CERTAIN CLASSES OF ANALYTIC FUNCTIONS ASSOCIATED WITH SAIGO'Sq-NTEGRAL OPERATOR

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ABSTRACT: - In the present paper we investigate several classes of analytic functions by using Saigo's integral operator in q-calculus and also find some inequalities forfunctions of defined classes.

Key Words: - Analytic functions, Fractional q-calculus, operators (Saigo's) Coefficient bounds.

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INTRODUCTION: The q-analysis theory has recently been utilized in numerous disciplines of science and engineering. In q-theory, the fractional q-calculus is an extension of the regular fractional calculus. A great work with q-calculus and fractional q-calculus operators has been investigated by Srivastava [4].In a previous paper Purohit and Raina [8], Investigated applications of fractional q-calculus operators to defined certain new classes of functions which are analytic in the open disk U={ξϵC‖ξ|<1}.Several others have previously released new classes of analytical functions with the help ofq-calculus operators. Purohit [7], Purohit and Raina [9]-[11] gived related work and added various classes of univalent and multi valently analytic functions in open unit disk U.

For any more inquiries on the analytic classes, we refer to [1], [5]-[6] and [12][16]for functions described by applying q-calculus operators and subject related to the this work. In the current inquiry, we are planning to develop few new classes of analytic functions applying the Saigo integral operator in q calculus. The results obtained must also provide the coefficient in equalities. First we use the main notations and definitions intheq-calculus which are relevant to grasp the object of the study.

The q-shifted factorials for any complex number σ, are delimited y

$$(\sigma;q)_m=\prod_{j=0}^{m-1}(1-q^j\sigma);m\in N \text{ and } (\sigma;q)_0=1, \tag{1.1}$$

And with regard to the basic analog of the gamma function

$$(q^\sigma;q)_m=\frac{\Gamma_q(\sigma+m)}{\Gamma_q(\sigma)},\quad (m>0) \tag{1.2}$$

In which the q-gamma function is set by

$$\Gamma_q(\sigma)=\frac{(q;q)_\infty(1-q)^{1-\sigma}}{(q^\sigma;q)_\infty},\quad (0<q<1) \tag{1.3}$$

The recurrence relationship specified by Gaspar and Rahman[3] for the q-gamma function is

$$\Gamma_q(1+\sigma)=\frac{(1-q^\sigma)\Gamma_q(\sigma)}{(1-q)} \tag{1.4}$$

If|q|<1, equation (1.1) shall continue to play a role m=∞ as an infinite product of convergence

$$(\sigma;q)_\infty=\prod_{j=0}^{\infty}(1-\sigma q^j),$$

And we have

$$(\sigma;q)_m=\frac{(\sigma;q)_\infty}{(\sigma q^m;q)_\infty};\ (m\in N\cup\{\infty\}).$$

$$\int_0^{\xi} f(y)\,d_q y = \xi(1-q)\sum_{k=0}^{\infty} q^k\ f(\xi q^k). \tag{1.5}$$

$$\int_0^{\xi} f(y)\,d_q y = \xi(1-q)\sum_{k=1}^{\infty} q^{-k}\ f(\xi q^{-k}). \tag{1.6}$$

The Fractional q-calculus operators

Definition2.1. For $\mathrm{R}(\kappa) > 0$, σ and J be real or complex, the Saigo fractional integral operators inq-calculus is defined by Garg and Chanchlani [2] as

$$\mathrm{I}_q^{\kappa,\sigma,J}\mathrm{f(x)} = \frac{\mathrm{x}^{-\sigma-1}}{\Gamma_q(\kappa)}\int_0^x (tq/x;q)_{\kappa-1}\sum_{m=0}^{\infty}\frac{(q^{\kappa+\sigma};q)_m (q^{-J};q)_m}{(q^{\kappa};q)_m (q;q)_m}\, q^{(J-\sigma)m}(-1)^m\, q^{-\left(\frac{m(m-1)}{2}\right)}\left(\frac{t}{x}-1\right)_m f(t)d_q t, \tag{1.7}$$

and

$$\mathrm{K}_q^{\kappa,\sigma,J}\mathrm{f(x)} = \frac{\mathrm{q}^{\left(\frac{\kappa(\kappa+1)}{2-\sigma}\right)}}{\Gamma_q(\kappa)}\int_0^x (x/t;q)_{\kappa-1}\, t^{-\sigma-1}\sum_{m=0}^{\infty}\frac{(q^{\kappa+\sigma};q)_m (q^{-J};q)_m}{(q^{\kappa};q)_m (q;q)_m}\, q^{(J-\sigma)m}(-1)^m\, q^{-\left(\frac{m(m-1)}{2}\right)}\left(\frac{x}{qt}-1\right)_m f(tq^{1-\kappa})d_q t, \tag{1.8}$$

