Cubic Observer Based Robust Tracking Control of Single Link Manipulator

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Abstract: The work presented here is aimed to design a cubic observer based state feedback controller for tracking control of single link manipulator . A cubic observer based novel state feedback controller is designed for angular position control. The dynamic model of the manipulator involves modeling of the flexible link and servo motor dynamics. The overall design has been elaborated under state space framework. A cubic error correction term is introduced in the Luenberger observer dynamics for fast convergence of steady state error. Robust stability criteria is obtained for the overall system using Lyapunov equation. Simulation study is performed using Python . An experimental validation of the proposed controller is carried out by Controller -In-Loop simulation using Raspberry Pi 3B model and a geared DC servo motor . Experimental results are presented at the end of this paper.
Keywords: Cubic Observer, State feedback , Tracking, Raspberry Pi, CIL(Controller-in-Loop).

1. Introduction

In recent years controlling of a more complex flexible link manipulator model is becoming a widely popular research problem for testing new control strategies due to its high nonlinearities. A single link manipulator system consist of two subsystems: mechanical link-arm and a DC servo motor at the rotary joint. By applying proper input voltage to the motor, desired torque can be produced to regulate the angular position of the link-arm. There are lots of different approaches available to control the angular position of link manipulators[1-2]. From conventional PID to different advanced control methods like Fuzzy gain-scheduling, Neural network based control etc has been very popular control techniques for link manipulators[3-5]. With the recent development of advanced control strategies state-space based control, sliding mode control, H∞ control and hybrid control received much research attention for controlling such highly nonlinear system having disturbances and unknown dynamics[6]. Observer based control strategy is well practiced state space controller for such systems whose output state cannot be directly measured from the plant [6-9]. Therefore, estimation of those states is required for implementing feedback control. Observers are also used in fault detection, disturbance rejection. In the literature, use of unknown input, robust, functional and optimal observer based control as well as sampled, delayed and nonlinear observer based sliding mode control have been reported [10-13]. In case of observer based control the main design objective of the observer must ensure that the estimated states should converge to the actual state values within shorter period of time. Now for the sake of maintaining the stability of the system, nonlinear error dynamics of observers are given less attention. A nonlinear observer was implemented as a generalization to the linear observers. This observer has a cubic nonlinear term in its error dynamics which converges faster that the linear term[14-15]. When error norms are equal, this cubic observer yields a faster descending Lyapunov function. In this paper, Cubic Luenberger-type nonlinear observer model is implemented to estimate state values of the single link manipulator model and a reference tracking operation of that model is simulated . Also software simulation results are verified by performing Controller-in-loop simulation .

The proposed controller in the present work attains the following control objectives

i) An cubic observer based state feedback controller is designed for asymptotic tracking of a desired trajectory.

ii)Faster state estimation in presence of model uncertainties present in the system ensuring robust controller performance.

This paper has been organized as follows, section 2 elaborates the state-space modeling of the single link manipulator system . In section 3, the cubic observer based state feedback controller design is discussed. In section 4, simulation study of the single link manipulator system is shown. Section 5 presents the CIL simulation results where controller signal is generated using Raspberry Pi and a dc servo motor is driven by that control signal. Last section contains the conclusive summary of the work.

2. Modelling of the Single link manipulator system

Single link manipulator system is combination of two sub systems,Manipulator system and Servo motor system. The mathematical modelling of single link manipulator system can be achieved by mathematically analysing these two systems independently and then combining them both. Block diagram for the same is given in Figure 1.

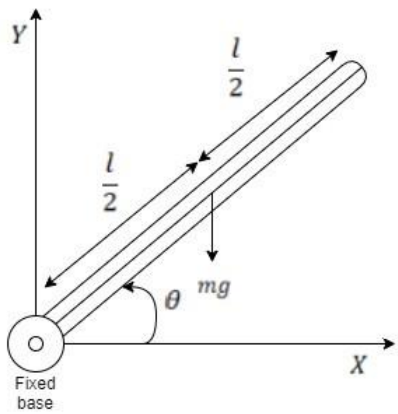


Figure.1 Single link manipulator model.

