Review Article: Game Theory with Neutrosophic Application

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Abstract: The number of research papers focusing on applications of game theory, as well as applications on Neutrosophic, has increased significantly in recent years. These applications may fall into different fields such as: economics, politics, social sciences, and others. This paper presents some of the work that has been done on these two topics, starting with a background in game theory, followed by moving on to the concept of fuzzy sets and neutrosophic sets, and then bringing us to some research papers with different applications in these two topics. The final part of the paper uses the two major scientific databases (Web of Science and Scopus) to analyze the work by topic, country, years, etc. The analysis shows that there is a significant gap in the research conducted in the field of game theory with neutrosophic application.
Keywords: Game Theory; Iterated Prisoner's Dilemma Game; Neutrosophic Set; Fuzzy Set; Intuitionistic Fuzzy Sets.

1. Introduction

Operations research - also known as the study of optimization strategies - is often referred to as a scientific method of decision-making [1]. The mental processes that result in choosing a course of action from a variety of options are referred to as decision making. Every decision-making process ends with a final choice. In many real-world situations, choices must be made in a setting of conflict between two or more opposing parties, each of whose actions is dependent on the others. Such a competitive environment is referred to as a "game." [2, 3, 4].

The game pits players against one another in a race to achieve goals. There are two primary categories of games: games of chance, like roulette, and games of strategy, like poker. Game of strategies is the game of interest in this paper. Finding the best strategy that maximizes the gain and minimizes each player's loss is the key objective. Mathematicians must develop new methods because of the intricacy of many mathematical models' calculations. One of the modern strategies used to overcome these challenges in conflict settings is game theory [5].

Game theory is a mathematical study of conflict and strategy in which a player's ability to make decisions is influenced by those of the other players. It was first created in economics to help researchers better understand a variety of economic behaviors. Any strategy is the interaction of two or more decision-makers (players), each given two or more options (strategies) so that the outcome is determined by all players' strategic choices. All potential outcomes are clearly preferred by each player, allowing for the customization of the corresponding facilities (rewards). The foundation for game theory science was laid by Von Neumann and Morgenstern's publication of "Game Theory and Economic Behavior" in 1944, which is regarded as the key source that created the field of game theory[6].

Game theory uses a variety of games to analyze various types of problems. The most noticeable element of a game is how many players there are, followed by how similar the game is and player collaboration. Depending on how the game is played, it can be categorized into a variety of models, including cooperative and non-cooperative games, perfect and imperfect information games, normal form games and extensive form games, simultaneously moving games and sequential moving games, constant sum games, zero-sum games, and nonzero-sum games, as well as symmetric and anti-symmetric games [7, 8].

Game theory has been widely used and reviewed in numerous fields and applications such as political science [9]–[11], economics [12], [13], finance [14], [15], water management [16]–[19], supply chain [20], [21], medicine and disease [22], [23], timber raw market [24], construction management [25] and selection of the pedagogical method in education [26]. Not only in human activities but the game theory can also be used in animals' interactions [27]. It also has increased its use in the social and biological sciences [28].

1. Preliminaries

In this section, we describe the main concepts of game theory, classification of game theory techniques, fuzzy set, intuitionistic fuzzy set, and neutrosophic set.

2.1 Game theory

A branch of mathematics which is concerned with the analysis of strategies for dealing with competitive circumstances in which the outcome of a participant's choice of action is critically depend on the actions of other participants. While it does not provide instructions on how to play the game, it does discuss some of the qualities that certain strategies offer which could be appealing. Even when an analysis recommends the ideal strategy for the game, it only does so under the presumption that everyone is playing to the best of their abilities. It never allows the player to penalize the player opponent for making mistakes, which is what most players do. Game theory can be classified in several ways depending on the type of game as shown in table 1[33]:

Table 1: Definitions and models of game theory

Type		Definition
Static Game	**Perfect Information**	The player is aware of both its own payoff function and the payoff functions of its opposing players.
	Imperfect Information	There isn't much information available to players regarding their opponents. Due to the lack of information, the calculation will thus consider the predicted payoff function when playing a game. A Bayesian Game is another name for this game.
Dynamic Game	**Perfect Information**	The player is aware of the actions taken by the previous player at each terminal. The first player's strategy is known as the terminal history, which can be used by the next player to fully guide their next move. All possible combinations of the chosen strategies made up a set of terminal histories.
	Imperfect Information	At that stage, the player is unaware of the previous player's selected strategy. It is possible that the next player's movement won't be perfectly informed by the first player's chosen strategy.
Cooperative Game	A cooperative game is a game in which the participants form a coalition and come to a legally binding agreement to strengthen their relationship. It is used to divide the reward of cooperation among its player members. This form of cooperation is based on a real-valued function known as the characteristic function of the game. All players involved in this bond will benefit more from any possible cooperation. Instead of acting independently, it is at least to ensure that the return obtained through cooperation is more profitable or is defined as superadditivity.	
Evolutionary Game	Evolutionary games are mathematical objects with different rules, payoffs, and mathematical behaviours. Each "game" represents different problems that organisms have to deal with, and the strategies they	

	might adopt to survive and reproduce. Evolutionary games are often given colorful names and cover stories which describe the general situation of a particular game. Evolutionary game theory started with the problem of how to explain ritualized animal behavior in a conflict situation; "why are animals so 'gentlemanly or ladylike' in contests for resources?"

2.2 Classification of Game Theory Techniques

The following subsections explain the main classification of the game in this work.

2.2.1 Simultaneous and Sequential Games

In simultaneous games, both players move at the same time or if they don't, the latter players are unaware of the earlier players' movements (making them effectively simultaneous). Sequential or dynamic games are those in which latter players are aware of some of the earlier players' activities. This doesn't have to be comprehensive knowledge of every move made by players before; it might just be very little knowledge. For example, a player might know that a previous player didn't do something, but he or she might not know which of the other available actions the first player actually did.

2.2.2 Zero-Sum or Non-Zero–Sum Games

In zero-sum games, which are a specific example of constant-sum games, players' decisions have no effect on the number of resources that are accessible, either up or down. In zero-sum games, any feasible combination of strategies always yields a net benefit for all participants. Poker is a prime example of a zero-sum game because one wins exactly the same amount as one's opponents lose. Zero-sum games include the majority of classic board games like chess and go as well as matching pennies.

Many of the games explored by game theorists (including the well-known prisoner's dilemma) are non-zero-sum games because the outcome has net results bigger or less than zero. Informally, a gain by one player does not always result in a loss by another in a non-zero-sum game.

Constant-sum games are comparable to illegal activities like stealing and gambling but not to the fundamental economic situation where earnings from commerce might be realized. By adding a fake player (often known as "the board") whose losses counterbalance the players' net victories, any game can be transformed into a (perhaps asymmetric) zero-sum game. The matrix in the example below is a zero-sum game.

$$\begin{array}{cc} & \begin{array}{cc} A & \quad B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0,0 & -5,5 \\ 4,-4 & 3,-3 \end{pmatrix} \end{array}$$

2.2.3 Cooperative and non-cooperative games

The game is cooperative if the players have the ability to create binding legal contracts. For instance, the legal system requires them to honor commitments they make. Games without cooperative play make this impossible. It's a common misconception that while communication between players is allowed in cooperative games, it's not allowed in non-cooperative ones. This categorization, which relies on two binary criteria, has, however, been challenged and occasionally rejected [34].