For q→ 1, the operators (2.1) and (2.2) reduce to Saigo's fractional integral operators $\mathrm{I}^{\kappa,\sigma,J}$ and $\mathrm{K}^{\kappa,\sigma,J}$ respectively which are defined by Saigo [17].

$$\mathrm{I}_q^{\kappa,\sigma,J}\mathrm{f(x)} = \mathrm{x}^{-\sigma}(1-q)^{\kappa}\sum_{m=0}^{\infty}\frac{(q^{\kappa+\sigma};q)_m (q^{-J};q)_m}{(q;q)_m}\, q^{(J-\sigma+1)m}\sum_{n=0}^{\infty} q^n\,\frac{(q^{\kappa+m};q)_n (q^{-J};q)_m}{(q;q)_n} f(xq^{n+m}), \tag{1.9}$$

and

$$\mathrm{I}_q^{\kappa,\sigma,J}\mathrm{f(x)} = \mathrm{x}^{-\sigma} q^{-\kappa(\kappa+1)/2}(1-q)^{\kappa}\sum_{m=0}^{\infty}\frac{(q^{\kappa+\sigma};q)_m (q^{-J};q)_m}{(q;q)_m}\, q^{Jm}\sum_{n=0}^{\infty} q^{n\sigma}\,\frac{(q^{\kappa+m};q)_n}{(q;q)_n} f(xq^{-\kappa-n-m}), \tag{2.0}$$

Now we define fractional integral operators for a function of complex variable z in q- Calculus as

Definition2.2. For R (k) > 0 ,σ and J be real or complex, the fractional q-integral operators for a function of complex variable z define by

$$\mathrm{I}_q^{\kappa,\sigma,J}\mathrm{f(z)} = \frac{\mathrm{z}^{-\sigma-1}}{\Gamma_q(\kappa)}\int_0^z (tq/z;q)_{\kappa-1}\sum_{m=0}^{\infty}\frac{(q^{\kappa+\sigma};q)_m (q^{-J};q)_m}{(q^{\kappa};q)_m (q;q)_m}\, q^{(J-\sigma)m}(-1)^m\, q^{-\left(\frac{m(m-1)}{2}\right)}\left(\frac{t}{z}-1\right)_m f(t)d_q t, \tag{2.1}$$

and

$$\mathrm{K}_q^{\kappa,\sigma,J}\mathrm{f(z)} = \frac{\mathrm{q}^{\left(\frac{\kappa(\kappa+1)}{2-\sigma}\right)}}{\Gamma_q(\kappa)}\int_0^z (z/t;q)_{\kappa-1}\, t^{-\sigma-1}\sum_{m=0}^{\infty}\frac{(q^{\kappa+\sigma};q)_m (q^{-J};q)_m}{(q^{\kappa};q)_m (q;q)_m}\, q^{(J-\sigma)m}(-1)^m\, q^{-\left(\frac{m(m-1)}{2}\right)}\left(\frac{z}{qt}-1\right)_m f(tq^{1-\kappa})d_q t,$$

Remark 2.1.(i) If we put $\sigma = -\kappa$ in (2.5) the n integral operator $I_q^{\kappa,\sigma,J}$ reduce in to Integral operator I_q^{κ} defined by Purohit and Raina in[8],that is

$$I_q^{\kappa,-\kappa,J} f(z) = I_q^{\kappa,} f(z)$$

(ii) If we put σ=0 in (2.5) then integral operator $I_q^{\kappa,\sigma,J}$ reduce in to integral operator $I_q^{J,\kappa}$. Recently defined by Purohit etal. That is

$$I_q^{\kappa,0,J} f(z) = I_q^{J,\kappa,} f(z)$$

Now, we introduce the image of the power function under fractional q– Integral operators $I_q^{\kappa,\sigma,J}$ and $K_q^{\kappa,\sigma,J}$

