

The Ideal optimized strategy of Transportation Model

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Abstract: Transportation Problem is a linear programming problem. Like LPP, transportation problem has basic feasible solution (BFS) and then from it we obtain the optimal solution. Among these BFS the optimal solution is developed by constructing dual of the TP. By using complimentary slackness conditions the optimal solutions is obtained by the same iterative principle. The method is known as MODI (Modified Distribution) method. In this paper we have discussed all the aspect of transportation problem.[1]

Keywords: Transportation Problem, Transportation Algorithm, North West Corner Rule, Least Cost Method, Vogel's Approximation Method, Modi Method, Initial Basic Feasible Solution, Optimal Solution

“We want these assets to be productive. We buy them. We own them. To say we care only about the short term is wrong. What I care about is seeing these assets in the best hands.”

– Carl Icahn

1. INTRODUCTION

Transportation Problem is a special structure of Linear Programming Problem (LPP), that is frequently encountered in the Operation Research literature. The model was first presented by F.L. Hitchcock in 1941. In 1950's simplex- based solution techniques were developed for the transportation problem exploiting its special structure. In the meantime, we come across varieties of transportation problems such as bottleneck problem, minimax and maximin problem, time minimization transportation problem, volume minimization transportation problem, etc. The bottleneck transportation problem was first discussed by Fulkerson, Glickberg, and Guss (1953) and subsequently by Guss in 1959. Later on, varieties of new theoretical and methodological development were made by Hammer in 1969. Edmond and Fulkerson 1970, Garfinkle and Rao 1979 and 1976, Kaplan 1976 and Poisner and Wu 1981, etc. Bottleneck models were mathematically

formulated with a special type of objective function in which the maximal cost coefficient of any variable with strictly positive value is minimized concerning a given set of constraints.

2. Literature Review

As an example suppose that the origin represents a military depot in which certain supplies, say ammunitions are stored and let the destination represent a combat zone at which there is specified demand for ammunition. To every depot combat zone pair, a coefficient c_{ij} is assigned indicating the amount of time required to ship any number of units i th origin to j th destination[16]. A military operation will start in all combat zone simultaneously at the earliest possible instance, it is necessary that the requested amount of ammunition must be available at all combat zones.[15] In other words, the operation cannot start before the last shipment of goods arrives and the problem is to schedule the shipment so that the operation can start as soon as possible

One important application of linear programming is in the area of physical distribution (transportation) of goods and services from several supply centres to several demand centres. A transportation problem when expressed in terms of an LP model can also be solved by the simplex method. However a transportation problem involves a large number of variables and constraints, solving it using simplex methods takes a long time. Two transportation algorithms, namely *Stepping Stone Method* and the *MODI* (modified distribution) *Method* have been developed for solving a transportation problem.[2]

The structure of transportation problem involves a large number of shipping routes from several supply centres to several demand centres. Thus, objective is to determine shipping routes between supply centres and demand centres in order to satisfy the required quantity of goods or services at each destination centre, with available quantity of goods or services at each supply centre at the minimum transportation cost and/or time.

The transportation algorithms help to minimize the total cost of transporting a homogeneous commodity (product) from supply centres to demand centres. However, it can also be applied to the maximization of total value or utility.

There are various types of transportation models and the simplest of them was first presented by F L Hitchcock (1941). It was further developed by T C Koopmans (1949) and G B Dantzig (1951). Several extensions of transportation models and methods have been subsequently developed.[3]

Throughout the thesis we will assume that the following structure is given: Let a company own " m " warehouses in which each of which there is a given amount of certain commodity in stock, and let there also be " n " consumer is with a given demand for this commodity. The " m " warehouses of the company are sources and " n " consumers are known as destinations. Moreover the unit transportation cost between each warehouse -consumer pair is known. The objective of the company is to transport units from the warehouses to the consumers, such that

- no more units leave a warehouse than there are instocks,
- the demands of the consumers are satisfied, and
- the total transportation cost is minimize