Non-cooperative games are better at simulating real-world scenarios in-depth and delivering accurate results than cooperative games. Cooperative games emphasize the overall game. There have been significant efforts to link the two methodologies. According to the so-called Nash-program, which is the research agenda for analyzing both axiomatic bargaining solutions and the equilibrium outcomes of strategic bargaining processes [35], many cooperative solutions have already been shown to have non-cooperative equilibria.

Cooperative and non-cooperative features both appear in hybrid games. In a cooperative game, for instance, groups of players may come together to play in coalitions, but these groups do not cooperate.

2.2.4 Perfect and imperfect information games

Games with perfect knowledge are a significant subset of sequential games. When every participant in a game is fully aware of each other's previous moves, it is said to have perfect information. Simultaneous games cannot be games with perfect knowledge because simultaneous movements are translated to extensive form and following moves in the sequence are unknown. Most game studies focus on games with insufficient knowledge. Two fascinating examples of perfect information games are the ultimatum game and the centipede game. Two excellent information games that are enjoyed by players are chess and checkers. Numerous card games are games with partial information, including contract bridge and poker.

Complete information and perfect information are related concepts that are sometimes confused. Every participant must have complete knowledge of the options and rewards offered to the other players, but not necessarily the specific actions that were chosen. However, by introducing "moves by nature," games with imperfect knowledge can be reduced to games of incomplete information.

2.2.5 Symmetric game

A symmetric game is a game in which the benefits for employing a specific strategy depend only on the other strategies being employed and not on the players. If player identities may be changed without affecting how effectively the strategies work, the game is said to be symmetrical. Numerous symmetric games are regularly investigated.

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0,0 & 3,2 \\ 2,3 & 0,0 \end{pmatrix} \end{array}$$

The prisoner's dilemma, the stag hunt, and the traditional versions of chicken are examples of asymmetric games. Some academics consider specific asymmetric games to be examples of these games. However, symmetric payoffs occur more frequently in each of these games.

Asymmetric games are those in which neither player's set of strategies is the same as the others. For instance, each participant has a different approach to the dictator game and the ultimatum game. However, a game could be asymmetrical while still having identical strategies for both sides.

***2.2.6* Discrete and Continuous Games**

Much of game theory is concerned with finite, discrete games having a finite number of players, actions, events, and outcomes, etc. however. Continuous games allow players to choose a strategy from a continuous strategy set. It expands on the concept of a discrete game, in which players select from a finite set of pure strategies.

2.2.7 Normal Form and Extensive Form

In normal form games, the matrix includes all perceptible and conceivable strategies, and their corresponding payoffs, for each player. Extensive form games, on the other hand, are those in which the game is described using a decision tree. Extensive form games aid in the representation of occurrences that may occur by chance. These games have a tree-like structure in which the names of the players are represented on separate nodes. According to the game's principle, there are always at least two players, often known as "players designated by I, II. etc.

$$\begin{array}{c} \\ a_1 \\ \\ a_n \end{array}\begin{array}{c} \begin{array}{ccc} b_1 & \quad & b_m \end{array} \\ \begin{pmatrix} a_{11}, b_{11} & \cdots & a_{1m}, b_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1}, b_{n1} & \cdots & a_{nm}, b_{nm} \end{pmatrix} \end{array}$$

The first step in the analysis of the game is to list every potential move for each player, such as strategies a_1, $a_2 \ldots a_n$ for a player *I* and strategies b_1, $b_2 \ldots b_m$ for player *II* (a strategy for a player is a description of the decisions he will make in all the possible situations that can arise in the game). If player *I* is playing strategy a_i and player *II* is playing strategy b_j, then player *I* gets payoff a_{ij} and player *II* gets payoff b_{ji}. The outcome, in this case, is (a_{ij}, b_{ji}). This is the normal form of the game (asymmetric game or two population game). The payoff values of the game can be recorded as $n \times m$ matrix, which contains the pairs (a_{ij}, b_{ji}) as elements. This matrix is called payoff matrix. The outcome of each player is called one round, and if the game consists of more than one round, then the game is called a repeated or iterated game.

The normal form is usually used to describe simultaneous games, while the extensive form is frequently utilized to depict sequential games. Since the extensive from extended to normal form is straightforward, many extensive form games correlate to the same normal form.

2.2.8 Repeated or Iterated Game.

An iterated (or repeated) game occurs when players interact by playing a comparable stage game (such as the prisoner's dilemma) numerous time. It is an extensive form game that comprises of a number of repetitions of some base game (called a stage game).

Repeated games encapsulate the concept that a player must consider the impact of his or her current action on the future actions of other players; this impact is frequently referred to as his or her reputation. Single stage game or single shot game are names for non-repeated games.

Depending on how long the game is played for, repeated games can be categorized into two categories: finite and infinite.

2.2.8.1 Iterated Prisoner's Dilemma Game (IPD)

One of the most interesting themes in evolutionary game theory, or even in classic game theory, is the so-called prisoner's dilemma (PD) game, which can be applied to such fields as political science and environmental problems [36].

PD is a contest between two participants (prisoners), each of whom is charged with a crime. Utilizing two players, the data were standardized. The two options available to each player are to cooperate (deny) (C) or defect (confess) (D) with the other player. If the players work together, both will receive rewards (R). If both of them make a mistake, they will both be penalized (P) [37-40]. If the decision is made differently, the cooperator will be duped (S) and the defector will be enticed (T).

These two key conditions, $T > R > P > S$ and $2R > T + S$, According to Nowak [41] switching between mutual cooperation and defection results in a lower payoff for the player than choosing mutual cooperation. The payoff matrix that follows is an excellent illustration of how the PD game works.

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix} \end{array}$$

The Nash equilibrium states that defection is the dominant strategy in a single game of the prisoner's dilemma (often referred to as one shot) [42]. Thus, both players will defect earning rewards of P points rather than the R points that mutual cooperation could have yielded. When considering the decisions of other players, each player's decision in the Nash equilibrium is optimal. Every player wins because everyone gets what they want.

Iterated Prisoner's Dilemma (IPD) is an interesting variant of PD where, the dominant mutual defection strategy relies on the fact that it is a one-shot game with no future while, the key of the IPD is that the two players may meet each other again, and develop their strategies based on the previous game interactions [43]. The single dominant strategy of mutual defection is thus eliminated because players will now use more sophisticated strategies that depend on the game's history to maximize their payoffs. As a result, a player's move today may have an impact on how his or her opponent behaves in the future and, consequently, on the player's future payoffs. In truth, reciprocal collaboration can arise in the right circumstances [44, 45].

Repeated game studies have a long history. The production of the new state utilizing the outcome of the just-preceded state (initial memory) has been the subject of extensive prior study on IPD [46]. One of the most frequently cited issues with memory is ignorance of the earlier condition or delay. The generation of the new state from the preceding second state is examined in [47]. The conflict between the two businesses is a famous illustration of a two-length memory. if two businesses are vying with one another for three projects. The first project's choice and award are known. The second project, which the other company is not aware of, was then decided. Last but not least, a new choice must be taken on the third project.