Remark 2.2. (i) If R(μ+1)>0 and R(μ−σ+ J+1)>0, then

$$I_q^{\kappa,\sigma,J}(z^{\mu}) = \frac{\Gamma_q(\mu+1)\Gamma_q(\mu-\sigma+J+1)}{\Gamma_q(\mu-\sigma+1)\Gamma_q(\mu+\kappa+J+1)} z^{\mu-\sigma}, \tag{2.2}$$

(ii) If R (σ−μ)>0 and R (J−μ)>0, then

$$K_q^{\kappa,\sigma,J}(z^{\mu}) = \frac{\Gamma_q(\sigma-\mu)\Gamma_q(J-\mu)}{\Gamma_q(-\mu)\Gamma_q(\sigma+\kappa+J-\mu)} z^{\mu-\sigma} q^{-\kappa\mu-\kappa(\kappa+1)/2}, \tag{2.3}$$

Where R (k)>0, σ and J be real or complex.

New classes of functions Let A_m represents the function class of the form

$$f(\xi) = \xi + \sum_{k=m+1}^{\infty} a_k \xi^k, (m \in N), \tag{2.4}$$

Which are analytic and univalent in open unit disk U. Above, let A_m^- highlights the sub class of A_m imposing of analytical and univalent functions particulate in the form

$$f(\xi) = \xi - \sum_{k=m+1}^{\infty} a_k \xi^k, (a_k \geq 0, m \in N),$$

We announce here the alike classes of functions connecting the operator (2.5):

$$G_m^{\kappa,\sigma,J}(\lambda,\zeta,q) = \left\{ f \in A_m^-; \left| \frac{1}{\zeta} \left\{ \frac{\xi D_{q,\xi} F_q^{(\kappa,\sigma,J,\lambda)}(\xi)}{F_q^{(\kappa,\sigma,J,\lambda)}(\xi)} - 1 \right\} \right| < 1 \right\}, \tag{2.5}$$

Where $F_q^{(\kappa,\sigma,J,\lambda)}(\xi) = \lambda \xi D_{q,\xi}\left(I_q^{(\kappa,\sigma,J)} f(\xi)\right) + (1-\lambda) I_q^{(\kappa,\sigma,J} f(\xi),$ (2.6)

And $R(\kappa) > 0, \zeta \in C \setminus \{0\}, 0 \leq \lambda < 1, 0 < q < 1$ and $\xi \in U$.

Also

$$H_m^{\kappa,\sigma,J}(\lambda,\zeta,q) = \left\{ f \in A_m^-; \left| \frac{1}{\zeta} \left\{ D_{q,\xi}\left(I_q^{(\kappa,\sigma,J)}(\xi)\right) + \lambda \xi D_{q,\xi}^2 \left(I_q^{(\kappa,\sigma,J)}(\xi)\right) - 1 \right\} \right| < 1 \right\}, \tag{2.7}$$

Where $R(\kappa) > 0, \zeta \in \mathrm{C} \setminus \{0\}, 0 \le \lambda < 1, 0 < \mathrm{q} < 1$ and $\xi \in \mathrm{U}$.

We now attain the subsequent coefficient bounds for functions of the form (3.2) to belong to the classes $\mathrm{G}_{\mathrm{m}}^{\kappa,\sigma,J}(\lambda,\zeta,\mathrm{q})$ and $\mathrm{H}_{\mathrm{m}}^{\kappa,\sigma,J}(\lambda,\zeta,\mathrm{q})$ (marked above).

Theorem2.1. A function $\mathrm{f} \in \mathrm{A}_{\mathrm{m}}^{--}$ lies in the class $\mathrm{G}_{\mathrm{m}}^{\kappa,\sigma,J}(\lambda,\zeta,\mathrm{q})$ if

$$\frac{1}{\mathrm{A}_1 B_1}\sum_{k=m+1}^{\infty} A_k B_k a_k \le 1,$$

Where $\mathrm{A_k}$ and $\mathrm{B_k}$ are given by

$$\mathrm{A_k} = \frac{\Gamma_q(k+1)\Gamma_q(k-\sigma+J+1)}{\Gamma_q(k-\sigma+1)\Gamma_q(k+\kappa+J+1)},$$

And

$$\mathrm{B_k} = \frac{[\lambda_q(1-q^{k-\sigma-1})+1-q][q(1-q^{k-\sigma-1})+|\varsigma|(1-q)]}{(1-q)^2},$$

The result is sharp.

