TWO SELF WEAKLY COMPATIBLE MAPPING SATISFYING (E.A) CONDITION BY AN IMPLICIT RELATION IN b-METRIC SPACE

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Abstract: We establish common fixed-point theorems for two self-mapping satisfying the (E.A) condition. The mappings are weakly compatible and include a point of coincidence in b −metric space(X, d, s). The mappings are satisfying contractive conditions defined by a category of implicit relations in six variables. Our theorems might be considered because the extensions of the key results on b-metric space of Mohamed Akkouchi, common fixed-point theorems for 2 self-mappings of a b-metric space under an implicit relation, Hacettepe Journal of mathematics and statistics volume 40(6) (2011), 805-810 [1].

Keywords: b-metric space; Implicit relations, Point of coincidence, (E.A) Property.

1. Introduction:

Fixed point theory came into existence with the well-designed results of contraction mapping principle given by Banach [2] in 1922. Researchers have generalized this result by decontaminating the contraction condition and replacing the metric space in several spaces. With the aim of generalization of the Banach contraction principle [2], Bakhtin [3] and Czerwik [4] was observed a weaker condition rather than the triangular inequality of metric spaces. They called these spaces as b −metric spaces.

On the additional hand, in 1997 Popa [5] introduced the concept of an implicit function within the contractive condition. This thought was the source of several common fixed point and theorems of coincidence point in numerous ambient spaces. In 1912, Berinde [6] obtained some contractive fixed-point theorems for strong contractions satisfying an implicit relation. Several conventional and common fixed-point theorems which unified via self-mappings satisfying implicit relation were proved in [7-19]. The aim of this paper is to analyze possible extension of Mohamed Akkouchi work on b-metric space.

2. Preliminaries:

Definition 2.1 In [20] Jungck defined two self-mappings $S \, and \, T$ of metric space(X, d) to be compatible if $\lim_{n\to\infty}(ST_n\,, TS_n) = 0$,whenever $\{x_n\}$is a sequence in X

and $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t, for\ some\ t \in X.$
The concept of compatibility was recycled by many authors to prove existence theorems in common fixed-point theory.
Definition 2.2 [7] Let metric space (X, d) has two self-mappings S and T.We say that T and S satisfy property (E.A) if there exists a sequence $\{x_n\}$ in X
and

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t, for\ each\ t \in X.$$

Definition 2.3 [20] Two self-mappings S and T of a metric space (X, d) are said to be weakly compatible if $Tu = Su$, for $u \in X$ implies $TSu = STu$.
Popa [21] introduced a system of implicit functions to prove new common fixed-point theorems. To designate the implicit function of Popa [21], let ψ be the family of real lower Semi-continuous funtions F $(t_1, t_2, \dots., t_6): \Re_+{}^6 \to \Re$ satisfying the subsequent conditions:

(F1) F is non-increasing within the variables t_5 and t_6 ,
(F2) There exist $h \in (0,1)$specified for each u, $v \geq 0$ with
    (F2)$_a$ F (u, v, v, u, u + v, 0) $\leq$ 0, or
    (F2)$_b$ F(u, v, u, v, 0, u+v) $\leq$ 0 we have got$u \leq hv$, and
(F3) F (u, u, 0, 0, u, u) >0, $\forall\ u > 0$.

The method of implicit relations has been extensively used in metric fixed point theory.
Many common fixed-point theorems were integrated and generalized by this method. Now, in metric fixed point theory, we will find a huge number of papers which are using several kinds of implicit relations. The method in powerful and effective in the study of common fixed points.
In 2008, Imdad and Ali [22] use the class ψ and established the following result
Theorem: 2.4 [22]. Let T and I are two self-mappings of a metric space (X, d)satisfied

(i) Mappings T and I satisfy the (E, A) property,
(ii) $F(d(Tx, Ty), d(Ix, Iy), d(Ix, Tx), d(Iy, Ty), d(Ix, Ty), d(Iy, Tx)) \leq 0$, for each $x, y \in X$ where $F \in \psi$,
(iii) I(X) may be a complete subspace of X, then
    (a) The pair (T, I) consumes a point of coincidence,
    (b) The pair (T, I) include a common fixed point provided it should be weakly compatible.