From the Kinematic analysis of any link manipulator system, the dynamics of the system can be expressed by the following equation,

$$M\ddot{\theta} + C\dot{\theta} + g = \tau \qquad (1)$$

For single link manipulator system after linearizing, equation (1) becomes,

$$2ml^2\ddot{\theta} + 6b\dot{\theta} + 3m\lg = 6\tau_m \qquad (2)$$

Here, m = mass of the link-arm, l = length of the link-arm, b = damping factor/friction constant, g = gravitational acceleration, τ_m = torque produced by the motor, θ = angular position of the link-arm.

From the torque equation of motor,

$$\tau_m = K_\tau i_m \qquad (3)$$

Here, K_τ = Torque constant of motor, i_m = current in motor's armature circuit. Substituting (3) in equation (2) and simplifying it is obtained as below

$$J\ddot{\theta} + 6b\dot{\theta} + p\theta = 6K_\tau i_m \qquad (4)$$

Here, $J = 2ml^2$ and p = 3mlg

Using KVL on the motor armature loop,

$$v = L_m \frac{di_m}{dt} + K_b\dot{\theta} + R_m i_m \qquad (5)$$

Here, V = voltage applied to the armature circuit, L_m = inductance of motor, K_b = back-emf constant, R_m = resistance of motor.

In a Single link manipulator system, the angular position of the arm can be controlled by controlling the torque produced by the driving servo motor. Now for controlling the torque, a proper controlled voltage should be applied to the armature circuit. So here we chose $\upsilon(t)$ as controlled variable or controller input to the system. Now the output of the system is the angular position of the link-arm.

The system states have been chosen as,

$$X_1 = \theta$$
$$X_2 = \dot{\theta}$$
$$X_3 = i_m$$

Now the state equations can be derived by equation (4) and (5),

$$\dot{X}_1 = \dot{\theta}$$
$$\dot{X}_2 = \ddot{\theta} = -\frac{p}{J}\theta - \frac{6b}{J}\dot{\theta} + 6\frac{K_\tau}{J} i_m$$

$$\dot{X}_3 = \dot{i}_m = -\frac{K_b}{L_m}\dot{\theta} - \frac{R_m}{L_m} i_m + \frac{1}{L_m} v$$

Output equation of the system,

$$Y = X_1 = \theta$$

From the above state-equations the state-space model of the single-link manipulator system becomes,

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{p}{J} & -\frac{6b}{J} & 6\frac{K_\tau}{J} \\ 0 & -\frac{K_b}{L_m} & -\frac{R_m}{L_m} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_m} \end{bmatrix} v \tag{6}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

For simulating the single link manipulator model, the model parameters taken is given in Table. 1 ,

Table.1. Values of model parameters

Parameter	Value
R_m	10 Ω
L_m	0.5 H
m	1 kg
b	5 N.m.s/rad
K_T	140 N.m/A
K_b	140 V.s/rad
l	2.07 m
J	8.6
p	126.42

3. Cubic observer based state feedback controller design

Luenberger-type linear observer design

Consider an LTI system described by the state-equation,

$$\begin{aligned} \dot{X}(t) &= AX(t) + Bu(t) \\ Y(t) &= CX(t) \end{aligned} \tag{7}$$

Here vector $X(t) \in R^n$, $Y(t) \in R^m$, $u(t) \in R^u$ represents the state variable , measured output and exogenous input vector respectively and $A_{n*n}, B_{n*u}, C_{m*n}, B_{n*u}$ represents the state, input ,output and disturbance input matrices of the plant respectively .