Proof. Let $\mathrm{f}(\xi) \in \mathrm{G}_{\mathrm{m}}^{\kappa,\sigma,J}(\lambda,\zeta,\mathrm{q})$ then on using (3.3),we have

$$\mathrm{R}\left\{\frac{\xi D_{q,\xi} F_q^{(\kappa,\sigma,J,\lambda)}(\xi) - F_q^{(\kappa,\sigma,J,\lambda)}(\xi)}{F_q^{(\kappa,\sigma,J,\lambda)}(\xi)}\right\} > -|\zeta|,$$

Since

$$D_{q,\xi}\xi^{\mu} = \left(\frac{1-\mathrm{q}^{\mu}}{1-\mathrm{q}}\right)\xi^{\mu-1} = [\mu]_{\mathrm{q}}\xi^{\mu-1}$$

$$\mathrm{F}_{\mathrm{q}}^{(\kappa,\sigma,J,\lambda)}(\xi) = \mathrm{A}_1\left[\frac{\lambda q(1-q^{-\sigma})+1-q}{1-q}\right]\xi^{1-\sigma} - \sum_{k=m+1}^{\infty} A_k a_k \left[\frac{\lambda q(1-q^{k-\sigma-1})+1-q}{1-q}\right]\xi^{k-\sigma},$$

And

$$\xi D_{q,\xi} F_q^{(\kappa,\sigma,J,\lambda)}(\xi) = \mathrm{A}_1\left[\frac{\lambda q(1-q^{-\sigma})+1-q}{(1-q)^2}\right](1-q^{1-\sigma})\xi^{1-\sigma} - \sum_{k=m+1}^{\infty} A_k a_k \left[\frac{\lambda q(1-q^{k-\sigma-1})+1-q}{(1-q)^2}\right]\xi^{k-\sigma}$$

Now on making use of above relations, we get

$$R\left\{\frac{\mathrm{A}_1\left[\frac{q(\lambda q(1-q^{-\sigma})+1-q)(1-q^{\sigma})}{(1-q)^2}\right]-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{q(\lambda q(1-q^{k-\sigma-1})+1-q)(1-q^{k-\sigma-1})}{(1-q)^2}\right]\xi^{k-1}}{\mathrm{A}_1\left[\frac{\lambda q(1-q^{-\sigma})+1-q)}{(1-q)}\right]-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{\lambda q(1-q^{k-\sigma-1})+1-q}{(1-q)}\right]\xi^{k-1}}\right\}>|\varsigma|$$

On putting ξ=r with noting that the denominator is positive for r=0 and also remains positive for 0<r<1 so that on letting $r\to 1^{-}$,we get

$$-\mathrm{A}_1\left[\frac{q(\lambda q(1-q^{-\sigma})+1-q)(1-q^{-\sigma})}{(1-q)^2}\right]+\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{q(\lambda q(1-q^{k-\sigma-1})+1-q)(1-q^{k-\sigma-1})}{(1-q)^2}\right]$$
$$<|\zeta|\left(\mathrm{A}_1\left[\frac{\lambda q(1-q^{-\sigma})+1-q}{(1-q)}\right]-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{\lambda q(1-q^{k-\sigma-1})+1-q}{(1-q)}\right]\right), \qquad \text{………………(2.8)}$$

On simplifying above inequality, we have

$$\frac{1}{\mathrm{A}_1 B_1}\sum_{k=m+1}^{\infty}A_k B_k a_k<1,$$

Which is desired in equality (2.6)?

Conversely suppose that inequality (3.6) holds and letting |ξ|=1,we have

$$\left|\frac{\xi D_{q,\xi}F_q^{(\sigma,\gamma,\lambda)}(\xi)-F_q^{(\sigma,\gamma,\lambda)}(\xi)}{F_q^{(\sigma,\gamma,\lambda)}(\xi)}\right|$$
$$=\left|\frac{\mathrm{A}_1\left[\frac{q(\lambda q(1-q^{-\sigma})+1-q)(1-q^{-\sigma})}{(1-q)^2}\right]-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{q(\lambda q(1-q^{k-\sigma-1})+1-q)(1-q^{k-\sigma-1})}{(1-q)^2}\right]\xi^{k-1}}{\mathrm{A}_1\left[\frac{\lambda q(1-q^{-\sigma})+1-q}{(1-q)}\right]-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{\lambda q(1-q^{k-\sigma-1})+1-q}{(1-q)}\right]\xi^{k-1}}\right|$$
$$<\frac{|\zeta|\left(\mathrm{A}_1\left[\frac{\lambda q(1-q^{-\sigma})+1-q}{(1-q)}\right]-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{\lambda q(1-q^{k-\sigma-1})+1-q}{(1-q)}\right]\right)}{\mathrm{A}_1\left[\frac{\lambda q(1-q^{-\sigma})+1-q}{(1-q)}\right]-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{\lambda q(1-q^{k-\sigma-1})+1-q}{(1-q)}\right]\xi^{k-1}}=|\zeta|$$