In their paper [25], J. Ali and M. Imdad have established some general common fixed-point theorems by employing a class of implicit relations with weaker conditions than those of the system ψ.

3. Implicit relations

Let $s \geq 1$ be fixed and $\mathcal{F}_s$ is that the set of all real lower semi-continuous functions $F(t_1, \dots, t_6) : R_+{}^6 \to R$ satisfying the succeeding conditions:

(P1) (s) F is nondecreasing in the variable t_1 and nonincreasing in the variable t_6,
(P2) $(s)\ F\ (1/s\ ,0,0,t,st,0)\ >\ 0, for\ all\ t\ >\ 0, and$
(P3) $F(t,t,0,0,t,t)\ >\ 0, for\ all\ t\ >\ 0.$
In particular the category F_1 is that the set of all real lower semi continuous functions $F\ (t_1, \dots, t_6): {R_+}^6 \rightarrow R$ satisfying the subsequent conditions:
(P1) F is nondecreasing within the variable t_1 and nonincreasing within the variable t_6,
(P2) $F(t,0,0,t,t,0) >\ 0, for\ all\ t\ >\ 0, and$
(P3) $F(t,t,0,0,t,t)\ >\ 0, for\ all\ t\ >\ 0.$

Let s be a given number in the set $[1,\infty)$.
Example 3.1: Let s be a given number in the set $[1,\infty)$.
and $F(t_1, \dots\ ,\ t_6) := \ t_1 - \ qsm\ max\ \{t_2 \dots , t_6\}$.
Where m is any nonnegative integer and $< \frac{1}{s^{m+2}}$.
(P1) : clear.
(P2) : $F\ (\frac{1}{s}\ t,0,0,t,st,0)\ =\ \frac{t}{s}\ (1\ -\ qsm+2)\ >\ 0$, for all t > 0.
(P3) : $F(t,t,0,0,t,t)\ =\ t\ (1 - \ qsm)\ >\ 0, for\ all\ t\ >\ 0.$

Example 3.2: Let s be a given number in the set $[1,\infty)$.
and $F(t_1, \dots t_6) :=\ t_1 t_2 - \ at_2 t_3 - \ bt_4 t_5 - \ ct_5 t_6,$
where $a\ >\ 0, b\ < \frac{1}{s^3}\ and\ c\ <\ 1.$
(P1) : Clear
(P2) : $F(\frac{1}{s}\ t,0,0,t,st,0)\ =\ \frac{t^2}{s^2}\ \ (1 - \ bs^3)\ >\ 0. For\ all\ t\ >\ 0.$
(P3) : $F(t,t,0,0,t,t)\ =\ t_2\ (1-c)\ >\ 0, for\ all\ t\ >\ 0.$

4. A general result on symmetric spaces:

Definition 4.1: Let X be a nonempty set. A symmetric on X could be a non-negative real function on X x Xsatisfying
(i) $d(x,y)\ =\ 0$ if and on condition that$x\ =\ y$,
(ii) $d(x,y)\ =\ d(y,x)$, for all $x,y\ \in X$.
Some fixed-point theorems in symmetric spaces for occasionally weakly compatible mappings are proved in [26].
Let X be a nonempty set. Let A belongs to self-mappings X. we note that Fix (A) denotes the set of common fixed point of A and coin (A) the set of coincidence points of A.
Point of coincidence: Let $S,T: X\ \rightarrow X$ be two self mappings of X. A point $p\ \in X$ is claimed to be a point of coincidence of mappings S and T if there exists $u\ \in X$in which$p = Su = Tu$.
The following lemma was proved by V. Popa in [23].