If the system (7) is observable then the simple Luenberger observer dynamics for state estimation of the system is given by,

$$\dot{\hat{X}}(t) = (A - LC)\hat{X}(t) + Bu(t) + LY \quad (8)$$

Here, $\hat{X}(t) \in R^n$ is the estimated state-vector and L is the linear observer gain matrix.

Now the estimation error and error dynamics of this linear observer is described by the following equations,

$$\begin{aligned} e(t) &= X(t) - \hat{X}(t) \\ \dot{e}(t) &= (A - LC)e(t) \end{aligned} \quad (9)$$

Luenberger-Type cubic observer design

The proposed cubic observer for a LTI system is given by

$$\dot{\hat{X}}_C(t) = (A - L_C C)\hat{X}_C(t) + Bu(t) + L_C Y - e_c^{\,T}(t)C^T \phi C e_C(t) M C e_C(t) \quad (10)$$

Now the estimation error and error dynamics of this cubic observer is described by the following equations,

$$\begin{aligned} e_C(t) &= X_C(t) - \hat{X}_C(t) \\ \dot{e}_C(t) &= (A - L_C C)e_C(t) + e_c^{\,T}(t)C^T \phi C e_C(t) M C e_C(t) \end{aligned} \quad (11)$$

From the error dynamics it is clear that the cubic term will vanish sooner than the linear term as t→∞ and near the origin i.e. $e_c(0) = e(0) = 0$, this cubic observer converges to the linear observer.

Cubic observer based full-state feedback controller design

For controlling the system, the estimated state is fed to the input via a state-feedback gain matrix $K^{n_u \times n}$. So, the control law becomes

$$u(t) = -K\hat{X}_C(t) \quad (12)$$

If the closed loop control system with linear observer is stable then the closed loop control system with cubic observer will also be stable.

Let the reference signal is r(t) = r,

Now the control law becomes,

$$u(t) = r - K\hat{X}_C(t) \quad (13)$$

To eliminate steady-state error from the closed-loop system, the reference signal is fed to the system via a Proportional gain $\bar{N}$.

$\bar{N}$ is defined as follows,

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 \end{bmatrix}^T \quad (14)$$

$$\bar{N} = N_u + KN_x$$

The modified control law,

$$u = \bar{N}r - K\hat{X}_c \quad (15)$$

Convergence and robust stability criteria

The convergence and stability criteria along with parameter formulation given for the proposed cubic observer has been summarized below. Block diagram realization of the proposed controller is shown in Figure.2

For given $C^T \phi C \geq 0$ the global stability of the error dynamics can be achieved by defining,

$$M = -\gamma P^{-1} C^T \phi \qquad (16)$$

Here, γ>0 is an arbitrary scalar, $\phi \geq 0$ is a performance scalar and P is a positive definite symmetric matrix obtained by solving Lyapunov equation,