Although the majority of game theory studies on the prisoner's dilemma have focused on two-player models, it is conceivable to create it with three or even more players.

2.3. <u>Background of Game Theory</u>

The history of game theory is impressive and long. This story began in 1713 when Waldegrave used a minimax mixed strategy to solve a two-player card game [48]. Madison then developed the first game-theoretic analysis in 1787 [49]. Additionally, Cournot used Nash equilibrium to solve a duopoly in 1838 [50]. And in 1881, the mathematical physician Edgeworth's publish an essay on the application of Mathematics to the Moral Sciences [51]. In this context Zermelo first demonstrated in 1913 [52] that the optimal strategy for a chess game is strictly determined.

When Von Neumann published his first work in 1928, a turning point in game theory history occurred [53]. In this paper, Von Neumann established the game extensive form and demonstrated the minimax theorem. Harsanyi later shown that Zeuthen's answer to the bargaining problem is equal to Nash's approach from 1930 [54, 55]. Then, in 1938 [56, 57], Borel proved the minimax theorem for the two-person zero-sum symmetric matrix game.

Von Neumann published the first comprehensive discussions and analyses of cooperative games in 1944 [58]. In 1946, Loomis used algebra to demonstrate the minimax theorem [59]. The prisoner dilemma was first expressed mathematically by Flood and Dresher in 1950 [60]. Additionally, Nash's initial investigations identified non cooperative games in 1950 [61-64].

George Brown used the iterative method to approximately solve the zero-sum games in 1951 [65, 66]. The first book on game theory was published in 1952 by Charles and McKinsey [67]. Also in 1952 The first famous Seminar on rational decision making, funded by the Ford university [68]. The Shapley value solution and optimal strategies for strictly competitive Stochastic Games were figured out by Shapley in 1953 [69-73]. Extensive form games were developed using the same method Kuhn did in 1953 [74, 75]. Isaacs illustrated differential games in 1954 [76-83]. Submissive game theory developed by Braithwaite was applied to philosophy in 1955 [84]. Aumann improved cooperative N-Person Games in 1959 [85, 86].

The non-transferable utility games are investigated in 1960 by Aumann and Peleg [87]. Schelling's first described the focal-point effect in 1960 [88]. Lewontin made the first official declaration of evolutionary biology applications in 1961 [89]. In 1962, Shubik used game theory to solve applications involving cost allocations [90]. Applications of game theory to insurance are first noted in 1962 by Borch's [91]. Bondareva connected game theory and linear programming in 1963 [92]. The Bargaining Set for Cooperative Games and Infinitely Repeated Games is clarified by Aumann in 1964 [93, 94].

Nash equilibrium research was first undertaken in 1964 by Carlton, Lemke, and Howson [95]. Then, in 1965, Selten refined Nash equilibrium [96]. The Kernel of a Cooperative Game introduced in 1965 by Davis and Maschler [97]. The theory of games of incomplete information arises in 1967 by Harsanyi [98-100]. In 1969 The Nucleolus of a Characteristic Function Game highlighted by Schmeidler [101].

Smith began using game theory to study biology in 1972. In The Logic of Animal Conflict and The Evolution of Fighting, Smith emphasized Evolutionarily Stable Strategy (ESS) [102-105]. In 1973, Harsanyi provided clarification in Games with Randomly Disturbed Payoffs [106]. In 1974, Aumann proposed the concepts of correlation equilibrium and correlation in randomized strategies [107]. Selten developed the trembling hand perfect equilibrium in 1975 [108]. Little child conducted studies on reducing takeoff and landing fees in 1977 [109].

Kreps and Wilson examined the extensive form of sub games with imperfect information in 1982 [110].Also; in 1982, Rubinstein thought about an alternating-offer game in which offers are made one after the other until one is accepted [111]. Smith's article Evolution and the Theory of Games was published in 1982 [112]. The automata idea is expanded upon by Neyman 1985 [113] and Rubinstein in 1986 [114] to include bounded rationality in repeated games and the prisoner's dilemma. Kohlberg and Mertens refine the standard Nash equilibrium in 1986 [115]. In 1988, Fudenberg and Kreps published two papers on the refinements of equilibrium [116, 117].

In 1990, Crawford [118, 119] presented a Nash equilibrium of mixed strategies. Fudenberg and Tirole examined perfect Bayesian equilibrium in 1991 [120]. In 1992, Binmore and Samuelson investigated the iterated games stability [121]. Lindgren and Mordahl in 1994 illustrated the spatial games from the point of view of evolutionary dynamics [122]. The behavior of strategies was investigated in the Prisoner Dilemma game in 1995 by Nowak and El-Seidy [123]. Equilibrium selection of evolutionary games was studied in 1997 by Samuelson [124]. In 1999, Sigmund and Nowak investigated the natural selection using game theory [125].

In 2000, Knez and Camerer [126] explained how to increase cooperation in Prisoner Dilemma (PD) game. In 2001, Goeree and Holt demonstrated the laboratory data for games that are played only once. These games span the standard categories: static and dynamic games with complete and incomplete information [127]. In

2003 [128], Cressman conducted extensive research on the dynamics of game evolution. In 2005 [129], Nowak and Sigmund conducted a study on direct reciprocity and natural selection. Roca, Cuesta, and Sánchez in 2009 explored the temporal and spatial effects of replicator dynamics [130]. Experimental data on how cooperation changes in endlessly replayed prisoner's dilemma games as participants gain experience are presented by Bó and Fréchette in 2011 [131]. Direct reciprocity in the alternating Prisoner's Dilemma environment was examined in 2013 by Zagorsky, Reiter, Chatterjee, and Nowak [132]. Then, in 2015 and 2016 Essam et al studied three players, relatedness and memory change of prisoner dilemma [133-135].

Ibrahim et al., 2021 [136] investigated a comprehensive review of the MCDM method and hybrid game theory technique. They explained the fundamental concepts and models of game theory to make game theory principles understandable to readers. Additionally, the definitions and models are discussed and classified into the categories of evolutionary game, cooperative game, dynamic game, and static game. As a result, the MCDM method and the hybrid game theory technique are reviewed, and a number of applications from earlier works of literature are emphasized.

2.4 Fuzzy Set

Fuzzy set theory has been shown to be a useful tool for describing situations with imprecise or ambiguous data. Fuzzy sets deal with such situations by assigning a degree to which an object belongs to a set. Zadeh proposed fuzzy set theory in 1965 [137], which is an applied approach to dealing with uncertainty. so that is assigned a membership function to any non-deterministic event.

In fuzzy sets (FS), the membership degree of an element in [0, 1] expresses the degree of belongingness of an element to a FS. Fuzzy set uses only real value $\mu_{\tilde{A}}(x) \in [0,1]$ to indicate the grade of truth membership $T_{\tilde{A}}(x)$ of FS $\tilde{A}$ defined on universe X. Zadeh, 1975 [138] proposed Interval Fuzzy Sets (IFSs) to express the uncertainty in the membership function. An interval-valued fuzzy set is FS in which the membership degree is assumed to belong to an interval.

2.4.1 Fuzzy Representation

We review the fundamental notions of fuzzy set theory, initiated by Bellman and Zadeh [137].

