dss_2 of Strong 2-Vertex Duplication Self Switching of Some Special Graphs

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Abstract

Let G be a graph and let $\sigma \subseteq V$ be a non-empty subset of $V - \sigma$ is said to be a self switching if $G \cong G^{\sigma}$ where G^{σ} is obtained from G by removing all edges between σ and $V - \sigma$ and adding edges between all non-adjacent vertices of σ and $V - \sigma$. A vertex v' is the duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in (vG), which is the duplication graph of G. A vertex v is called a duplication self vertex switching of a graph G if the resultant graph obtained after duplication v has v as a self vertex switching.

Keywords: Switching, 2-vertex self switching, $SS_2(G), ss_2(G), dSS_2(G), dss_2(G)$.

1.Introduction

For a finite undirected simple graph $G(V, E)$ with $|V(G)| = p$ and a non-empty set $\sigma \subseteq V$, the switching of G by σ is defined as the graph $G^{\sigma}\ (V, E')$ which is obtained from G by removing all edges between σ and its complement, $V - \sigma$ and adding as edges all non-edges between σ and $V - \sigma$. Switching has been defined by Seidel [1,6] and is also referred to as Seidel switching. When $\sigma = \{v\} \subset V$, the corresponding switching $^{\{v\}}$ is called a *vertex switching* and is denoted by G^{v}. A non-empty set $\sigma \subseteq V$ is said to be self switching if $G \cong G^{\sigma}$ .We also call it as $|\sigma|$ $-$vertex self switching. The set of all k $-$vertex self switchings of G each with cardinality k is denoted by $SS_k\ (G)$ and its cardinality by$ss_k\ (G)$. If $k = 1$, then we call the corresponding self switching as self vertex switching .. When $|\sigma| = 2$, we call it as $2 -$ vertex self switching. The set of all $2 -$ vertex self switchings sets of a graph G is denoted by $SS_2(G)$and its cardinality by $ss_2(G)$.The number $ss_2(G)$ for the graph G, when G is a Path P_n, a cycle C_n, and a complete bipartite graph $K_{m,n}$ has been found

C. Jayasekaran and G. Sumathy [2] has done a survey on self-vertex switching of graphs. The existence of self vertex switching like trees, path, complete graph unicycle, two cyclic, bicyclic but not a two cyclic graph with given number of vertices are analyzed.

A vertex v' is said to be a duplication of v if all the vertices which are adjacent to v in G are also adjacent to v' in G' . The concept of duplication self vertex switching was introduced by C. Jayasekaran and Prabhavathy [3,4] A vertex v is called a duplication self vertex switching of a graph G if the resultant graph obtained after duplication of v has v as a self vertex switching. The set of duplication self vertex switching is denoted $dSS_1(G)$.The number of duplication self vertexswitching is denoted $dss_1(G)$.Let $\sigma = \{x, y\} \subseteq V(G)$ is called a **2 $-$vertex duplication self switching** of a graph G if $D((u, v)G) \cong D((u, v)G)^{\sigma}$. If $\sigma = \{u, v\}$, then σ is called the **strong 2 $-$ vertex duplication self switching** of G. Already we discussed about some results of strong 2-vertex duplication self switching of graphs [8].In this paper, we deals about the wheel graph, cycle graph of G and $\bar{G}$ and helm graph.

2.Strong 2-vertex duplication self switching graphs

In this paper, we are analyzing only the non-adjacency vertices of G to find $ss_2(G)$.

Theorem 2.1: If $\sigma=\{u,v\} \subseteq V$ is a strong $2-$vertex duplication self switching of a graph G, then $d_G(u)+ d_G(v) = p$ if $uv \notin E(G)$.

Proof:

Let $\sigma=\{u,v\} \subseteq V$ be a strong $2-$vertex duplication self switching of a graph G. By the definition, $D((u,v)G)=D((u,v)G)^{\sigma}$ and hence $|E(D(u,v)G)|=|E(D(u,v)G)^{\sigma}|$.