$$(A - L_C C)^T P + P(A - L_C C) = -Q < 0, \forall Q > 0, L_C = L \qquad (17)$$

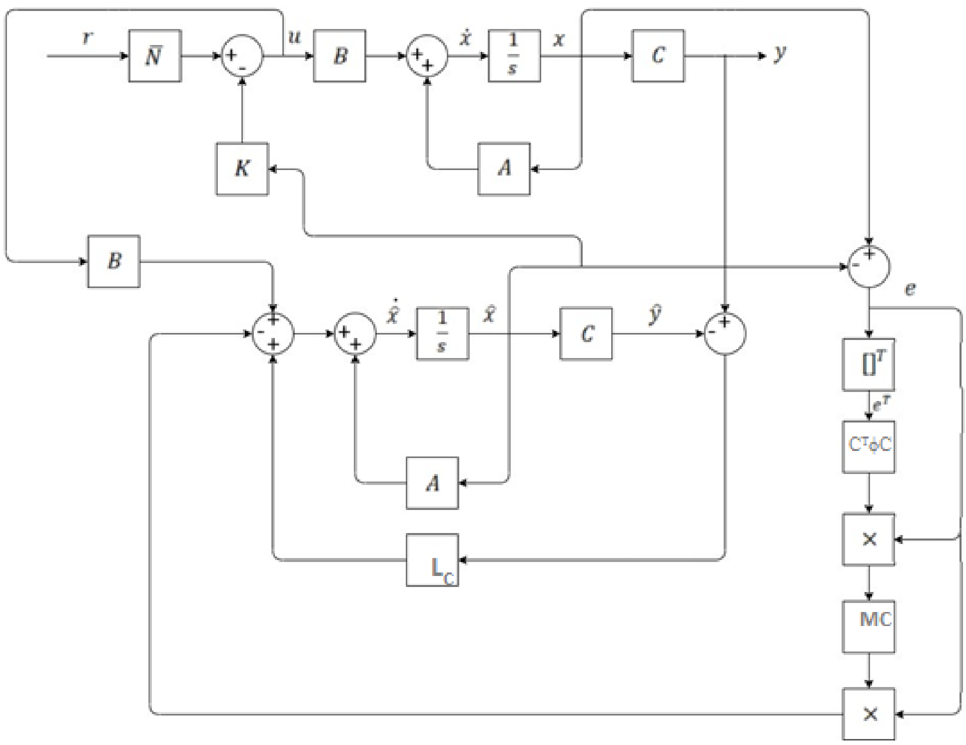


Figure.2 Block diagram of cubic observer based control model.

4. Simulation Study

The single-link manipulator state space model is expressed with the following state matrices.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ -14.70 & -3.49 & 97.67 \\ 1 & -16.29 & -20 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Controllability matrix of the system is,

$$M_C = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 195.348 \\ 0 & 195.348 & 4588.12 \\ 2 & -40 & -2380.07 \end{bmatrix}$$

Here we find $|M_c| \neq 0$ and rank of $M_c = 3$. Therefore the system is controllable.

Observability matrix of the system is,

$$M_O = \begin{bmatrix} C & CA & C^2A \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -14.7 & -.4884 & 97.674 \end{bmatrix}$$

Here we find $|M_o|\neq 0$ and rank of M_o = 3. Therefore the system is observable.

Controller parameters

The controller gain parameters has been obtained below.

State feedback gain matrix

The open-loop system poles are at -0.176, -11.66+39.17j and -11.66-39.17j. We want to adjust the feedback gains for the system poles to be so placed that its response has damping factor ζ = 0.707 and settling time τ_s = 0.7sec. Poles required for desired response are -5.71, -5.72+5.71j and -5.72-5.71j.

Now, the state feedback gain matrix is obtained as below

$$K = \begin{bmatrix} 0.8813295 & -7.78993013 & -3.1692 \end{bmatrix}$$

Cubic Observer parameters

The observer gain (L) has been obtained by pole place placement method as detailed below.

For designing observer gain matrix, desired pole locations are taken 4-10 times of the controller poles to make it converge the estimation error to 0 faster.

Poles required for desired observer response are -57.1, -57.2+57.1j and -57.2-57.1j.

So the desired polynomial equation

$$(s+57.1)(s+57.2-57.1j)(s+57.2+57.1j) \tag{18}$$

Now, the observer gain matrix

$$L = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix}$$

Desired characteristics equation is

$$|sI - A + LC| = \left| \begin{pmatrix} s+a & -1 & 0 \\ 14.7+b & s+3.4884 & -97.674 \\ c & 16.279 & s+20 \end{pmatrix} \right| \tag{19}$$

$$= s^3 + (l_1 + 23.4884)s^2 + (23.4884 l_1 + l_2 + 1674.503046)s + (1659.803046 l_1 + 20 l_2 + 97.674 l_3 + 294)$$