Definition 2.4.1 A convex fuzzy set $\tilde{A}$ on $\mathbb{R}$ is a fuzzy number if the following conditions hold [139]:

(a) Its membership function is piecewise continuous.

(b) There exist three intervals [a, b], [b, c], [c, d] such that $\mu_{\tilde{a}}$ is increasing on [a, b], equal to 1 on $[b, c]$, decreasing on $[c, d]$ and equal to 0 elsewhere.

Definition 2.4.2 [139] A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)} & , a_1 \le x \le a_2 \\ 1 & , a_2 \le x \le a_3 \\ \dfrac{(a_4 - x)}{(a_4 - a_3)} & , a_3 \le x \le a_4 \end{cases}$$

2.4.2 Arithmetic operations between two trapezoidal fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. Then, the results of applying fuzzy arithmetic on the trapezoidal fuzzy numbers as shown in the following:

- **Addition:** $\tilde{A}+\tilde{B} = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4]$
- **Subtraction:** $\tilde{A} - \tilde{B} = [a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4]$
- **Scalar Multiplication:**

- $x\tilde{A} = (xa_1, xa_2, xa_3, xa_4)$; $x > 0, x \in \mathbb{R}$
- $x\tilde{A} = (xa_2, xa_1, -xa_4, -xa_3)$; $x < 0, x \in \mathbb{R}$

- **Image of $\tilde{A}$:** $-\tilde{A} = (-a_2, -a_1, a_4, a_3)$

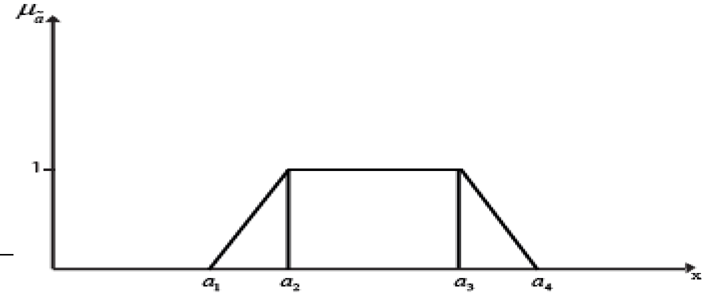

Fig (1): Truth membership function of trapezoidal fuzzy numbers

2.5. Intuitionistic Fuzzy Sets (IFSs)

Atanassov introduced intuitionistic fuzzy sets, which are extensions of fuzzy sets, in 1986 [140]. Intuitionistic fuzzy sets consider both truth membership function $T_{\tilde{A}}(x)$ and falsity membership function $F_{\tilde{A}}(x)$. Intuitionistic fuzzy sets are incapable of dealing with inconsistent and indeterminate information, both of which are common in belief systems

2.5.1 Intuitionistic Fuzzy Representation

We review the fundamental notions of intuitionistic fuzzy set theory, initiated by Atanassov (1986) [140].

Definition 2.5.1 Let X be a non-empty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{< x, T_{\tilde{A}}(x),\ F_{\tilde{A}}(x) >: x \in X\}$, where the functions $T_{\tilde{A}}(x),\ F_{\tilde{A}}(x)$:X→ $[0, 1]$, define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$, $T_{\tilde{A}}(x) + F_{\tilde{A}}(x) \leq 1$ [142].

2.5.2 Arithmetic operations between two intuitionistic fuzzy set

Let $A = \{< x, T_{\tilde{A}}(x),\ F_{\tilde{A}}(x) >: x \in X\}$, $B = \{< x, T_{\tilde{B}}(x),\ F_{\tilde{B}}(x) >: x \in X\}$, are introduced by Atanassov, 1999 [142].

Addition: A⊕B= $\{< x, T_{\tilde{A}}(x) + T_{\tilde{B}}(x) - T_{\tilde{A}}(x)\, T_{\tilde{B}}(x), F_{\tilde{A}}(x) F_{\tilde{B}}(x) >: x \in X\}$,

Difference: A⊖B= $\{x, \min(T_{\tilde{A}}(x),\ F_{\tilde{B}}(x), \max\ (F_{\tilde{A}}(x)\, T_{\tilde{B}}(x)) : x \in X\}$,

Multiplication: A⊗B= $\{< x, T_{\tilde{A}}(x)\, T_{\tilde{B}}(x),\ F_{\tilde{A}}(x) + F_{\tilde{B}}(x) - F_{\tilde{A}}(x) F_{\tilde{B}}(x) >: x \in X\}$.

2.6 Ranking function

The ranking function is a viable and efficient method of ordering fuzzy numbers. There have been several types of ranking functions introduced that are used to solve linear programming problems with fuzzy parameters. The ranking function is denoted by $F(\mathfrak{R})$, where $\mathfrak{R} :\ F(\mathfrak{R}) \rightarrow \mathfrak{R}$, and $F(\mathfrak{R})$ is the set of fuzzy numbers defined on a real line, where a natural order exist [143].

When comparing fuzzy linear programming problems using a ranking function. Typically, a crisp model that is equivalent to the fuzzy linear programming problem is defined, and then the optimal solution of this model is used as the optimal solution for the fuzzy linear programming problem. Suppose that $\tilde{a}$ and $\tilde{b}$ be two trapezoidal fuzzy numbers, then the ranking function of $F(\mathfrak{R})$ is as following:

If $\tilde{a} \geq_{\mathfrak{R}} \tilde{b}$ then $\mathfrak{R}(\tilde{a}) \geq_{\mathfrak{R}} \mathfrak{R}(\tilde{b})$

If $\tilde{a} \leq_{\mathfrak{R}} \tilde{b}$ then $\mathfrak{R}(\tilde{a}) \leq_{\mathfrak{R}} \mathfrak{R}(\tilde{b})$

If $\tilde{a} =_{\mathfrak{R}} \tilde{b}$ then $\mathfrak{R}(\tilde{a}) =_{\mathfrak{R}} \mathfrak{R}(\tilde{b})$

Where $\tilde{a}$ and $\tilde{b}$ are in $F(\mathfrak{R})$ also in the same way we can write $\tilde{a} \leq_{\mathfrak{R}} \tilde{b}$ iff $\tilde{b} \geq_{\mathfrak{R}} \tilde{a}$.

Lemma 2.6.1: let $\mathfrak{R}$ be any ranking function, then:

- $\tilde{a} \geq_{\mathfrak{R}} \tilde{b}$ iff $\tilde{a} - \tilde{b} \geq_{\mathfrak{R}} 0, iff - \tilde{b} \geq_{\mathfrak{R}} - \tilde{a}$.
- $\tilde{a} \geq_{\mathfrak{R}} \tilde{b}$ and $\tilde{c} \geq_{\mathfrak{R}} \tilde{d}$, then $\tilde{a} + \tilde{c} \geq_{\mathfrak{R}} \tilde{b} + \tilde{d}$.

2.7 Neutrosophic Set

We frequently encounter incomplete and indeterminate information in real-world scenarios, so the information cannot be represented solely by the membership and non-membership functions. to get out of this situation Smarandache [144, 145] proposed the neutrosophic set for dealing with incomplete and indeterminate information. It is distinguished by three independent degrees: truth membership degree (T), indeterminacy-membership degree (I), and falsity membership degree (F). The decision makers want to increase the degree of truth-membership while decreasing the other ones.