Lemma: 4.2 [23] let X be a nonempty set with s symmetric d and f, g, S and T self-mappings of X specified
(3.1) $F(d(fx,gy),d(Sx,Ty),d(fx,Sx),d(gy,Ty),d(fx,Sy),d(gy,Sx) \leq 0$ for all $x,y \in X$, where F satisfies property (P_3). If there are $x,y \in X$ such as $fx = Sx$ and $gy = Ty$, then f and S have a unique point of coincidence $u = fx = Sx$, and g and T have a unique point of coincidence $v = gy = Ty$.The concept of a b-metric space was introduced by S. Czerwik (see [4] and [24]). We recall the resulting definition.

b-metric space:

Definition 4.3 [4] let X be a (nonempty) set and a given real number$s \geq 1$. A function $d: X \times X \to R_+$ (nonnegative real numbers) is entitled a b-metric providing: for all $x,y,z \in X$,
(bm-1) $d(x,y) = 0$ iff $x = y$,
(bm-2) $d(x,y) = d(y,x)$,
(bm-3) $d(x,z) \leq s\,[d(x,y) + d(y,z)]$.

The pair (X,d) is called a b-metric space with parameter s.
We remark that a metric space is evidently a b-metric space. However, S. Czerwik (see [4],[24]) has shown that a b-metric on X need not be a metric on X.
Let d be a b-metric with parameter s on a set X. As with in the metric case, the b-metric d induces a topology. The space X will be furnished with this topology associated to d.Particularly a sequence$\{x_n\}$converges to a point $x \in X$ if $\lim_{n\to\infty} d(x_n,x) = 0$. Almost all the concepts and result obtained for metric spaces can be extended to the case of b-metric spaces. For a large number of results concerning b-metric spaces, the reader is invited to refer the papers [4] and [24].
Theorem: 4.4 (Mohamed Akkouchi [1]) Let (X,d) be a b-metric space with parameter s. let S and T be self-mappings of X such that:

(i) T and S satisfy the (E.A) property,
(ii) $F(d(Tx,Ty),d(Sx,Sy),d(Sx,Tx),d(Sy,Ty),d(Sx,Ty),d(Sy,Tx)) \leq 0$ for each $x,y \in X$, where $F \in F_s$,
(iii) S(X) is a closed subspace of X. then
    (a) The pair (T,S) has a point of coincidence,
    (b) For all $x,y \in$ coin ({S, T}), we have $Sx = Sy = Tx = Ty$,
    (c) The pair (T, S) has a unique common fixed point provided it is weakly compatible.

Main Results

The aim of this paper is to analyze a possible extension of **Mohamed Akkouchi [1]**to the case of b-metric spaces (introduced by S, Czerwik [4] and [24], by employing a suitable system of implicit relations.

We establish the existence of unique common fixed point for a weakly compatible pair of self-mappings of a b-metric space in main results of theorem 4.1 and 4.2 of this paper. The paper contains three sections.

Theorem 4.5 Let (X, d) be a b-metric space with parameter s, where $s \geq 1$.
Let S *and* T be two self-mapping of X such that:

(i) T *and* S satisfy (E.A) property,

(ii) $F\{ d(Tx,Ty), d(Sx,Sy), d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx)\} =$
$F\{d(Tx,Ty) - q\, max\, (d(Sx,Sy), d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx))\} \leq 0$
Where, $q < \frac{1}{s^2}$ for each $x, y \in X$, $F \in Ƒ_s$,

(iii) $S(X)$ is a closed subspace of X.

Then

(a) The pair of (T, S) has a point of coincidence,

(b) For all $x, y \in coin\, (\{S,T\})$, we have $Sx = Sy = Tx = Ty$,

(c) The pair (T, S) has a unique common fixed point provided it is weakly compatible.