Formally the model can be described as follows: let N_1 be a set of "m" location called origins so that there is a supply of s_i at the i^{th} origin, $i = 1, 2, \dots, m$ and let N_2 be a set of n locations called destinations so that there is demand d_j at the j^{th} destination, $j = 1, 2, \dots, n$. In the above example, the origin corresponds to the warehouses whereas the retailers are represented by the destinations. we assume that the total

$$\text{demand equal to the total supply i.e. } \sum_{i=1}^m s_i = \sum_{j=1}^n d_j$$

total supply exceeds total demands exactly one dummy destination is created to absorb the excess supply; its demand equals $\sum_{i=1}^m s_i - \sum_{j=1}^n d_j$. The case of excess demand is handled similarly using exactly one dummy origin. Finally, it is assumed that exists exactly one connection between each origin-destination pair, that, detours are not allowed and that each of these connections has infinite capacity. The cost for shipping one quantity unit from the i th origin to the j th destination is given by c_{ij} . All unit transportation costs from a dummy origin or to a dummy destination are assumed to be equal. Usually, we will set them equal to zero but sometimes, due to the specific nature of the problem considered, they will be assigned some other value. If the connection between i th origin and j th destination for some reason is forbidden, the corresponding cost is assigned an extremely high value, i.e. we set [4]

$$C_{ij} : M \gg 0$$

In these cases, the existence of feasible solutions is no longer guaranteed. Note that sometimes c_{ij} denotes the distance per quantity unit; then the objective is to minimize the total distance of the shipments, assuming that each unit is transported separately. As in all linear programming problems, the objective function is linear. The above structure may be visualized in the following figure I :

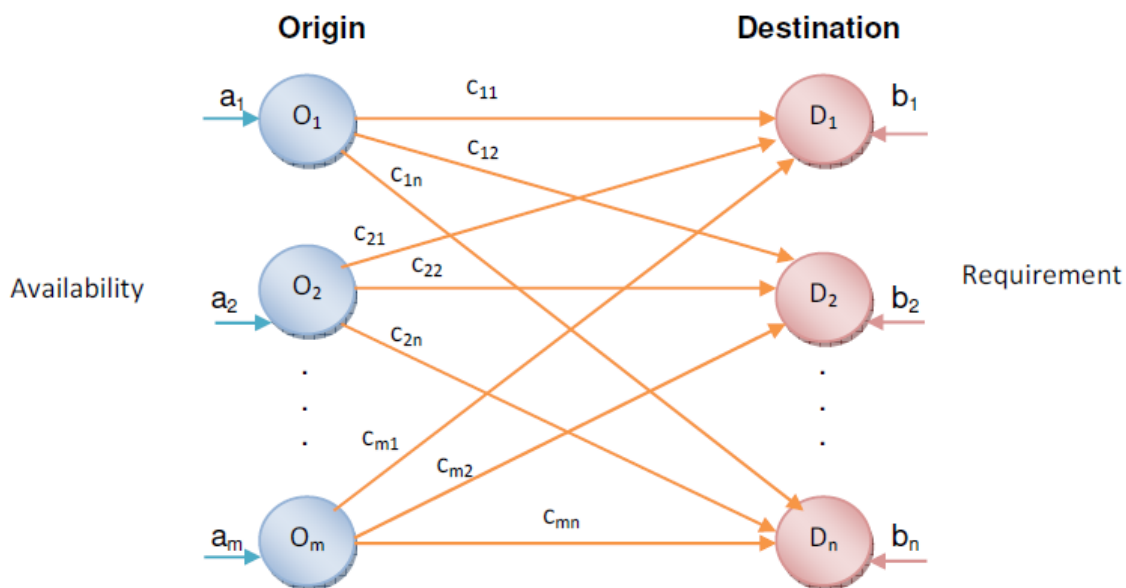


Fig I

"Formulate this transportation problem as an LPP model to minimize the total transportation cost".

3. Research Methodology

Let there be m sources of supply, S_1, S_2, \dots, S_m having a_i ($i = 1, 2, \dots, m$) units of supply (or capacity), respectively to be transported to n destinations, D_1, D_2, \dots, D_n with b_j ($j = 1, 2, \dots, n$) units of demand (or requirement), respectively. Let c_{ij} be the cost of shipping one unit of the commodity from source i to destination j . If x_{ij} represents number of units shipped from source i to destination j , the problem is to determine the transportation schedule so as to minimize the total transportation cost while satisfying the supply and demand conditions.[5] Mathematically, the transportation problem, in general, may be stated as follows:

$$\text{Minimize (total cost) } Z = \sum_{i=1}^n a_{ij} \sum_{j=1}^m c_{ij}$$

subject to the constraints

$$\sum_{i=1}^m x_{ij} = a_{ij} \quad i = 1, 2, \dots, m \text{ (supply constraints)}$$