Smarandache demonstrated in 2005 [146] the neutrosophic sets are generalizations of intuitionistic fuzzy sets and inconsistent intuitionistic fuzzy sets (picture fuzzy set, ternary fuzzy set). He also stated that the result obtained by applying neutrosophic operators differs from the result obtained by applying intuitionistic fuzzy operators for the same problem. He explains that indeterminacy in intuitionistic fuzzy operators is ignored when transforming from neutrosophic components to intuitionistic fuzzy components. As a result, Neutrosophic Set (NS) [146] is a generalization of both the fuzzy set [137] and the intuitionistic fuzzy set [138].

NS is a philosophical branch that studies the origin, nature, and scope of neutralities, as well as their interactions with various ideational spectra [147]. It has the potential to be a general framework for analyzing uncertainty in data sets, including big data sets [148].

2.7.1 Neutrosophic Representation

This subsection introduces some fundamental definitions in neutrosophic set theory [149].

Definition 2.7.1 Let U be the universe of discourse, then the neutrosophic set A is on object having the form A={ $x: T_{\tilde{A}(x)}, I_{\tilde{A}(x)}, F_{\tilde{A}(x)}, x \in U$ }where the function $T, I, F: U \to]^{-}0, 1^{+}[$ respectively the degree of membership, the degree of indeterminacy and degree of non-membership of the element $x \in U$ to the set A with the condition.

$$0^{-} \leq T_{\tilde{A}(x)}, I_{\tilde{A}(x)}, F_{\tilde{A}(x)} \leq 3^{+}$$

Definition 2.7.2 Let U be the universe of discourse, then the single valued neutrosophic set A is on object having the form A={$x: T_{\tilde{A}(x)}, I_{\tilde{A}(x)}, F_{\tilde{A}(x)}, x \in U$ }where the function $T, I, F: U \to]^{-}0, 1^{+}[$ respectively the degree of membership, the degree of indeterminacy and degree of non-membership of the element $x \in U$ to the set A with the condition [149].

$$0 \leq T_{\tilde{A}(x)}, I_{\tilde{A}(x)}, F_{\tilde{A}(x)} \leq 3$$

Definition 2.7.3 [150] The trapezoidal neutrosophic number $\widetilde{A}$ is a neutrosophic set in $\mathbb{R}$ with the following truth (T), indeterminacy (I) and falsity (F) membership functions as shown in Fig (2):

$$T_{\widetilde{A}}(x) = \begin{cases} \frac{\alpha_{\widetilde{A}}(x-a_1)}{a_2-a_1} & : a_1 \leq x \leq a_2 \\ \alpha_{\widetilde{A}} & : a_2 \leq x \leq a_3 \\ \alpha_{\widetilde{A}}\left(\frac{x-a_3}{a_4-a_3}\right) & : a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{\widetilde{A}}(x) = \begin{cases} \frac{(a_2 - x + \theta_{\widetilde{A}}(x - a_1'))}{a_2 - a_1'} & : a_1' \leq x \leq a_2 \\ \theta_{\widetilde{A}} & : a_2 \leq x \leq a_3 \\ \frac{(x - a_3 + \theta_{\widetilde{A}}(a_4' - x))}{a_4' - a_3} & : a_3 \leq x \leq a_4' \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\widetilde{A}}(x) = \begin{cases} \frac{(a_2-x+\beta_{\widetilde{A}}(x-a_1''))}{a_2-a_1''} & : a_1'' \leq x \leq a_2 \\ \beta_{\widetilde{A}} & : a_2 \leq x \leq a_3 \\ \frac{(x-a_3+\beta_{\widetilde{A}}(a_4''-x))}{a_4''-a_3} & : a_3 \leq x \leq a_4'' \\ 1 & \text{otherwise} \end{cases}$$

Where $\alpha_{\widetilde{A}}$, $\theta_{\widetilde{A}}$ and $\beta_{\widetilde{A}}$ represent the maximum degree of truthiness, minimum degree of indeterminacy and minimum degree of falsity, respectively, $\alpha_{\widetilde{A}}$, $\theta_{\widetilde{A}}$ and $\beta_{\widetilde{A}} \in [0,1]$.

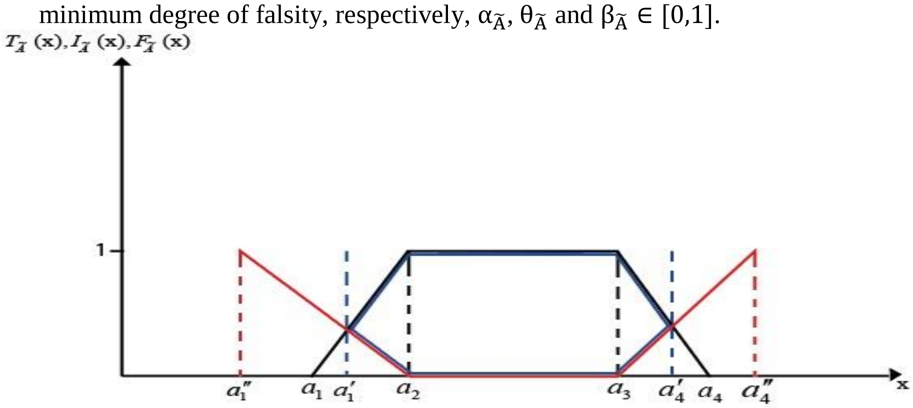

Fig (2): Truth, indeterminacy and falsity membership functions of trapezoidal neutrosophic number $\widetilde{A}$.

Definition 2.7.4 The mathematical operations on two trapezoidal neutrosophic numbers [150].

$\widetilde{A} =< a_1, a_2, a_3, a_4; \alpha_{\widetilde{A}}, \theta_{\widetilde{A}}, \beta_{\widetilde{A}} >$ and $\tilde{B} =< b_1, b_2, b_3, b_4; \alpha_{\widetilde{B}}, \theta_{\widetilde{B}}, \beta_{\widetilde{B}} >$ are as follows:

$$\widetilde{\mathbf{A}} + \widetilde{\mathbf{B}} = \; < (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} >$$

$$\widetilde{\mathbf{A}} - \widetilde{\mathbf{B}} = \; < (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} >$$

$$\widetilde{\mathbf{A}}^{-1} = \; < (\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}); \alpha_{\widetilde{A}}, \theta_{\widetilde{A}}, \beta_{\widetilde{A}} > \text{Where } \widetilde{A} \neq 0)$$

$$\boldsymbol{\lambda}\widetilde{\mathbf{A}} = \begin{cases} < \lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \alpha_{\widetilde{A}}, \theta_{\widetilde{A}}, \beta_{\widetilde{A}} > : \lambda > 0 \\ < \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; \alpha_{\widetilde{A}}, \theta_{\widetilde{A}}, \beta_{\widetilde{A}} > : \lambda < 0 \end{cases}$$

$$\widetilde{\boldsymbol{A}}\widetilde{\boldsymbol{B}} = \begin{cases} < (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} > \\ \qquad \text{if } (a_4 > 0, b_4 > 0) \\ < (a_1 b_4, a_2 b_3, a_3 b_2, a_4 b_1); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} > \\ \qquad \text{if } (a_4 < 0, b_4 > 0) \\ < (a_4 b_4, a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} > \\ \qquad \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

$$\frac{\widetilde{\mathbf{A}}}{\widetilde{\mathbf{B}}} \begin{cases} < \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} > \\ \qquad \text{if } (a_4 > 0, b_4 > 0) \\ < \left(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} > \\ \qquad \text{if } (a_4 < 0, b_4 > 0) \\ < \left(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}\right); \alpha_{\widetilde{A}} \wedge \alpha_{\widetilde{A}}, \theta_{\widetilde{A}} \vee \theta_{\widetilde{A}}, \beta_{\widetilde{A}} \vee \beta_{\widetilde{A}} > \\ \qquad \text{if } (a_4 < 0, b_4 < 0) \end{cases}$$