Comparing (418) and (19) we get the observer gain matrix

$$L = \begin{pmatrix} 148.0116 \\ 7913.43128856 \\ -319.85231786 \end{pmatrix}$$

Cubic observer gain,

$$L_c = L = \begin{pmatrix} 148.0116 \\ 7913.43128856 \\ -319.85231786 \end{pmatrix}$$

Let, a positive definite symmetric matrix

$$Q = 10 I_3$$

Now solving Lyapunov equation (3.7) we get,

$$P = \begin{pmatrix} -0.030973 & 0.415624 & 0.0617675 \\ 0.415624 & -0.0188625 & 0.270504 \\ 0.0617675 & 0.270504 & 0.212799 \end{pmatrix}$$

Considering γ = 2, ϕ = 10,M is obatined as

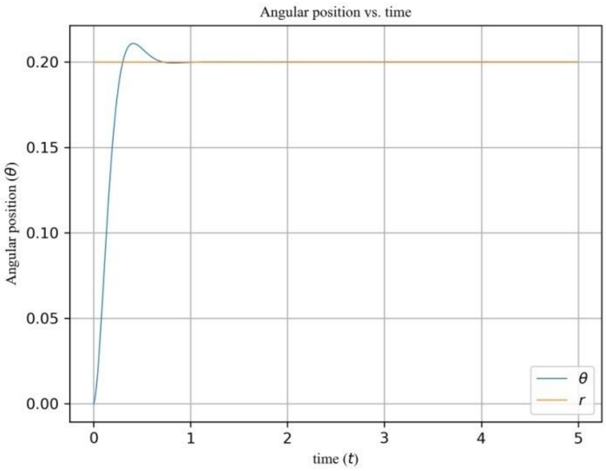
$$M = \begin{bmatrix} 684.68 \\ 2.19 \\ 4.77 \end{bmatrix}$$

Calculating precompensated Proportional gain

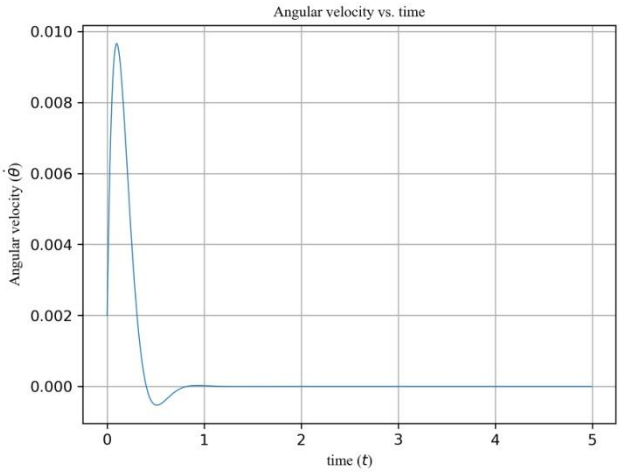
The precompensated forward Proportional gain is obtained,

$$\bar{N} = 0.0024999999999999933$$

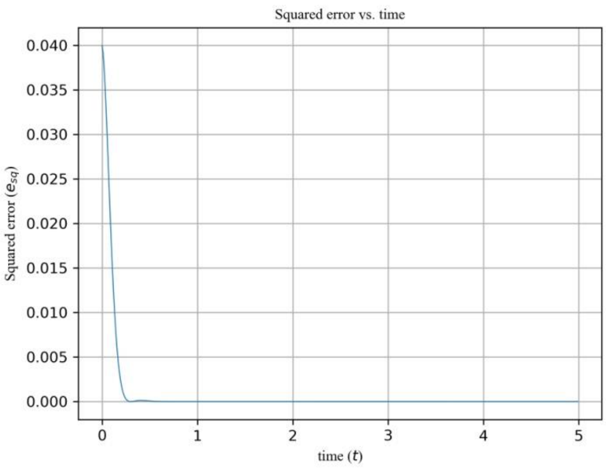
A simple reference tracking operation is performed to test the performance of the cubic observer based control model. A step signal of magnitude 0.2 is used as reference. For simulation initial angular position was taken 0 and the initial angular velocity was taken 0.002 rad/s. All initial states for cubic observer were set to 0. Simulation is carried out. The simulation results are presented below. From figure 3(a) it is clear that the controlled system is giving desired response. Figure 3(b) shows how fast angular velocity of the system is estimated at the very beginning of the simulation and with the control course it eventually settled to 0 while reaching the set-point. Figure 3(c) shows variation of squared error of the whole control process within the simulation time. Figure 3(d) presents the control effort required to control the single link manipulator plant.



