Neutrosophic Digraph with Generalized Modus Tollens

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Article History: *Do not touch during review process(xxxx)*

Abstract: In this paper, we generalize modus tollens method for neutrosophic digraph by using neutrosophic rule and define degree of a vertex in neutrosophic digraph, Indegree of a vertex in neutrosophic digraph, out degree of a vertex in neutrosophic digraph and generalized modus tollens are discussed.

Keywords: Generalized Modus Tollens, Neutrosophic graph, Neutrosophic digraph.

1. Introduction

Neutrosophical logic is a new branch that studies the origin, nature, and field of indeterminacy as well as the interaction of all the different spectra that a person can imagine in a case so that this logic takes into account every idea with its opposite (its opposite) with the spectrum of indeterminacy. The main idea of neutrosophical logic is to distinguish each logical statement in three The dimensions are truth (T) in degrees, false (F) in degrees, and indeterminacy (I) in degrees. We express it in the form (T, I, F) and put them under the field of study, which gives a more accurate description of the data of the phenomenon studied, as this reduces the degree of randomness in the data that would reach high-accuracy results that contribute to making the most appropriate decisions for decision makers. For A is an element, it may be an idea, an adjective, a proposition, a theory, or... we express it in the neutrosophic as. A is the anti-event of anti A where (A, neut A, anti A) While neut A is not A and also not anti A, it is an indefinite event related to A. For example, if A represents a team's victory, anti A represents its loss and neut A represents its tie with the other team. Also, if A represents voting for a candidate then anti A represents voting against that candidate while neut A represents never voting or voting with a blank card or a void card.

2.Basic Preliminaries:

Let $G = (V, E)$ such that V is the set of vertices. A vertex is also called a node or element, and E *is the* set of edges. An edge is pair (x, y) of vertices in V.

Definition2.1. (Neutrosophic Set) [1]:

Let V be a given set. A *neutrosophic set* A in V is characterized by a truth membership function $T_A(x)$, an indeterminate membership function $I_A(x)$ and a false membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are fuzzy sets on V. That is, $T_A(x)$: $V \to [0,1]$, $I_A(x)$: $V \to [0,1]$ and $F_A(x)$: $V \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition2.2. (Neutrosophic Graph) [1]: Let V be a given set. Also, assume E be a given set with respect to V. A *neutrosophic graph* is a pair $G = (A, B)$, where $A: V \to [0,1]$ is a neutrosophic set in V and $B: E \to [0,1]$ is a neutrosophic set in E such that

- $T_B(xy) \le min\{T_A(x), T_A(y)\}$,
- $I_B(xy) \le min\{I_A(x), I_A(y)\}$,
- $F_B(xy) \le max\{F_A(x), F_A(y)\}$, for all $\{x, y\} \in E$. V is called vertex set of G and E is called *edge set* of G, respectively.

Example 2.2. Let *G: (A, B)* be a (4,5) neutrosophic graph, Where $V(G) = \{v_1, v_2, v_3, v_4\}$ and $E(G) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1, v_4v_2\}$.

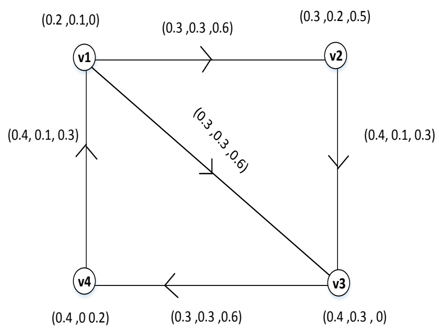

Figure 1" A neutrosophic graph with A and B"

Definition2.3[3]:

Mathematical model a self-contained set of formulas and/or equations based on an approximate quantitative description of real phenomena and created in the hope that the behavior it predicts will be consistent with the real behavior on which it is based.

- Is indispensable in many applications
- Is successful in many further applications
- Gives precision and direction for problem solution
- Enables a thorough understanding of the system method.
- Prepares the way for better design or control of a system.
- Allows the efficient use of modern computing capabilities.