(2)

$$\sum_{j=1}^n x_{ij} = b_{ij} \quad j = 1, 2, \dots, n \text{ (demand constraints)}$$

(3)

and

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j. \tag{4}$$

Existence of feasible solution A necessary and sufficient condition for a feasible solution to the transportation problems is:

$$\text{Total supply} = \text{Total demand}$$

$$\sum_{i=1}^n a_{ij} \sum_{j=1}^m c_{ij} \quad \text{(also called } \textit{rim conditions})$$

To From	D_1		D_2			D_n		Supply a_i
S_1	c_{11}	x_{11}	c_{12}	x_{12}		c_{1n}	x_{1n}	a_1

S_2	c_{21}	x_{21}	c_{22}	x_{22}		c_{2n}	x_{2n}	a_2
.								
S_m	c_{m1}	x_{m1}	c_{m2}	x_{m2}		c_{mn}	x_{mn}	A_m
Demand B_j	b_1		b_2			b_n		$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$

In this problem, there are $(m + n)$ constraints, one for each source of supply, and distinction and $m \times n$ variables. Since all $(m + n)$ constraints are equations, therefore, one of these equations is extra (redundant). The extra constraint (equation) can be derived from the other constraints (equations), without affecting the feasible solution. It follows that any feasible solution for a transportation problem must have exactly $(m + n - 1)$ non-negative basic variables (or allocations) x_{ij} satisfying the rim conditions.[6]

• **THE TRANSPORTATION ALGORITHM**

The algorithm for solving a transportation problem may be summarized into the following steps:

Step 1: Formulate the problem and arrange the data in the matrix form

The formulation of the transportation problem is similar to the LP problem formulation. In transportation problem, the objective function is the total transportation cost and the constraints are the amount of supply and demand available at each source and destination, respectively.

Step 2: Obtain an initial basic feasible solution In this chapter, following three different methods are discussed to obtain an initial solution:

- North-West Corner Method,
- Least Cost Method, and
- Vogel's Approximation (or Penalty) Method.

The initial solution obtained by any of the three methods must satisfy the following conditions:

- The solution must be feasible, i.e. it must satisfy all the supply and demand constraints (also called *rim conditions*).
- The number of positive allocations must be equal to $m + n - 1$, where m is the number of rows and n is the number of columns.

Any solution that satisfies the above conditions is called *non-degenerate basic feasible solution*, otherwise, *degenerate solution*.

Step 3: Test the initial solution for optimality In this chapter, the Modified Distribution (MODI) method is discussed to test the optimality of the solution obtained in Step 2. If the current solution is optimal, then stop. Otherwise, determine a new improved solution.

Step 4: Updating the solution Repeat Step 3 until an optimal solution is reached.[8]

METHODS OF FINDING INITIAL SOLUTION:

There are several methods available to obtain an initial basic feasible solution. In this chapter, we shall discuss only following three methods:

Find the initial basic feasible solution for given problem by using following methods:

- North-west corner rule
- Least cost method
- Vogel’s approximation method

3. Findings

Example 3.1

1) Determine basic feasible solution to the following transportation problem using NWCR

P	4	1	2	6	9	s_i
Q	6	4	3	5	7	100
R	5	2	6	4	8	120
d_j	40	50	70	90	90	120

Solution:

P	4	1	2	6	9	s_i
	40	50	10			
Q	6	4	3	5	7	100
			60	60		
R	5	2	6	4	8	120
			30	90		
d_j	40	50	70	90	90	120

Since $\sum a_i = \sum b_j = 340$

The given Transportation problem is balanced

Allocation = 7

$$m + n - 1 = 3 + 5 - 1 = 7$$

Allocation = m + n - 1

$$\begin{aligned} \text{The initial cost} &= (4 \times 40) + (1 \times 50) + (10 \times 2) + (3 \times 60) + (5 \times 60) + (4 \times 30) + (8 \times 90) \\ &= 160 + 50 + 20 + 180 + 300 + 120 + 720 \end{aligned}$$

The initial transportation cost = Rs 1550 /-

- **Least Cost Method (LCM)**

Since the main objective is to minimize the total transportation cost, transport as much as possible through those routes (cells) where the unit transportation cost is lowest. This method takes into account the minimum unit cost of transportation for obtaining the initial solution and can be summarized as follows:

Example 3.2

The initial basic feasible solution for following transportation problem by LCM method

P	4	1	2	6	9	s_i
Q	6	4	3	5	7	100
R	5	2	6	4	8	120
d_j	40	50	70	90	90	120

Solution:

