-\mu_A(x,q)$. Clearly, $\mu_B(x+y,$

$q) \geq \mu_B(x,q) \wedge \mu_B(y,q)$, for all x and $y \in R$ and $q \in Q$ and $\mu_B(xy,q) \geq \mu_B(x,q) \wedge \mu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. Also, $\mu_B(x \vee y,q) \geq \mu_B(x,q) \wedge \mu_B(y,q)$, for all x and $y \in R$ and $q \in Q$ and $\mu_B(x \wedge y,q) \geq \mu_B(x,q) \wedge \mu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. Since A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of R, we have $\mu_A(x+y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\nu_B(xy,q) \geq \{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\}$, which implies that $\nu_B(x+y) \leq 1-\{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\} = \nu_B(x,q) \vee \nu_B(y,q)$. Therefore, $\nu_B(x+y,q) \leq \nu_B(x,q) \vee \nu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. And $\mu_A(xy,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\nu_B(xy,q) \geq \{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\}$ which implies that $\nu_B(xy,q) \leq 1-\{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\} = \nu_B(x,q) \vee \nu_B(y,q)$. Therefore, $\nu_B(xy,q) \leq \nu_B(x,q) \vee \nu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. Also, $\mu_A(x \vee y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\nu_B(x \vee y,q) \geq \{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\}$, which implies that $\nu_B(x \vee y) \leq 1-\{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\} = \nu_B(x,q) \vee \nu_B(y,q)$. Therefore, $\nu_B(x \vee y,q) \leq \nu_B(x,q) \vee \nu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. And, $\mu_A(x \wedge y,q) \geq \mu_A(x,q) \wedge \mu_A(y,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\nu_B(x \wedge y,q) \geq \{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\}$, which implies that $\nu_B(x \wedge y) \leq 1-\{(1-\nu_B(x,q)) \wedge (1-\nu_B(y,q))\} = \nu_B(x,q) \vee \nu_B(y,q)$. Therefore, $\nu_B(x \wedge y,q) \leq \nu_B(x,q) \vee \nu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. Hence $B = \Box A$ is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

2.7 Theorem: If A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R, then ◊A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. That is $A=\{\langle (x,q),\mu_A(x,q),\nu_A(x,q)\rangle\}$, for all $x \in R$ and $q \in Q$. Let $\Diamond A = B = \{\langle (x,q),\mu_B(x,q),\nu_B(x,q)\rangle\}$, where $\mu_B(x,q)=1-\nu_A(x,q)$, $\nu_B(x,q)=\nu_A(x,q)$. Clearly $\nu_B(x+y,q) \leq \nu_B(x) \vee \nu_B(y)$, for all x and $y \in R$ and $\nu_B(xy,q) \leq \nu_B(x,q) \vee \nu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. Since A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R, we have $\nu_A(x+y,q) \leq \nu_A(x,q) \vee \nu_A(y,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\mu_B(x+y,q) \leq \{(1-\mu_B(x,q)) \vee (1-\mu_B(y,q))\}$,which implies that $\mu_B(x+y,q) \geq 1-\{(1-\mu_B(x,q)) \vee (1-\mu_B(y,q))\} = \mu_B(x,q) \wedge \mu_B(y,q)$. Therefore, $\mu_B(x+y,q) \geq \mu_B(x,q) \wedge \mu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. And $\nu_A(xy,q) \leq \nu_A(x,q) \vee \nu_A(y,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\mu_B(xy,q) \leq \{(1-\mu_B(x,q)) \vee (1-\mu_B(y,q))$, which implies that $\mu_B(xy,q) \geq 1-\{(1-\mu_B(x,q)) \vee (1-\mu_B(y,q))\} = \mu_B(x,q) \wedge \mu_B(y,q)$. Therefore, $\mu_B(xy,q) \geq \mu_B(x,q) \wedge \mu_B(y,q)$, for all x and $y \in R$ and $q \in Q$. Also, $\nu_A(x \vee y,q) \leq \nu_A(x,q) \vee \nu_A(y,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\mu_B(x \vee y,q) \leq \{(1-\mu_B(x,q)) \vee (1-\mu_B(y,q))\}$ which implies that