There are many other works in this field, such as:

Hussein et al. [151] studied a NLP problem to handle with both incomplete and indeterminate information, where they converted the neutrosophic model into the corresponding crisp based on the NS parameters. Darehmiraki [152] proposed a new ranking method for solving the LP problem that incorporates neutrosophic numbers into all objective function and constraint coefficients. Edalatpanah et al [153] proposed a new algorithm for addressing the Single-Valued NLP problem. They also presented a numerical example to demonstrate the efficacy of the new algorithms. Khatteret al [154] presented the α, β, γ cut of single-valued triangular neutrosophic numbers and introduced the arithmetic operations of triangular neutrosophic numbers using α, β, γ cut. The proposed approach converts each triangular neutrosophic number in linear programming problem to weighted value using possibilistic mean to determine the crisp linear programming problem. Badr et al. [155] contributed streamlined neutrosophic linear programming models and proposed ranking functions for both maximization and minimization neutrosophic linear programming.

2. Background of Game Theory with Neutrosophic

Bhattacharya et al. [156] explored the possibilities and developed justifications for applying neutrosophic game theory principles as a generation of the fuzzy game theory model, in order to gain a better understanding of the Israel-Palestine conflict in terms of the goals and governing strategies of both sides. They expanded on an earlier attempt by Yakir Plessner (2001) to provide a game-theoretic explanation of this problem and go on to argue a neutrosophic adaptation of the standard 2x2 zero-sum game-theoretic model in order to determine an optimal outcome.

Pramanik et al. (2014) [157] investigates deals with the enduring conflict between India and Pakistan since 1947 over Jammu and Kashmir. By taking into consideration the influence of the USA and China on crisis dynamics, they examined the evolution and status of the dispute as well as the dynamics of the India-Pakistan relationship. They talked about the various study groups' and persons' proposed solutions. They have expanded on the concept of the game-theoretic model of the Jammu and Kashmir conflict in a neutrosophic environment. To correctly understand the Jammu and Kashmir conflict in terms of the goals and strategies of each side, they have investigated the possibilities and developed arguments for the application of the principle of neutrosophic game theory. Standard 2×2 zero-sum game theoretic model used to identify an optimal solution.

In 2017, Abu-Faty et al. [158] proposed a new approach for solving Multi Criteria Group Decision Making (MCGDM) problems that involve participant competition in the presence of data ambiguity. In this approach, the data uncertainty is first represented using Single Valued Neutrosophic Sets (SVNs). The SVN environment is then handled via the TOPSIS technique. Due to its proficiency in handling situations involving competition, game theory is then employed to determine the optimal solution. For the competitive MCGDM issue in a neutrosophic environment, they discovered the Nash equilibrium (two-player non-constant sum game). The suggested technique makes it possible to use neutrosophic sets conjunction with game theory principles to solve competitive MCGDM problems under uncertain conditions.

The Neutrosophic soft group discussions were the focus of Selvakumari et al, 2018 [159] study. Neutrosophic fuzzy soft game, a brand-new game model that is based on Neutrosophic soft group theory, has been proposed. They concentrated on discussing a class of two person zero-sum games with Neutrosophic fuzzy soft payoffs.

Khalifa created two-person zero-sum matrix games in a single valued neutrosophic environment in 2019 [160]. They suggested a method for solving the game problem given ambiguous and contradictory information.

Debnath introduced the idea of interval-valued neutrosophic soft sets in game theory strategy in 2020 [161]. Interval-valued neutrosophic soft sets (in short ivn-soft sets) is a generalization of interval-valued intuitionistic fuzzy soft sets. They described the two-person ivn-soft game, which can be used to resolve problems involving ambiguous, insufficient, inconsistent, and inaccurate data. Then they presented a method of solving the games based on ivn-soft saddle points, ivn-soft lower values, ivn-soft upper values, ivn-soft dominated strategy, and ivn-soft Nash equilibrium. They also extended the two-person ivn-soft game to n-person ivn-soft game.

Arias et al. in 2020 [162] provided a neutrosophic model in matrix form for non-cooperative games that generalizes a previous solution utilizing triangular intuitionistic fuzzy payoffs. The indeterminacy membership function, which is not restricted to any condition of dependency between the membership and non-membership functions, was defined due to this generalization. Particularly, the matrix's elements are payoffs of triangular neutrosophic numbers with a single value. The neutrosophic solution has the advantage of allowing for a more precise expression of the ambiguity that is common in political conflicts.

A neutrosophic method was put up by Martinez et al. in 2021 [163] to manage the contradictions that arise during project negotiation and execution. In addition to explicitly incorporating the indeterminacy that exists in the modeling of this type of activity, this methodology has the advantages that data can be entered in the form of language phrases. When parties are negotiating qualitative content, this methodology is helpful.

In 2021 [164] Bhaumik et al. developed and analyzed a matrix game with many objectives, and they used a linguistic technique to solve the problem in a single-valued neutrosophic environment. In place of the crisp data utilized in previous research, they offered a problem-oriented example to support their designed methodologies with successful real-life implications.

For the classification of criminal data and its optimization, Remani et al. 2022 [165] extended the prisoners' dilemma. Based on the neutrosophic logic principle, a confusion matrix-based optimization technique for crime data using a game theory model predicts the individual involved in the crime and clusters them into confess, not confessed, and in deterministic stages. With this method, they first mapped neutrosophic logic into the criminal justice system to remove the uncertainty surrounding crime clustering. They then divided the crime data pairs with two neutrosophic values for the offenders into three clusters. They improved the disjoint clustering, which will be carried out based on the ratio of intra-cluster to inter-cluster distances. They have put the suggested strategy into practice.

3. Literature Analysis

In this section we analysis 69,256 papers in the fields of game theory, 2741 papers in the fields of neutrosophic, and 18 papers in the fields of game theory with neutrosophic based on Scopus and Web of Science databases, which are the world's largest use of citation databases. It is also the most respected platform for the analysis of peer-reviewed literature: scholarly publications, conference proceedings and books. As shown in Table 2 below, the articles on game theory or Neutrosophic groups have different fields of applications such as social sciences, chemistry, physics, medicine, mathematics,...etc. It also shows an increase in publication in recent years in these two topics, while articles on game theory with Neutrosophic groups have applications only in more relevant fields such as engineering, mathematics and computer science and are published as articles. Egypt and some other Arab countries rank in neural game theory with Neutrosophic applications.