P	4	1	2	6	9	s_i
Q	6	4	3	5	7	100
R	5	2	6	4	8	120
d_j	40	50	70	90	90	120

Since $\sum a_i = \sum b_j = 340$

The given transportation problem is balanced

Allocation = 7

$m + n - 1 = 3 + 5 - 1$

$= 7$

Allocation = $m + n - 1$

The initial cost = $(1 \times 50) + (2 \times 50) + (6 \times 10) + (3 \times 20) + (7 \times 90) + (5 \times 30) + (4 \times 90)$

$= 50 + 100 + 60 + 60 + 630 + 150 + 360$

The initial Transportaion cost = Rs 1410/-

3.3 Vogel's Approximation Method (VAM)

Vogel's approximation (penalty or regret) is preferred over NWCR and LCM methods. In this method, an allocation is made on the basis of the opportunity (or penalty or extra) cost that would have been incurred if the allocation in certain cells with minimum unit transportation cost were missed. Hence, allocations are made in such a way that the penalty cost is minimized. An initial solution obtained by using this method is nearer to an optimal solution or is the optimal solution itself. The steps of VAM are as follows:

Example 3.3

Find the initial basic feasible solution for following transportation by VAM

P	4	1	2	6	9	s_i
Q	6	4	3	5	7	100
R	5	2	6	4	8	120
d_j	40	50	70	90	90	120

Solution :

P	4	1	2	6	9	s_i
	30		70			
Q	6	4	3	5	7	100
				90	30	
R	5	2	6	4	8	120
	10	50			60	
d_j	40	50	70	90	90	120

$$\text{Since } \sum a_i = \sum b_j = 340$$

The given transportation problem is balanced

$$\text{Allocation} = 7$$

$$m + n - 1 = 3 + 5 - 1$$

$$= 7$$

$$\text{Allocation} = m + n - 1$$

$$\begin{aligned} \text{Initial transportation cost} &= (4 \times 30) + (2 \times 70) + (5 \times 90) + (7 \times 30) + (5 \times 10) + (2 \times 50) + (8 \times 60) \\ &= 120 + 140 + 450 + 210 + 50 + 100 + 480 \end{aligned}$$

The initial transportation cost = Rs1550/-

• Steps of MODI Method (Transportation Algorithm)

The steps to evaluate unoccupied cells are as follows:

Step 1: For an initial basic feasible solution with $m + n - 1$ occupied cells, calculate u_i and v_j for rows and columns. The initial solution can be obtained by any of the three methods discussed earlier.

To start with, any one of u_{is} or v_{js} is assigned the value zero. It is better to assign zero to a particular u_i or v_j where there are maximum number of allocations in a row or column respectively, as this will reduce the considerably arithmetic work. The value of u_{is} and v_{js} for other rows and columns is calculated by using the relationship.

Changing the shipping route involves adding to cells on the closed path with plus signs and subtracting from cells with negative signs.

$$c_{ij} = u_i + v_j, \quad \text{for all occupied cells } (i, j).$$

Step 2: For unoccupied cells, calculate the opportunity cost by using the relationship

$$d_{ij} = c_{ij} - (u_i + v_j), \quad \text{for all } i \text{ and } j.$$

Step 3: Examine sign of each d_{ij}

- If $d_{ij} > 0$, then the current basic feasible solution is optimal.
- If $d_{ij} = 0$, then the current basic feasible solution will remain unaffected but an alternative solution exists.
- If one or more $d_{ij} < 0$, then an improved solution can be obtained by entering an unoccupied cell (i, j) into the solution mix (basis). An unoccupied cell having the largest negative value of d_{ij} is chosen for entering into the solution mix (new transportation schedule).

Step 4: Construct a closed-path (or loop) for the unoccupied cell with largest negative value of d_{ij} . Start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell. Trace a path along

the rows (or columns) to an occupied cell, mark the corner with a minus sign (−) and continue down the column (or row) to an occupied cell. Then mark the corner with plus sign (+) and minus sign (−) alternatively. Close the path back to the selected unoccupied cell.

Step 5: Select the smallest quantity amongst the cells marked with minus sign on the corners of closed loop. Allocate this value to the selected unoccupied cell, add it to occupied cells marked with plus signs, and subtract it from the occupied cells marked with minus signs.