$\mu_B(x\vee y,q)\geq 1-\{(1-\mu_B(x,q))\vee(1-\mu_B(y,q))\}=\mu_B(x,q)\wedge\mu_B(y,q)$. Therefore, $\mu_B(x\vee y, q) \geq \mu_B(x,q)\wedge\mu_B(y,q)$, for all x and y$\in$R and q$\in$Q. And $\nu_A(x\wedge y,q)\leq\nu_A(x,q)\vee\nu_A(y,q)$, for all x and y$\in$R and q$\in$Q, which implies that $1-\mu_B(x\wedge y,q)\leq\{(1-\mu_B(x,q))\vee(1-\mu_B(y,q))\}$, which implies that $\mu_B(x\wedge y,q)\geq 1-\{(1-\mu_B(x,q))\vee(1-\mu_B(y,q))\}=\mu_B(x,q)\wedge \mu_B(y,q)$. Therefore, $\mu_B(x\wedge y,q)\geq\mu_B(x,q)\wedge\mu_B(y,q)$, for all x and y$\in$R and q$\in$Q. Hence B = ◊A is an Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. Hence B = ◊A is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

2.8 Theorem: Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring H and f is an isomorphism from a ℓ-semiring R onto H. Then A∘f is a Q-intuitionistic L-fuzzy ℓ-subsemiring of R.

Proof: Let x and y in R and q $\in$ Q, A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring H. Then we have, $(\mu_A\circ f)(x+y,q)=\mu_A(f(x+y),q)=\mu_A(f(x,q)+f(y,q))\geq \mu_A(f(x,q))\wedge\mu_A(f(y,q))\geq(\mu_A\circ f)(x,q)\wedge(\mu_A\circ f)(y,q)$, which implies that $(\mu_A\circ f)(x+y,q)\geq (\mu_A\circ f)(x,q)\wedge(\mu_A\circ f)(y,q)$. And $(\mu_A\circ f)(xy,q)=\mu_A(f(xy,q))=\mu_A(f(x,q)f(y,q))\geq\mu_A(f(x,q)) \wedge \mu_A(f(y,q)) \geq (\mu_A\circ f)(x,q) \wedge (\mu_A\circ f)(y,q)$, which implies that $(\mu_A\circ f)(xy,q)\geq (\mu_A\circ f)(x,q) \wedge (\mu_A\circ f)(y,q)$. Also, $(\mu_A\circ f)(x\vee y,q)=\mu_A(f(x\vee y),q)=\mu_A(f(x,q)\vee f(y,q))\geq \mu_A(f(x,q)) \wedge \mu_A(f(y,q))\geq(\mu_A\circ f)(x,q) \wedge (\mu_A\circ f)(y,q)$, which implies that $(\mu_A\circ f)(x\vee y,q)\geq (\mu_A\circ f)(x,q) \wedge (\mu_A\circ f)(y,q)$. And, $(\mu_A\circ f)(x\wedge y,q)=\mu_A(f(x\wedge y),q)=\mu_A(f(x,q)\wedge f(y,q))\geq \mu_A(f(x,q)) \wedge \mu_A(f(y,q))\geq(\mu_A\circ f)(x,q) \wedge (\mu_A\circ f)(y,q)$, which implies that $(\mu_A\circ f)(x\wedge y,q)\geq (\mu_A\circ f)(x,q) \wedge (\mu_A\circ f)(y,q)$. Then we have, $(\nu_A\circ f)(x+y,q)=\nu_A(f(x+y,q))=\nu_A(f(x,q)+f(y,q))\leq\nu_A(f(x,q)) \vee\nu_A(f(y,q))\leq(\nu_A\circ f)(x,q)\vee(\nu_A\circ f)(y,q)$, which implies that $(\nu_A\circ f)(x+y,q)\leq(\nu_A\circ f)(x,q) \vee (\nu_A\circ f)(y,q)$. And, $(\nu_A\circ f)(xy,q)=\nu_A(f(xy,q))=\nu_A(f(x,q)f(y,q))\leq\nu_A(f(x,q)) \vee\nu_A(f(y,q)) \leq (\nu_A\circ f)(x,q) \vee (\nu_A\circ f)(y,q)$, which implies that $(\nu_A\circ f)(xy,q)\leq(\nu_A\circ f)(x,q) \vee (\nu_A\circ f)(y,q)$. Also, $(\nu_A\circ f)(x\vee y,q)=\nu_A(f(x\vee y,q))=\nu_A(f(x,q)\vee f(y,q))\leq\nu_A(f(x,q))\vee\nu_A(f(y,q))\leq(\nu_A\circ f)(x,q) \vee (\nu_A\circ f)(y,q)$, which implies that $(\nu_A\circ f)(x\vee y,q)\leq(\nu_A\circ f)(x,q) \vee (\nu_A\circ f)(y,q)$. And, $(\nu_A\circ f)(x\wedge y,q)=\nu_A(f(x\wedge y,q))=\nu_A(f(x,q)\wedge f(y,q))\leq\nu_A(f(x,q)) \vee\nu_A(f(y,q))\leq(\nu_A\circ f) (x,q) \vee (\nu_A\circ f)(y,q)$, which implies that $(\nu_A\circ f)(x\wedge y,q) \leq (\nu_A\circ f)(x,q) \vee (\nu_A\circ f)(y,q)$. Therefore (A∘f) is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