Definition2.4:

A neutrosophic directed graph is a directed graph which has at least one edge to be an indeterminacy. A neutrosophic oriented graph is neutrosophic directed graph having no symmetric pair of directed indeterminacy lines. A neutrosophic subgraph *H* of neutrosophic *G* is a subgraph *H* which itself a neutrosophic graph

Example2.3: "Neutrosophic Digraph"

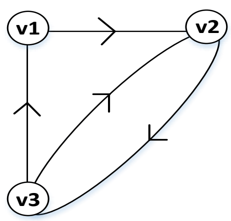

Figure 2" Neutrosophic digraph"

Example 2.4: The following example shows an example of neutrosophic graph represented as neutrosophic relation matrix M.

Table (1)" Neutrosophic relation matrix M"

M_G	b_1	b_2
a_1	(0.6,0.2,0.3)	(0.3,0.6,0.5)
a_2	(0.5,0.2,0.4)	(0,0,0,)
a_3	(0.6,0.3,0)	(0.3,0,0.8)

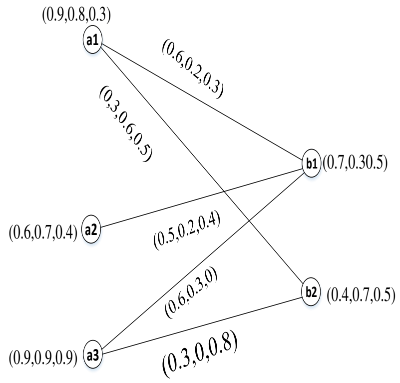

Figure3"Neutrosophic graph"

Example2.5: "Neutrosophic Digraph"

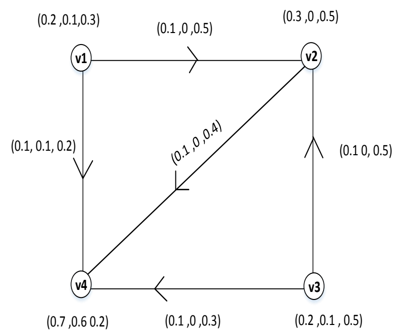

Figure4"Neutrosophic Digraph"

Table (2)"Degree of neutrosophic digraph**"**

vertices	id (v_i)	od (v_i)	Degree
v_1	0.5	0.465	1.36
v_2	0.44	0.275	0.715
v_3	0.5	0.56	1.06
v_4	0.86	0.5	1.36

Example2.6: "Neutrosophic Digraph"

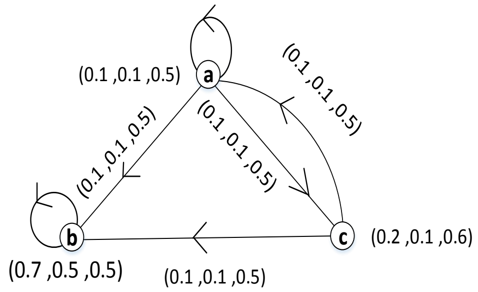

Figure5" Neutrosophic Digraph"

Table (3) "Degree of neutrosophic digraph**"**

Vertex	id[σ(a)]	od[σ(b)]	Degree
σ(a)	0.16	0.16	0.32
σ(b)	0.34	0.18	0.52
σ(c)	0.08	0.16	0.24

Definition2.6 [2]:

A homomorphism of neutrosophic graph $h: G \to G`$ is a map $h: V \to V`$ which satisfies $A(v_i) \leq A`\big(h(v_i)\big)\ \forall\ v_i \in V$. i.e.

$T_A(v_i) \leq T_{A`}(h(v_i))$ , $I_A(v_i) \leq I_{A`}(h(v_i))$, $F_A(v_i) \leq F_{A`}\big(h(v_i)\big)\ \forall\ v_i \in V$ and $\mathrm{B}(v_i, v_j) \leq B`(h(v_i)$, $h(v_j))\ \forall\ v_i\ , v_j \in V$ i.e. $T_B(v_i\ , v_j) \leq\ T_{B`}(h(v_i)\ , h(v_j))$,

$I_B(v_i\ , v_j) \leq\ I_{B`}(h(v_i)\ , h(v_j))$, $F_B(v_i\ , v_j) \leq\ F_{B`}(h(v_i)\ , h(v_j))\ \forall\ v_i\ , v_j \in V$.

Definition2.7:

The set of all vertices adjacent to a vertex *(v)* is called the neighborhood of *(v)* and is denoted by *N(v).*

Example2.10:
The neighborhood of (v_2) is $\{(v_1), (v_4), (v_5)\}$ and $\mu(e_1), \mu(e_4), \mu(e_5)$ are incident with (v_2).