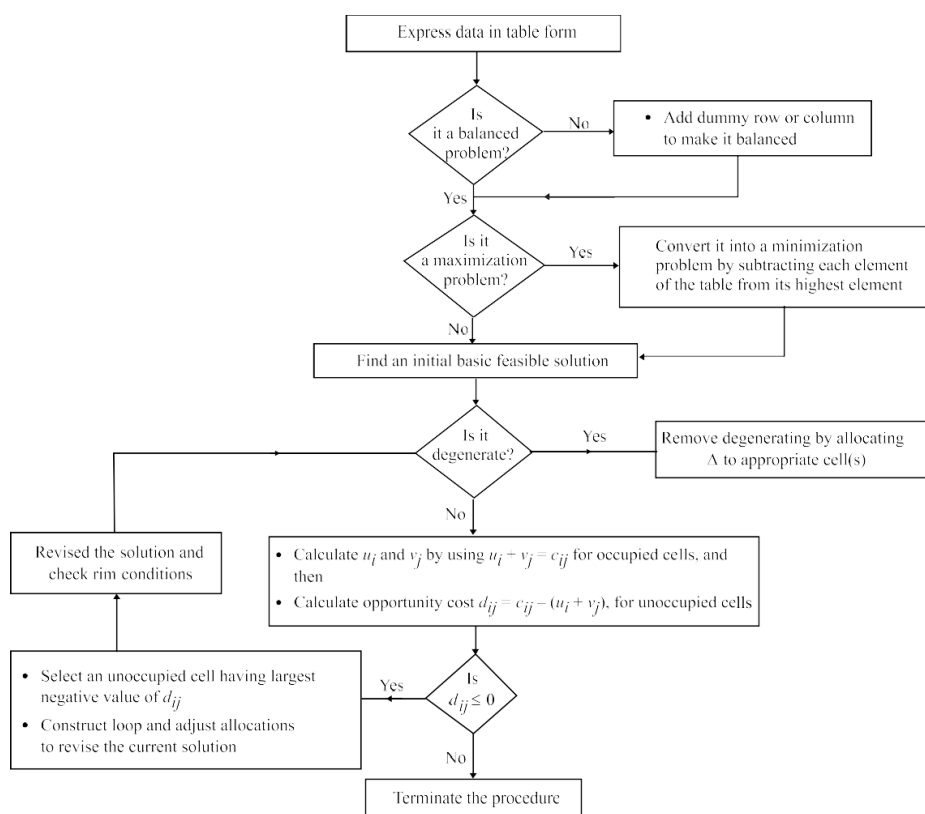
Step 6: Obtain a new improved solution by allocating units to the unoccupied cell according to Step 5 and calculate the new total transportation cost.

Step 7: Test optimality of the revised solution. The procedure terminates when all $d_{ij} \geq 0$ for unoccupied cells.

Remarks 1. The closed-loop (path) starts and ends at the selected unoccupied cell. It consists of successive horizontal and vertical (connected) lines whose end points must be occupied cells, except an end point associated with entering unoccupied cell. This means that every corner element of the loop must be an occupied cell.

It is immaterial whether the loop is traced in a clockwise or anti-clockwise direction and whether it starts up, down, right or left (but never diagonally). However, for a given solution only one loop can be constructed for each unoccupied cell.[12]

- There can only be one plus (+) sign and only one minus (−) sign in any given row or column.
- The closed path indicates changes involved in reallocating the shipments.



The steps of MODI method for solving a transportation problem are summarized in the flow chart shown in Fig. 9.1.

• **Close-Loop in Transportation Table and its Properties**

Any basic feasible solution must contain $m + n - 1$ non-zero allocations provided.

- any two adjacent cells of the ordered set lie either in the same row or in the same column, and
- no three or more adjacent cells in the ordered set lie in the same row or column. The first cell of the set must follow the last in the set, i.e. each cell (except the last) must appear only once in the ordered set.