2.9 Theorem: Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R, then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R, for every a in R.

Proof: Let A be a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R. For every x and y$\in$R and q$\in$Q, we have, $((a\mu_A)^p)(x+y,q)=p(a)\mu_A(x+y,q)\geq p(a)\{(\mu_A(x,q)\wedge\mu_A(y,q)\}=p(a)\mu_A(x,q)\wedge p(a)\mu_A(y,q)=((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$.Therefore, $((a\mu_A)^p)(x+y,q) \geq((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$. Now, $((a\mu_A)^p)(xy,q)=p(a)\mu_A(xy,q)\geq p(a)\{\mu_A(x,q)\wedge\mu_A(y,q)\}=p(a)\mu_A(x,q)\wedge p(a)\mu_A(y,q)=((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$.Therefore, $((a\mu_A)^p)(xy,q) \geq ((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$. Also, $((a\mu_A)^p)(x\vee y,q)=p(a)\mu_A(x\vee y,q)\geq p(a)\{(\mu_A(x,q)\wedge$

$\mu_A(y,q)\}=p(a)\mu_A(x,q)\wedge p(a)\mu_A(y,q)=((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$. Therefore, $((a\mu_A)^p)(x\vee y,q)\geq((a\mu_A)^p)(x,q)\wedge((a\mu_A)^p)(y,q)$. For every x and $y\in R$ and $q\in Q$, we have, $((a\nu_A)^p)(x+y,q)=p(a)\nu_A(x+y,q)\leq p(a)\{(\nu_A(x,q)\vee\nu_A(y,q)\}=p(a)\nu_A(x,q)\vee p(a)\nu_A(y,q) = ((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. Therefore, $((a\nu_A)^p)(x+y,q)\leq((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. Now, $((a\nu_A)^p)(xy,q)=p(a)\nu_A(xy,q)\leq p(a)\{\nu_A(x,q)\vee\nu_A(y,q)\}=p(a)\nu_A(x,q)\vee p(a)\nu_A(y,q) =((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. Therefore, $((a\nu_A)^p)(xy,q)\leq((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. Also, $((a\nu_A)^p)(x\vee y,q)=p(a)\nu_A(x\vee y,q)\leq p(a)\{(\nu_A(x,q)\vee\nu_A(y,q)\}=p(a)\nu_A(x,q)\vee p(a)\nu_A(y,q)=((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. Therefore, $((a\nu_A)^p)(x\vee y,q)\leq((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. And, $((a\nu_A)^p)(x\wedge y,q)=p(a)\nu_A(x\wedge y,q)\leq p(a)\{(\nu_A(x,q)\vee\nu_A(y,q)\}=p(a)\nu_A(x,q)\vee p(a)\nu_A(y,q)=((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. Therefore, $((a\nu_A)^p)(x\wedge y,q) \leq ((a\nu_A)^p)(x,q)\vee((a\nu_A)^p)(y,q)$. Hence $(aA)^p$ is a Q-intuitionistic L-fuzzy ℓ-subsemiring of a ℓ-semiring R.

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