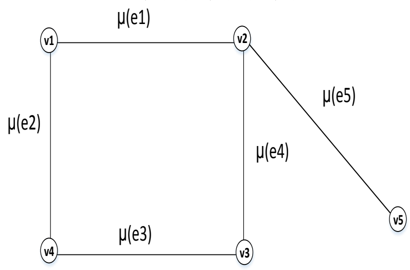

Figure6 "A simple graph"

Definition2.8: A generalized modus tollens, is expressed by the following schema, such that

Rule: If *x* is *A* Then *y* is *B*

Fact: *y* is *B'*

Conclusion: *x* is *A'*

Where *A'* is close to *A [i.e. A' = A]* and *B'* is close to *B [i.e. B' = B]* are neutrosophic sets of appropriate universes, it is also called Generalized Modus Tollens.

3. Characteristics of Neutrosophic Digraph with Generalized Modus Tollens Using Mathematical Models:

If two neutrosophic digraphs are isomorphic then corresponding vertices have the same degree.

Proof: Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be isomorphic under an isomorphism h.

Let $A(v) \epsilon A_1$. Let $N(A(v)) = \{A(w)/(A(w)) \epsilon A_1$ and $B(A(v), A(v)) \epsilon A_1\}$

And $N(h(A(v))) = \{A(w)/(A(w)) \epsilon A_2$and $[h(A(v)), h(A(v))] \epsilon A_2\}$ Now,

$A(w) \epsilon N(A(v))$ iff $B(A(v), A(w)) \epsilon A_1$ iff $(h(A(u), r(A(w))) \epsilon A_2$ iff $h(\sigma(w)) \epsilon N(h(A(v)))$ [by definition of $N(h(v))$] Hence $| N(A(v)) | = | N(h(A(v))) |$)

since h is a bijection Here the L.H. S and R.H. S are respectively the outdegree of A(v) and $h(A(v))$. Hence $A(v)$ and $h(A(v))$ have the same outdegree. Similarly, we can prove that $A(v)$ and $h(A(v))$ have the same indegree and hence $A(v)$ and $h(A(v))$ have the same degree pair.

Example3.1: " Neutrosophic directed graphs"

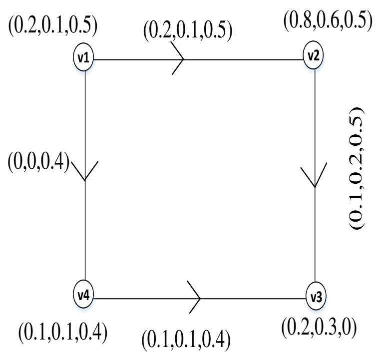

Figure7"Neutrosophic directed graph"

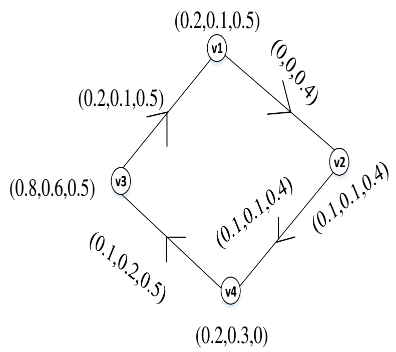

Figure8 "Neutrosophic directed graph"

Table (4)"Degree of neutrosophic digraph**"**

Vertices	Indegree	outdegree
V_1	0.5	0.355
$h(u_1)$	0.5	0.355

In this section, we present measurement of neutrosophic in degree centrality, neutrosophic out-degree centrality and neutrosophic degree centrality, respectively, in DNSN.

Definition 3.1[3]:

Let $G_{dn}^{\sim} = (V, E_{dn}^{\sim})$ be a DNSN and the single valued neutrosophic set (SVNS) is used to represent the arc lengths of $G^{\sim}_{dn}$. The sum of the lengths of the arcs that are adjacent to a social node v_x is calculated which is nothing but a SVNS. The neutrosophic value of node v_x, $d^{\sim}_I(v_x)$, is calculated as follows. $d_I^{\sim} = x\, e_{yx}^{\sim}$, $y=1$, $y_6=x$ The symbol $\sum$ refers to an addition operation of SVNS and $e^{\sim}_{yx}$ denotes a SVNS associated with the arc (i,j). $d_I^{\sim}(v_x) = (d_I^T(v_x) d_I^I(v_x);\ d_I^F(v_x))$ is an another SVNS which represents the neutrosophic in degree centrality (NIDC) of node v_x. The score value of the corresponding SVNS is determined and this score value is called as the NIDC of the node v_x. $d_I^{\sim s}(v_x) = \frac{1+pq}{2}$