Consider the following two cases represented in Tables 9.10(a) and 9.10(b). In Table 9.10(a), if we join the positive allocations by horizontal and vertical lines, then a closed loop is obtained. The ordered set of cells forming a loop is:

$$L = \{(a, 2), (a, 4), (e, 4), (e, 1), (b, 1), (b, 2), (a, 2)\}$$

The loop in Table 9.10(b) is not allowed because it does not satisfy the conditions in the definition of a loop. That is, the cell $(b, 2)$ appears twice.[13]

	1	2	3	4
a		•		•
b	•	•		
c				
d				
e	•			•

(a)

	1	2	3	4
a	•	•		
b	•	•		•
c				
d		•		•
e				

(b)

Example 3.4

Find the optimal solution for the following transportation problem by using MODI method

P	4	1	2	6	9	s_i
Q	6	4	3	5	7	100
R	5	2	6	4	8	120
d_j	40	50	70	90	90	120

Solution :

P	4	1	2	6	9	s_i
	30		70			

Q	6	4	3	5	7	100
				90	30	
R	5	2	6	4	8	120
	10	50			60	
d _j	40	50	70	90	90	120

Since $\sum a_i = \sum b_j = 340$

The given transportation problem is balanced

Allocation = 7

$$m + n - 1 = 3 + 5 - 1$$

$$= 7$$

Allocation = m + n - 1

$$\begin{aligned} \text{Initial transportation cost} &= (4 \times 30) + (2 \times 70) + (5 \times 90) + (7 \times 30) + (5 \times 10) + (2 \times 50) + (8 \times 60) \\ &= 120 + 140 + 450 + 210 + 50 + 100 + 480 \end{aligned}$$

The initial transportation cost = Rs1550/-

i) To find the Optimal Solution:

P	4	1	2	6	9	s _i
	30		70			
Q		4	3	5	7	100
	6			90	30	
R	5	2	6	4	8	120
	10	50			60	
d _j	40	50	70	90	90	120

ii) Find the Occupied Cells :

$$U_1 + V_1 = C_{11} ;$$

$$U_1 + V_3 = C_{13} ;$$

$$U_2 + V_4 = C_{24} ;$$

$$U_2 + V_5 = C_{25} ;$$

$$U_3 + V_1 = C_{31} ;$$

$$U_3 + V_2 = C_{32} ;$$

$$U_3 + V_5 = C_{35}.$$

iii) Find the U & V values:

$$U_3 = 0$$

$$U_3 + V_1 = C_{31}$$

$$U_3 + V_2 = C_{32}$$

$$U_3 + V_5 = C_{35}$$

$$0 + V_1 = 5$$

$$0 + V_2 = 2$$

$$0 + V_5 = 8$$

$$V1 = 5$$

$$V2 = 2$$

$$V5 = 8$$

$$U1 + V1 = C11$$

$$U1 + V3 = C13$$

$$U1 + 5 = 4$$

$$-1 + V3 = 2$$

$$U1 = -1$$

$$V3 = 3$$

$$U2 + V5 = C25$$

$$U2 + V4 = C24$$

$$U2 + 8 = 7$$

$$-1 + V4 = 5$$

$$U2 = -1$$

$$V4 = 6$$

iv) Unoccupied Cells:

$$U1 + V2 - C12 = -1 + 2 - 1 = 0$$

$$U1 + V4 - C14 = -1 + 6 - 6 = -1$$

$$U1 + V5 - C15 = -1 + 8 - 9 = -2$$

$$U2 + V1 - C21 = -1 + 5 - 6 = -2$$

$$U2 + V2 - C22 = -1 + 2 - 4 = -3$$

$$U2 + V3 - C23 = -1 + 3 - 3 = -1$$

$$U3 + V3 - C33 = 0 + 3 - 6 = -3$$

$$U3 + V4 - C34 = 0 + 6 - 4 = 2$$

Iteration I

P	4	1	2	6	9	s_i
	30		70			
Q		4	3	5	7	100
	6			30	90	
R	5	2	6	4	8	120
	10	50		60		
d_j	40	50	70	90	90	120

i) Occupied Cells:

$$U1 + V1 = C11$$

$$U1 + V3 = C13$$

$$U2 + V4 = C24$$

$$U2 + V5 = C25$$

$$U3 + V1 = C31$$

$$U3 + V2 = C32$$

$$U3 + V4 = C34$$

ii) Find the U & V values:

$$U3 = 0$$

$$\begin{array}{lll}
 U_3 + V_1 = C_{31} & U_3 + V_2 = C_{32} & U_3 + V_4 = C_{34} \\
 0 + V_1 = 5 & 0 + V_2 = 2 & 0 + V_4 = 4 \\
 V_1 = 5 & V_2 = 2 & V_4 = 4
 \end{array}$$

$$\begin{array}{lll}
 U_2 + V_4 = C_{24} & U_1 + V_1 = C_{11} & U_1 + V_3 = C_{13} \\
 U_2 + 4 = 5 & U_1 + 5 = 4 & -1 + V_3 = 2 \\
 U_2 = -1 & U_1 = -1 & V_3 = 3
 \end{array}$$