Hence , by the maximum modulus principle and the condition,we can say that

$\mathrm{f}(\xi)\in \mathrm{G}_{\mathrm{m}}^{\kappa,\sigma,J}(\lambda,\zeta,\mathrm{q})$ and the external function is assumed by

$$\mathrm{f}(\xi)=\xi-\frac{A_1B_1}{A_{m+1}B_{m+1}}\xi^{m+1};\ (\mathrm{m}\in \mathrm{N})$$

Where A(σ,γ,k,q) and B(λ,J,k,q) are given by(2.7) and (2.8)respectively.

Theorem2.2.A function f∈A_m^-belongs to the class $H_m^{\kappa,\sigma,J}(\lambda,\zeta,q)$ iff

$$\sum_{k=m+1}^{\infty} A_k E_k a_k \le |\zeta| + A_1 E_1 - 1,$$

Where A_k is given by (3.7) and E_k is given by

$$E_k = \frac{(1-q^{k-\sigma-1})[\lambda(1-q^{k-\sigma-1})+(1-q)]}{(1-q)^2}, \qquad \text{(2.9)}$$

The result is sharp.

Proof. First suppose that $f(\xi)\in H_m^{\kappa,\sigma,J}(\lambda,\zeta,q)$ the non using (3.5), we get

$$R\left\{D_{q,\xi}\left(I_q^{(\kappa,\sigma,J)} f(\xi)\right)+\lambda\xi D_{q,\xi}^2\left(I_q^{(\kappa,\sigma,J)} f(\xi)\right)-1\right\} > -|\zeta|,$$

Now we obtain

$$D_{q,\xi}\left(I_q^{\kappa,\sigma,J}f(\xi)\right)=A_1\left[\frac{1-q^{1-\sigma}}{1-q}\right]\xi^{-\sigma}-\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{1-q^{k-\sigma}}{1-q}\right]\xi^{k-\sigma-1},$$

And

$$\lambda\xi D_{q,\xi}^2\left(I_q^{\kappa,\sigma,J}f(\xi)\right)=\lambda\left[A_1\frac{(1-q^{1-\sigma})(1-q^{-\sigma})}{(1-q)^2}\xi^{-\sigma}-\sum_{k=m+1}^{\infty}A_k a_k\frac{(1-q^{k-\sigma-1})(1-q^{k-\sigma})}{(1-q)^2}\xi^{k-\sigma-1}\right],$$

Further on making use of above inequalities in (3.15), we get

$$R\left\{A_1\left(\frac{1-q^{1-\sigma}}{1-q}\right)\left(1+\frac{\lambda(1-q^{-\sigma})}{1-q}\right)\xi^{-\sigma}-\sum_{k=m+1}^{\infty}A_k a_k\left(\frac{1-q^{k-\sigma}}{1-q}\right)\left(1+\frac{\lambda(1-q^{k-\sigma-1})}{1-q}\right)\xi^{k-\sigma-1}-1\right\}>-|\varsigma|$$

On putting $\xi=r$ and lettingr$\to$1,weget

$$\sum_{k=m+1}^{\infty}A_k a_k\left[\frac{(1-q^{k-\sigma})[\lambda(1-q^{k-\sigma-1})+(1-q)]}{(1-q)^2}\right]<|\zeta|+A_1\left[\frac{(1-q^{1-\sigma})[\lambda(1-q^{-\sigma})+1-q]}{(1-q)^2}\right]-1,$$

that implies

$$\sum_{k=m+1}^{\infty}A_k E_k a_k<|\zeta|+A_1E_1-1.$$

Which is desired inequality (3.13).Conversely suppose that inequality (3.6) holds and letting$|\xi|=1$,wehave

$$\left|D_{q,\xi}\left(I_q^{\kappa,\sigma,J}f(\xi)\right)+\lambda\xi D_{q,\xi}^2\left(I_q^{\kappa,\sigma,J}f(\xi)\right)-1\right|=\left|A_1\left(\frac{1-q^{1-\sigma}}{1-q}\right)\left(1+\frac{\lambda(1-q^{-\sigma})}{1-q}\right)\xi^{-\sigma}\right.$$
$$\left.-\sum_{k=m+1}^{\infty}A_k a_k\left(\frac{1-q^{k-\sigma}}{1-q}\right)\left(1+\frac{\lambda(1-q^{k-\sigma-1})}{1-q}\right)\xi^{k-\sigma-1}-1\right|\le|J|$$