Table 2: Analysis using the Web of Science and Scopus databases of game theory and neutrosophic sets by subject, country, years, etc.

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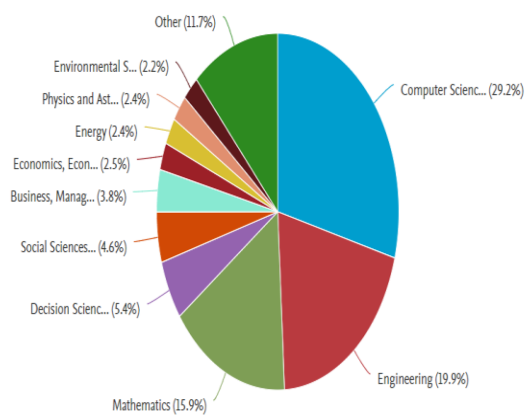
<td>
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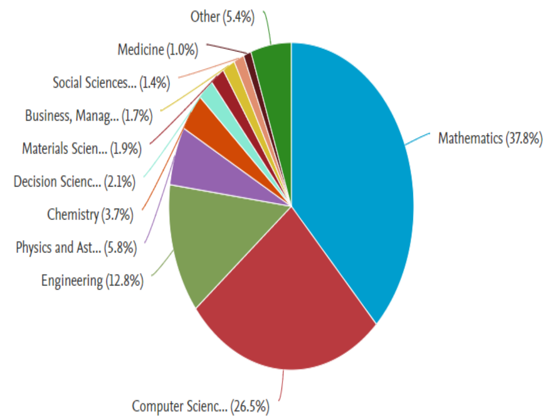
<td>
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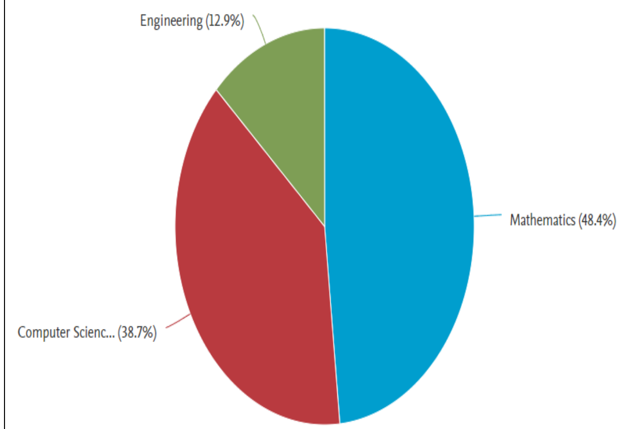
<td>
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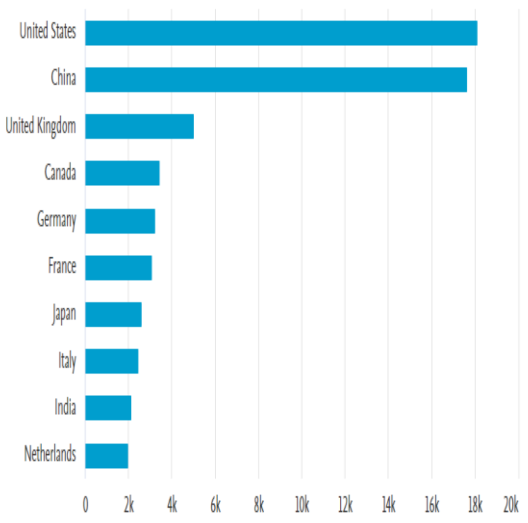
<td>
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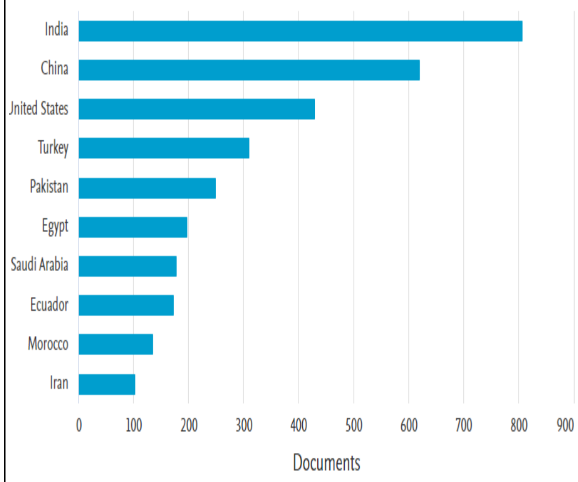
<td>
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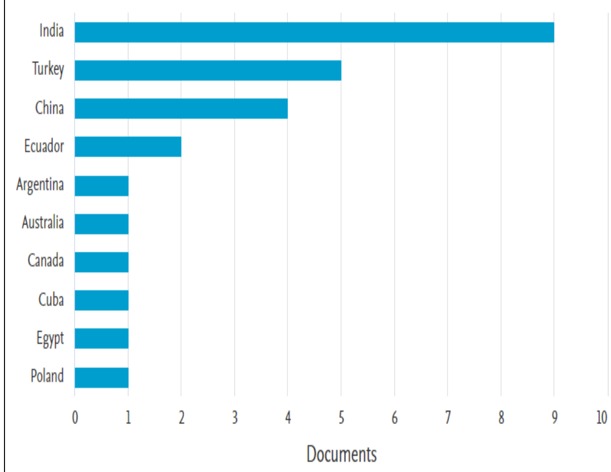
<td>
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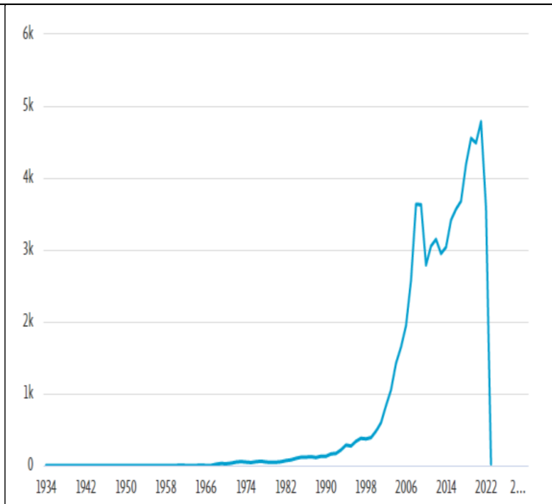
<td>
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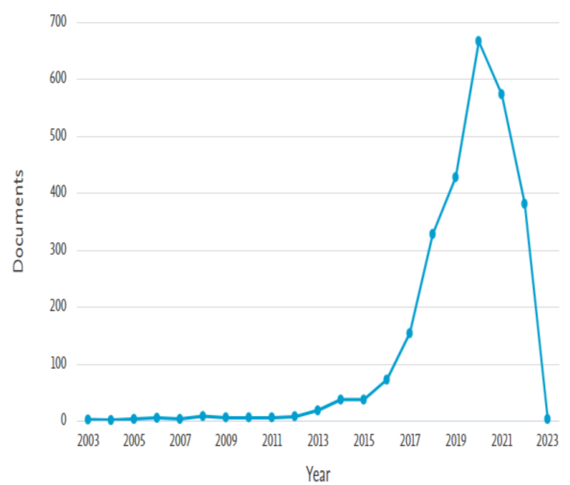
<td>
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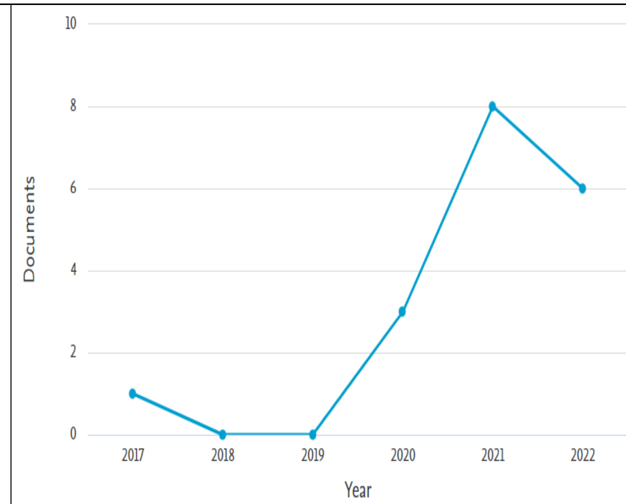
<td>
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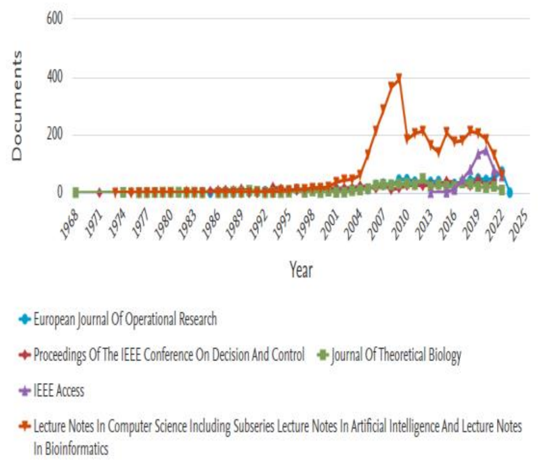
<td>
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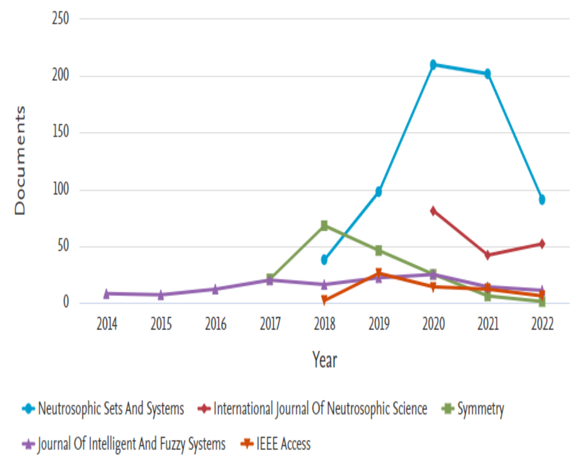
<td>
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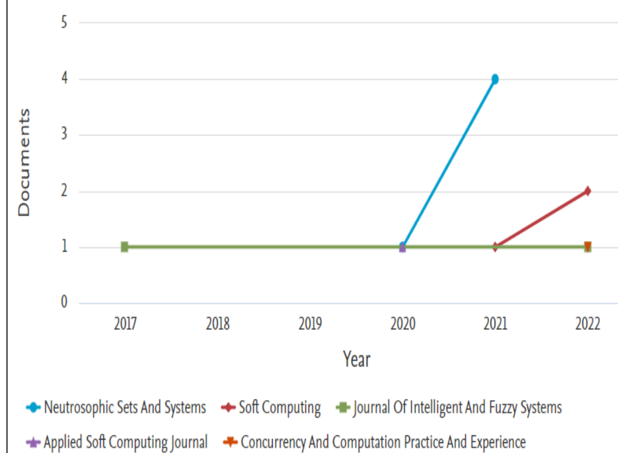
<td>