$p = \left(d_I^T(v_x) - 2d_I^I(v_x) - d_I^F(v_x)\right)$ $q = (2 - d_I^T(v_x) - d_I^F(v_x))$

Here, $d^{\sim s}_I(v_x)$ represents the NIDC of node v_x.

Definition 3.2[3]: Let $G_{dn}^{\sim} = (V, E_{dn}^{\sim})$ be a DNSN and the

SVNS is used to represent the arc lengths of $G_{dn}^{\sim}$.The sum of the lengths of the arcs that are adjacent from a social node v_x is calculated which is nothing but a SVNS. The neutrosophic value of node v_x, $d_o^{\sim}(v_x)$,is calculated as follows. $d_o^{\sim}(v_x) = \sum e_{yx}^{\sim}$

The symbol Σ refers to an addition operation of SVNS and $e^{\sim}_{yx}$ denotes a SVNS associated with the arc (i, j). $d_I^{\sim}(v_x) = (d_I^T(v_x) d_I^I(v_x);\ d_I^F(v_x))$ is an another SVNS

which represents the NIDC of node v_x. The score value of the corresponding SVNS is determined and this score value is called as the NIDC of the node v_x.

$d_o^{\sim s}(v_x) = \frac{1+pq}{2}$ $p = \left(d_I^T(v_x) - 2d_I^I(v_x) - d_I^F(v_x)\right)$ $q = (2 - d_I^T(v_x) - d_I^F(v_x))$

Here, $d^{\sim s}_O(v_x)$ represents the neutrosophic out degree centrality of node v_x.

Definition 3.3[3]: Let $G_{dn}^{\sim} = (V, E_{dn}^{\sim})$be a DFSN and the SVNS is applied to describe the edge weights of $G^{\sim}_{dn}$. The sum of NIDC and NODC of node v_x is called neutrosophic degree centrality(NDC) of v_x. The NDC of node v_x, $d^{\sim}(v_x)$, is described as follows. $d^{\sim}(vx) = d^{\sim}sI\,(vx) + d^{\sim}sO\,(vx)$ Here, $d^{\sim}(v_x)$ is the NDC of node v_x.

These three types of degree can be used to reflect the communication ability of a node in DNSN. The NIDC of any node can be applied to describe the receptivity or popularity and the NODC of any node can be applied to measure of expansiveness.

3.2 Result:

Characteristics of neutrosophic digraph with generalized modus tollens method using mathematical models.

Proof:

For a communication networks, set up the corresponding transition probability matrix and find the importance of each member in the network. In a neutrosophic digraph D, sum of the indegrees of all the vertices is equal to the sum of their outdegrees, each sum being equal to the number of arcs in *D*.

A neutrosophic digraph can serve as a model for a communication network. Consider the network given in fig8: If an (edge) arc is directed from $A(v_1)$to $A(v_2)$,it means that $A(v_1)$can communicate with $A(v_2)$.

In the given networks A (v_5) can communicate directly with $A(v_2)$,but

$A(v_2)$ can communicate with $A(v_5)$only indirectly through $A(v_3)$and $A(v_4)$.

In the given networks

$A(v_1)$can communicate directly with $A(v_2)$ &$A(v_3)$

$A(v_2)$can communicate directly with $A(v_1)$& $A(v_3)$. $A(v_3)$can communicate directly with $A(v_1)$, $A(v_2)$& $A(v_4)$.

$A(v_4)$can communicate directly with $A(v_3)$& $A(v_5)$.

$A(v_5)$can communicate directly with $A(v_2)$.

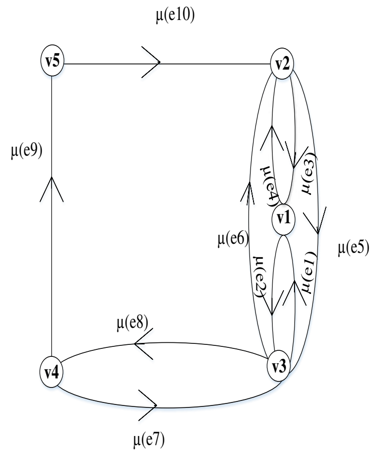

Figure9"Neutrosophic digraph"

However, every individual can communicate with every other individual.