Hence, by the maximum modulus principle and the condition (3.7),we can say that
$f(\xi)\in H_m^{\kappa,\sigma,J}(\lambda,\zeta,q)$ and the external function is assumed by

$$f(\xi)=\xi-\frac{|\zeta|+A_1E_1-1}{A_{m+1}E_{m+1}}\xi^{m+1};\ (m\in N), \qquad \text{……………………………………………………….(2.10)}$$

Where A_k and E_k are given by respectively.

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REFERENCES

[1] H.aldwe byandm. Darus, A sub class of harmonic univalent functions associated with q-analogue of dziok-Srivastava operator, isrnmath. Anal.,**2013**(2013), article id 382312.
[2] M.gargandl.Chanchalani, Kober fractional q-derivative operators,Matematiche,**16**(2011),13-26.
[3] G. Gasper and M. Rahman, Basic Hypergeometric Series, Encyclopedia Math. Appl. 35, Cambridgeuniv.Press,Cambridge(1990).
[4] H.M. Srivastava, Operators of basic (or q-) calculus and fractional q-calculus and their applications in Geometric Function Theorey of complex analysis, Iran. J. Sci. Technol., Trans. A,Sci.,**2020**(2020).
[5] Y.luchkoandj.J.Trujillo,Caputo-typemodificationoftheerdely-Kober fractional derivative, Fract.Calc.Appl. Anal.,**10** (2007),249-267.
[6] G. Murugusundaramoorthy, C. Selvaraj and O.S. Babu, Subclasses of starlike functions Associated with fractional q-calculus operators, J.complexanal.,**2013**(2013),articleid572718.
[7] S.D. Purohit, A new class of multivalently analytic functions associated with fractional q-calculusoperators, Fract. Differ. Calc.,**2**(2) (2012),129—138.
[8] S.D.purohit andr .K.Raina, Certain subclasses of analytic functions associated with fractional q-calculus operators, Math.Scand.,**109**(2011),55—70.
[9] S.D.purohit and r.K.Raina ,Fractional q-calculus and certain subclass of univalent analytic functions, Mathematica, **55**(78)(2013),62—74.
[10] S.D.purohit and r.K.Raina,Some classes of analytic and multivalent functions associatedwithq-derivative operators,actauniv.Sapientiae,Math.,**6**(1)(2014),5—23.
[11] S.D.purohit and r.K.Raina, On a subclass of p-valent analytic functions involving fractional q-calculus operators,kuwaitj.Sci.,**42**(1)(2015),1—15.
[12] S.D.Purohit,M.M.Gour, S.joshiandd.L.Suthar, certain classes of analytic functions bound with kober operators inq-calculus, journal of mathematics, **2021**(2021),articleid 3161275.
[13] R.K. Saxena, R.K. Yadav, S.D. Purohit and S.L. Kalla, Kober fractional q-integral operator ofthebasicanalogueoftheh-function,Rev.Tec.Fac.Ing.Univ.Zulia.,**28**(2)(2005),154—158.
[14] R.K. Saxena, S.D. Purohit and S.L. Kalla, On generalized Weyl fractional q-integral operator involving generalized basic hypergeometric functions, Fract.Calc.Appl.Anal.,**2**(2008),129—142.
[15] K.A. Selva kumaran, S.D. Purohit and A. Secer, Majorization for a class of analytic functions defined by q-differentiation, Math. Probl.Eng.,**2014**(2014),articleid 653917.
[16] K.A. Selva kumaran, S.D. Purohit, A. Secer and M. Bayram, Convexity of certain q-integral operators of p-valent functions, Abstr.Appl.Anal.,**2014**(2014),articleid 925902.
[17] M. Saigo, A remark on integral operators involving the Gauss hypergeometric functions, Math.Rep.Coll.Gen.Educ.,kyushuuniv.,**11**(1978)135–143.

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