iii) Unoccupied Cells:

$$\begin{array}{l}
 U_1 + V_2 - C_{12} = -1 + 2 - 1 = 0 \\
 U_1 + V_4 - C_{14} = -1 + 4 - 6 = -3 \\
 U_1 + V_5 - C_{15} = -1 + 6 - 9 = -4 \\
 U_2 + V_1 - C_{21} = 1 + 5 - 6 = 0 \\
 U_2 + V_2 - C_{22} = 1 + 2 - 4 = -1 \\
 U_2 + V_3 - C_{23} = 1 + 3 - 3 = 1 \\
 U_3 + V_3 - C_{33} = 0 + 3 - 6 = -3 \\
 U_3 + V_5 - C_{35} = 0 + 6 - 8 = 2
 \end{array}$$

Iteration II

P	4 40	1	2 60	6	9	s_i
Q	6	4	3 10	5 20	7 90	100
R	5	2 50	6	4 70	8	120
d_j	40	50	70	90	90	120

i) Occupied Cells:

$$\begin{array}{lll}
 U_1 + V_1 = C_{11} & U_1 + V_3 = C_{13} & U_2 + V_3 = C_{23} \\
 U_2 + V_4 = C_{24} & U_2 + V_5 = C_{25} & U_3 + V_2 = C_{32} \\
 U_3 + V_4 = C_{34} & &
 \end{array}$$

ii) U & V values:

Put U = 0

$$\begin{array}{lll}
 U_2 + V_3 = C_{23} & U_2 + V_4 = C_{24} & U_2 + V_5 = C_{25} \\
 0 + V_3 = 3 & 0 + V_4 = 5 & 0 + V_5 = 7 \\
 V_3 = 3 & V_4 = 5 & V_5 = 7
 \end{array}$$

$$\begin{array}{lll}
 U_1 + V_3 = C_{13} & U_1 + V_1 = C_{11} & U_3 + V_4 = C_{34} \\
 U_1 + 3 = 2 & -1 + V_1 = 4 & U_3 + 5 = 4
 \end{array}$$

$$U_1 = -1$$

$$V_1 = 5$$

$$U_3 = -1$$

$$U_3 + V_2 = C_{32}$$

$$-1 + V_2 = 2$$

$$V_2 = 3$$

iii) Unoccupied Cells:

$$U_1 + V_2 - C_{12} = -1 + 3 - 1 = 1$$

$$U_1 + V_2 - C_{14} = -1 + 5 - 6 = -2$$

$$U_1 + V_5 - C_{15} = -1 + 7 - 9 = -3$$

$$U_2 + V_1 - C_{21} = 0 + 5 - 6 = -1$$

$$U_2 + V_2 - C_{22} = 0 + 3 - 4 = -1$$

$$U_3 + V_1 - C_{31} = -1 + 5 - 5 = -1$$

$$U_3 + V_3 - C_{33} = -1 + 3 - 6 = -4$$

$$U_3 + V_5 - C_{35} = -1 + 7 - 8 = -2$$

Iteration III

P	4	1	2	6	9	s_i
	40	20	40			
Q		4	3	5	7	100
	6		30		90	
R	5	2	6	4	8	120
		30		90		
d_j	40	50	70	90	90	120

i) Occupied Cells:

$$U_1 + V_1 = C_{11}$$

$$U_1 + V_2 = C_{12}$$

$$U_1 + V_3 = C_{13}$$

$$U_2 + V_3 = C_{23}$$

$$U_2 + V_5 = C_{25}$$

$$U_3 + V_2 = C_{32}$$

$$U_3 + V_4 = C_{34}$$

ii) U & V values:

$$\text{put } U_1 = 0$$

$$U_1 + V_1 = C_{11}$$

$$U_1 + V_2 = C_{12}$$

$$U_1 + V_3 = C_{13}$$

$$0 + V_1 = 4$$

$$0 + V_2 = 1$$

$$0 + V_3 = 2$$

$$V_1 = 4$$

$$V_2 = 1$$

$$V_3 = 2$$

$$U_3 + V_2 = C_{32}$$

$$U_3 + V_4 = C_{34}$$

$$U_2 + V_3 = C_{23}$$

$$U_3 + 1 = 2$$

$$1 + V_4 = 4$$

$$U_2 + 2 = 3$$

$$U_3 = 1$$

$$V_4 = 3$$

$$U_2 = 1$$

$$U_2 + V_5 = C_{25}$$

$$1 + V_5 = 7$$

$$V_5 = 6$$

iii) Unoccupied Cells:

$$U_1 + V_4 - C_{14} = 0 + 3 - 6 = -3$$

$$U_1 + V_5 - C_{15} = 0 + 6 - 9 = -3$$

$$U_2 + V_1 - C_{21} = 1 + 4 - 6 = -1$$

$$U_2 + V_2 - C_{22} = 1 + 1 - 4 = -2$$

$$U_2 + V_4 - C_{24} = 1 + 3 - 5 = -1$$

$$U_3 + V_1 - C_{31} = 1 + 4 - 5 = 0$$

$$U_3 + V_3 - C_{33} = 1 + 2 - 6 = -3$$

$$U_3 + V_5 - C_{35} = 1 + 6 - 8 = -1$$

The initial total min TP cost = $(4 \times 40) + (1 \times 20) + (40 \times 2) + (30 \times 3) + (90 \times 7) + (30 \times 2) + (4 \times 90)$

The initial total min TP cost = Rs1400/-

CONCLUSION:

The paper survey mathematical models and algorithms used to solve different types of transportation modes by air, water, space, cables, tubes, and road. It presents the variants, classification, and the general parameters of the Transportation Problems.

In the paper have taken the four methods like NWCR, LCM, VAM and MODI method in this four methods, The MODI method is the best method to solve the transportation problem with minimum cost in business, marketing, economics and manufacturing industries. As future work, we propose to investigate mathematical models of the space transportation problems, maritime transportation issues, and the creation of new algorithms that solve these problems.[14]

REFERENCES:

- [1] Bit. A. K. and Alam, S. S., An additive fuzzy programming model for multi-objective transportation problem, Fuzzy Sets and Systems, 57, 1993, 313-319.
- [2] Bit, AK, Biswal MP, Alam, SS., Fuzzy programming approach to multi-criteria decision making transportation problem, Fuzzy Sets and Systems, 50, 1992, 35-41.
- [3] Charnes. A. and Klingman, D., The More-for-less paradox in distribution models, Cahiersdu Centre d'EtudesRechercheOperationnelle, 13, 1971, 11-22.
- [4] Charnes. A., S. Duffuaa and Ryan, M., The More- for-Less Paradox in Linear Programming, European Journal of Operation Research, 31, 1987, 194-197.
- [5] Charnes, A., Cooper, W.W., The stepping stone method for explaining linear programming calculation in transportation problem, Mgmt. Sci., 1, 1954, 49 - 69.
- [6] Charnes, A., Cooper, W.W., and Henderson, A., An Introduction to Linear Programming, Wiley, New York, 1953.
- [7] Chandra. S and P.K. Saxena., Time Minimizing Transportation Problem with impurities, Asia- Pacific J. Op.Res.,4, 1987, 19-27.
- [8] Currin.D.C.,Transportation problem with inadmissible routes. Journal of the Operational Research Society,37, 1986, 387-396.

- [9] Dantzig, G.B., *Linear Programming and Extensions*, Princeton University Press, Princeton, N J , 1963.
- [10] Das.S.K.,Goswami.A and Alam.S.S.,Multi-objective transportation problem with interval cost, source and destination parameters, *European Journal of Operational Research*, 117, 1999, 100-112.
- [11] Garfinkel.R.S and Rao.M.R., The bottleneck transportation problem, *Naval Research Logistics Quarterly*, 18, 1971, 465 – 472.
- [12] Gaurav Sharma,S. H. Abbas, Vijay kumar Gupta., Solving Transportation Problem with the help of Integer Programming Problem, *IOSR Journal of Engineering*, 2 , 2012, 1274-1277.
- [13] Gupta, A., Khanna, S., and Puri, M.C.,Paradoxical situations in transportation problems, *Cahiers du Centre d'EtudesRechercheOperationnelle*, 34, 1992, 37–49.
- [14] Hitchcock, F.L.,The distribution of a product from several sources to numerous localities. *Journal of Mathematics & Physics*, 20, 1941, 224-230.
- [15] *Achievements in Materials and Manufacturing Engineering*, 28, 2008, 71-74.
- [16] Zeleny.M., *Multiple criteria decision making*; Mc Graw-Hill Book Company, 1982.