Our problem is to determine the importance of each individual in this network. The importance can be measured by the fraction of the messages on an average that pass through him.

In the absence of any other knowledge, we can assume that if an individual can send message direct to n-individuals, he will send a message to any one of them with probability $\frac{1}{n}$.

In the present example, the communication probability matrix is

$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Here no individual is to send a message to himself and so all diagonal elements are zero. Since all elements of the matrix are non-negative and the sum of elements of every row is unity, the matrix

is a stochastic matrix and one of its eigenvalues is unity. The corresponding normalized eigenvector is $\left[\frac{11}{45}, \frac{13}{45}, \frac{3}{10}, \frac{1}{10}, \frac{1}{15}\right]$. In the long run, these fractions of messages will pass through A (v_1), A (v_2), A (v_3), A (v_4), A(v_5) respectively.

Thus we can conclude that in this network, σ(v_3) is the most important person. Let D = sum of the indegrees of all the vertices in D.

C = sum of the outdegrees of all the vertices in D. $\sum_i$ B(vi¸ vi + 1)
= sum of the number of arcs in D.

Table (4)"Degree of neutrosophic digraph**"**

Vertices	Indegree(NIDC)	Outdegree(NODC)	Degree(NDC)
σ(v_1)	0.82	0.905	1.725
σ(v_2)	1.585	1.085	2.67
σ(v_3)	1.405	1.405	2.81
σ(v_4)	0.585	1	1.585
σ(v_5)	0.5	0.5	1

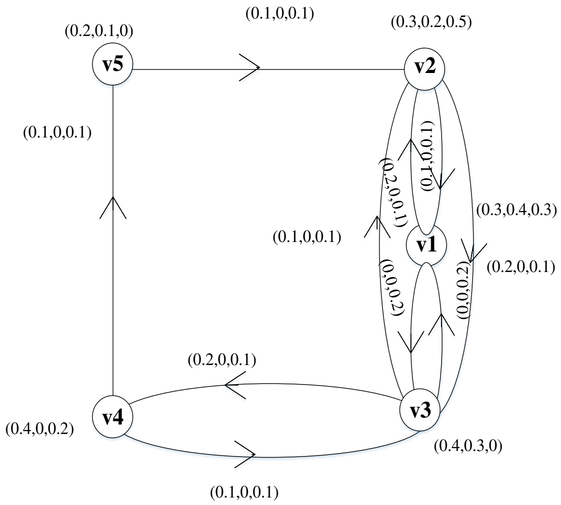

Figure 10" Neutrosophic digraph"

Rule1:

Rule: IF x is D THEN y is equal to C
Fact: y is 4.89
Conclusion: x is 4.89

Example:

D= $\sum_{v\in V}$ id[σ(v)] = 0.82+ 1.585+ 1.405+ 0.585+ 0.5= 4.89
C = $\sum_{v\in V}$ od[σ(v)] = 0.905+ 1.085+1.405+1+0.5=4.89

Rule2:

Rule: IF x is C THEN y is equal to $\sum_i \mathrm{B}(v_i, v_{i+1})$
Fact: y is 4.89
Conclusion: x is 4.89

Example:

$\sum_i \mathrm{B}(v_i, v_{i+1})$ = 4.89

Finally D = C =$\sum_i \mathrm{B}(v_i, v_{i+1})$ = 4.89

An arc (A(v_1), A (v_2)) contributes one to the outdegree of A (v_i) and one to the indegree of A(v_i). Hence each arc contributes one to the sum D and one to the sum C. Hence $D = C = \sum_i \mathrm{B}(v_i, v_{i+1})$. Hence proved.

4.Conclusion:

Finally, we have analyzed some concepts of neutrosophic digraph, outdegree of neutrosophic digraph, indegree of neutrosophic digraph, and we have easily obtained neutrosophic digraphs indegree and outdegree by mathematical model with generalized modus tollens method. We have been able to show that the neutrosophic is better than the fuzzy, as it studies the state of indeterminacy.

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