That implies, $q + d_G(u) + d(v) = q + d_G(u) + d_G(v) + [p + 2 - 1 - d_{D((u,v)G)}(u)] - d_{D((u,v)G)}(u) + [p + 2 - 1 - d_{D((u,v)G)}(v)] - d_{D((u,v)G)}(v) + 2.$

$0=p + 1 - [d_G(u) + 1] - (d_G(u) + 1) + p + 1 - [d_G(v) + 1] -(d_G(v) + 1) + 2.$

$0= 2p + 2 - 2d_G(u) - 2 - 2d_G(v) - 2 + 2.$

$0= 2p - 2d_G(u) - 2d_G(v)$

$d_G(u) + d_G(v) = p$

Note 2.2: The converse of the above theorem need not be true. For example, let us consider the graph G with 4 vertices given in figure 2.1. In this graph the vertices a and b are non- adjacent with $d_G(a) + d_G(b) = 4 = p$.Therefore the graph $D((a,b)G)$ and $D((a,b)G)^{\{a,b\}}$ is given in the figure 2.2 and figure 2.3. Thus $D((a,b)G) \ncong D((a,b)G)^{\{a,b\}}$

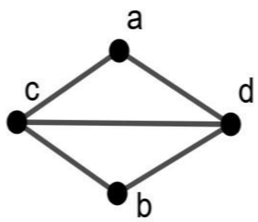

Fig. 2.1. G

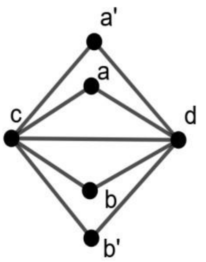

Fig. 2.2. D((a,b)G)

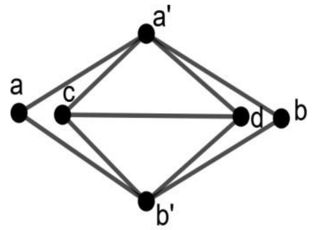

Fig. 2.3. $D((a,b)G)^{\{a,b\}}$

Definition 2.3: The Wheel graph W_p for each $p \geq 4$ is obtained by connecting an isolated vertex to all other vertices in a cycle C_{p-1}.That is, $W_p = K_1 + C_{p-1}, \forall\, i = 1,2, \dots, p-1$.

Theorem2.4: For $p\geq 4$, $dss_2(W_p) = 0$.

Proof:

Let G= W_p , Here $W_p = K_1 + C_{p-1}$. where K_1 be the vertex w and C_{p-1} be the cycle $v_1v_2 \dots v_{p-1}v_1$.
Then $V(W_p) = \{w, v_1, v_2, \dots, v_{p-1}\}$ and $E(W_p) = \{wv_i, wv_{p-1}, v_iv_{i+1}, v_1v_{p-1}/1 \leq i \leq p-2\}$.

In W_p , the central vertex w is of degree $p-1$ and $d(v_i)=3, \forall\ 1 \leq i \leq p-1$.

Let $G=W_{p.}$ If σ={u, v/u=v_i ,v=v_{i+2}, for some $1 \leq i \leq p-1$}, let us consider the case $uv \notin E(G)$ then $d_G(u) + d_G(v) \neq p$ and hence by theorem 2.1 σ is not a strong 2–vertex duplication self switching of G.

Therefore, $D((u,v)G\ ) \ncong ((u,v)G)^{\sigma}$.Hence $dss_2(W_p)=0$.

Theorem2.5: For p≥3, $dss_2\ (C_p)$={$\begin{matrix} 2, if p=4 \\ 0, otherwise \end{matrix}$

Proof:

Let $G=C_p$. Here $V(C_p)=\{v_1,v_2,\dots,v_n,v_1\}$ be the vertices of the cycle and $E(C_p)=\{e_1,e_2,\dots,e_n\}$ be the vertices of cycle C_p.
If $u_iu_{i+2} \notin E(G)$

Case 1: If p=4
Let $\sigma=\{u_1,u_3\}$ & $\sigma=\{u_2,u_4\}$
For any two non- adjacent vertices u and v in $C_{p,}\ d_G(u_1)+d_G(u_3)$ =4=p. Clearly
$D((u_1,u_3)G) \cong D((u_1,u_3)G)^{\{u_1,u_3\}}$.
Similarly, $D((u_2,u_4)G) \cong D((u_2,u_4)G)^{\{u_2,u_4\}}$.
Hence by the theorem 2.1,σ is a strong 2-vertex duplication self switching of C_p.
Case 2: If p≠4
For any two non-adjacent vertices u and v in $C_{p,}\ d_G(u)+d_G(v)=4 \neq p$. By the theorem 2.1, $\sigma$is not a strong 2-vertex duplication self switching of C_p.
Therefore, $dss_2(C_p)$=0.
Result 2.6: For any graph G, if $uv \notin$ E(G), then $dss_2(G) \neq dss_2(\bar{G})$