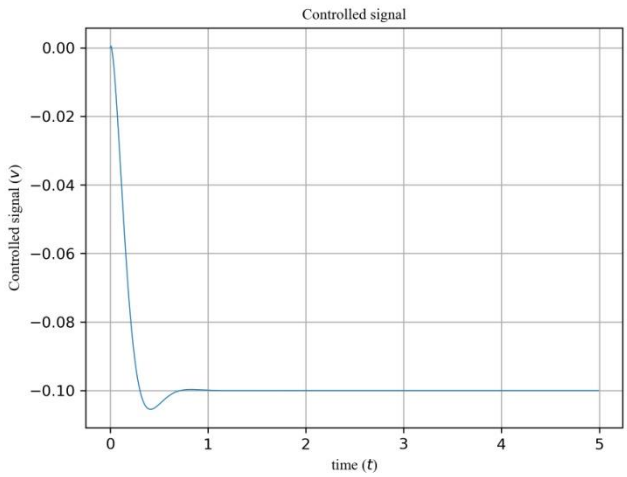
(a)



(b)



(c)



(d)

Figure.3 Simulation results of the cubic observer based single link manipulator reference tracking: (a) Reference tracking of angular position, (b) Variation of angular velocity during tracking, (c) Squared error plot, (d) Plot of generated control signal to perform tracking operation.

5. Controller-in-loop (CIL) simulation

CIL simulation of the proposed control model is carried out with a microcomputer module, Raspberry Pi. Raspberry Pi has been set up to act like a standalone embedded controller. A DC servomotor is taken as the real time plant and driven by generates appropriate control signal by proposed control methodology. Angular position of that dc servo motor is controlled depending upon the generated control signal. Experimental setup for the above CIL simulation model is shown below in Figure.4 and the real time control signal generated by the embedded controller is shown in Figure. 5. It almost matches with the control signal obtained from simulation study in Fig. 3(d). It can be said that the embedded cubic observer based state feedback controller can provide has attained desired control performance.

Figure.4 Experimental setup for CIL simulation

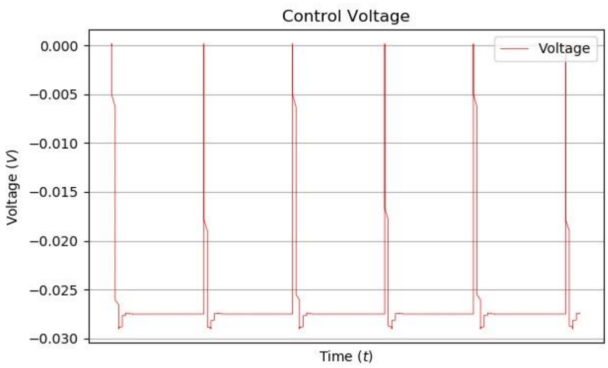


Figure.5 Real-time plot of control signal generated by the embedded controller.

8.Conclusion

This paper presented the implementation of a cubic observer based state estimation and state feedback control of a Single link manipulator system and showed how it can affect the performance of the system. Due to the nonlinear term in error dynamics of cubic observer ,state estimation error decays at a faster rate than the linear ones. Also a CIL simulation has been done to verify the performance of the controller in practice. This work can be formulated as an optimization problem while choosing different control parameters in the future work.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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