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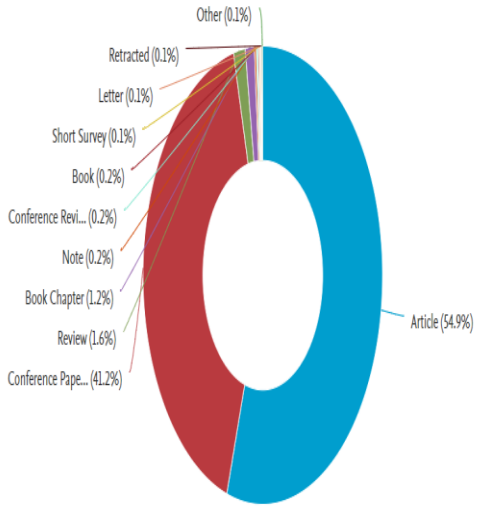
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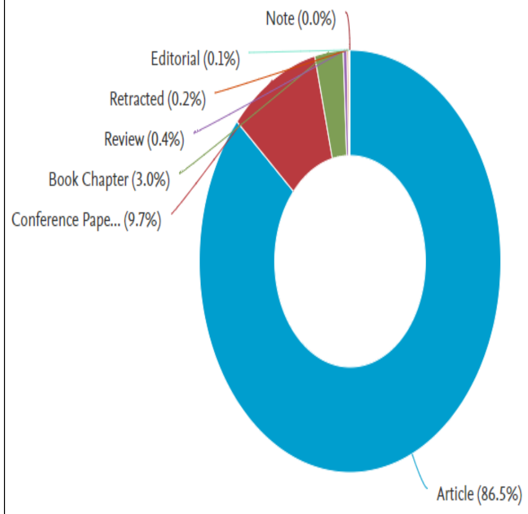
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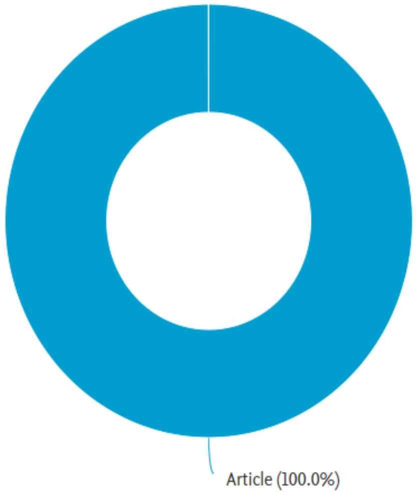
Article (100.0%)

Conclusion

In this paper, we provided a comprehensive background on game theory, fuzzy sets and neutrosophic sets. Then we demonstrated some of the work that has been done on game theory and neutrosophic sets. The last part of the paper dealt with the analysis of work using the two major scientific databases (Web of Science and Scopus) by subject, country, years, etc. The analysis showed that articles on game theory or neural groups have different fields of application such as social sciences, chemistry, physics, medicine and mathematics. ..etc. It also showed an increase in publication in recent years in these two topics, while articles on game theory with Neutrosophic groups have applications only in more related fields such as engineering, mathematics, and computer science and are published as articles. It also turned out that there is a significant gap in the research done in the field of game theory with neutrosophic application. Egypt and some other Arab countries may rank in neural game theory with Neutrosophic applications.

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