Note 2.7: Consider the graph $G=C_4$ and $dss_2(G)$= 2.
Now $\bar{G}=2K_2$ is given in the figure 2.4.Clearly G is disconnected.
Therefore $G \ncong \bar{G}$ and $dss_2(\overline{C_p})$= 0.
Thus, $dss_2(C_p) \neq dss_2(\overline{C_p})$
Example 2.8:

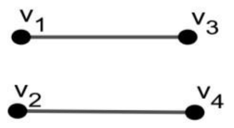

Fig. 2.4. $\bar{G}=2K_2$

Result 2.9: For any connected graph G, all the strong 2-vertex duplication self switchings are in one component.
Definition 2.10: The Helm graph H_p is obtained from the wheel graph $W_p=K_1+C_{p-1}$, by adding a pendant vertex to the other vertices in the cycle C_{p-1}.

Theorem2.11: For $p\geq 4$,$dss_2(H_p)=0$.
Proof:

The Helm graph H_p is the graph obtained from a $n-$wheel graph by joining a pendant edge at each vertex of the cycle. Thus, the graph H_p has $p=2n-1$vertices and $q=3(n-1)$edges. Let w be the central vertex of H_p. Let $v_1, v_2, \ldots, v_{n-1}$ be the pendantvertices of H_p and $u_1, u_2, \ldots, u_{n-1}$ be the vertices of degree 4.

Let $G=H_p$. Now let us consider the case $uv \notin$ E(G)

Let $\sigma=\{w,v_i\}$ for all $i=1\leq i\leq n-1$,then $d_G(w)+d_G(v_i)\neq p$ and hence by theorem 2.1 σ is not a strong 2 –vertex duplication self switching of G.

Also if $\sigma=\{v_i,v_{i+1}\}$ for all $i=1\leq i\leq n-1$,then $d_G(v_i)+d_G(v_{i+1})\neq p$ and hence by theorem 2.1 σ is not a strong 2 –vertex duplication self switching of G.

If $\sigma=\{u_i,u_{i+2}\}$ for all $i=1\leq i\leq n-1$,then $d_G(u_i)+d_G(u_{i+2})\neq p$ and hence by theorem 2.1 σ is not a strong 2 –vertex duplication self switching of G. Thus $dss_2(H_p)=0$.

Conclusion:

We have found the number of 2 $-$vertex duplication self switchings of wheel, cycle and Helm graphs in this paper. Further, we are analyzing some more results of 2 $-$vertex duplication self switching graphs.

References:

[1] Colbourn.C and Corneil.D, „On deciding switching equivalence of graphs‟, Discrete Appl. Math.,vol.2,pp.181-184,1980.

[2] C. Jayasekaran, and G. Sumathy, Self vertex switchings of connected two-cyclic graphs, accepted for publication in *Journal of Discrete Mathematical Sciences and Cryptograph*

[3] C. Jayasekaran & V. Prabavathy, „Some results on Duplication self vertex switchings‟, International Journal of Pure and Applied Mathematics, vol. 116, No. 2,427 − 435.

[4] C. Jayasekaran & V. Prabavathy, „A characterization of duplication self vertex switching in graphs‟, International Journal of Pure and Applied Mathematics, vol. 118, No.2.

[5] C. Jayasekaran, J. Christabel Sudha & M.Ashwin Shijo,„ 2 − vertex self switchings of some special graphs‟, International Journal of Scientific Research and Review, vol.7, Issue 12,2018. ISSN No:2279 − 543X

[6] C. Jayasekaran, J. Christabel Sudha, Some results on 2 −vertex self switching in joints, "Italian Journal of Pure and Applied Mathematics", communicated.

[7] G.Sumathy, Self vertex switching of two-cyclic and bicyclic graphs, Manonmaniam Sundaranar University, Tirunelveli, India, 2014.

[8] G.Sumathy and K.S Shruthi , "Some results on strong 2-vertex duplication self switching ofsome connected graphs", Malaya journal of Mathematik, Vol.8, No.4,ISSN No.2319-3786,2020

[9] J.J Seidal, A survey of two graphs, Proceedings of the international Coll.Theorie Combinatorie (Rome1973). Tomo I, Acca, Naz. Lincei pp. 481-511,1976.

[10] Selvam Avadayppan and M. Bhuvaneshwari, More results on self vertex switching, International journal of Modern sciences and Engineering Technology, Volume 1Issue 3, 2014, pp.10-17.