Proof: Since T *and* Ssatisfy the property (E.A), there exists in X a sequence $\{x_n\}$ satisfying $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t$, for some $t \in X$.
Since $S(X)$ is closed, there exists a point $a \in X$ such that $t = \lim_{n\to\infty} Sx_n = Sa$.Also, we have
$t = \lim_{n\to\infty} Tx_n = Sa$. To get a contradiction, suppose that $Sa \neq Ta$. Then by using (ii) for $x = x_n$ *and* $y = a$, we obtain that

$$F\{d(Tx_n, Ta) - q\, max\, (d(Sx_n, Sa), d(Sx_n, Tx_n), d(Sa, Ta), d(Sx_n, Ta), d(Sa, Tx_n))\} \leq 0$$

Since $d(Sa, Ta) - s\, d(Sa, Tx_n) \leq s\, d(Tx_n, Ta)$, and F is nondecreasing in the first variable, then we get

$$F\left\{\left(\frac{1}{s} d(Sa,Ta) - d(Sa,Tx_n)\right) - q\, max\big(d(Sx_n,Sa), d(Sx_n,Tx_n), d(Sa,Ta), d(Sx_n,Ta), d(Sa,Tx_n)\big)\right\} \leq 0$$

Since d is a b −metric with parameter s, then we have
$d(Sx_n, Ta) \leq s[\, d(Sx_n, Sa) + d(Sa, Ta)]$.
Since F is non increasing in the fifth variable then we get

$$F\left\{\left(\frac{1}{s} d(Sa,Ta) - d(Sa,Tx_n)\right) - q\, max\begin{pmatrix} d(Sx_n,Sa), d(Sx_n,Tx_n), d(Sa,Ta), \\ s[\, d(Sx_n,Sa) + d(Sa,Ta)], d(Sa,Tx_n)\end{pmatrix}\right\} \leq 0.$$

It is easy to show that $\lim_{n\to\infty} d(Sx_n, Tx_n) = 0$, so by $n \to \infty$ and using the continuity of F,
That is $t = \lim_{n\to\infty} Sx_n = Sa$ and $t = \lim_{n\to\infty} Tx_n = Sa$,
we get:

$$F\left\{\left(\frac{1}{s} d(Sa,Ta) - 0\right) - q\, max \begin{pmatrix} 0,0, d(Sa,Ta), \\ s[\,0 + d(Sa,Ta)], 0 \end{pmatrix}\right\} \leq 0.$$

$$\Rightarrow F\left\{\frac{1}{s} d(Sa,Ta) - q\, max(0,0, d(Sa,Ta), sd(Sa,Ta), 0\,)\right\} \leq 0$$

$$\Rightarrow F\left\{\frac{1}{s} d(Sa,Ta) - q\, sd(Sa,Ta)\right\} \leq 0$$

$$\Rightarrow \frac{1}{s} d(Sa,Ta) \leq q\, sd(Sa,Ta)$$

$$\Rightarrow \frac{1}{s} \leq q\, s \text{ for all } d(Sa,Ta) < 0$$

$$\Rightarrow \frac{1}{s^2} \leq q$$

Which is contradiction of $P_1(s)$. Hence,$Sa = Ta$. That is a is a coincidence point of the pair $\{S,T\}$. We set $z = Sa = Ta$. So, z is a point of coincidence of the pair $\{S,T\}$.
Suppose that $x, y \in coin(\{S,T\})$.As in the proof of Lemma 3.1, one can prove that $Sx = Sy$.
Suppose that $S\ and\ T$ are weakly compatible. Then $S\ and\ T$ commute at the point $z = Sa = Ta$.
Next, we show that z is a common fixed point of $T and\ S$. We have
$Tz = TSa = STa = Sz$.
By (ii) for $x = a\ \ and\ \ y = z$ we have successively:

$$F\big\{d(Ta,Tz) - \ q\, max\,\big(\, d(Sa,Sz), d(Sa,Ta), d(Sz,Tz), d(Sa,Tz), d(Sz,Ta)\big)\big\} \leq 0$$

$$\Rightarrow F\big\{d(z,Tz) - \ q\, max\,\big(\, d(z,Tz), d(z\,,z), d(Tz,Tz), d(z,Tz), d(Tz,z)\big)\big\} \leq 0$$

$$\Rightarrow F\big\{d(z,Tz) - \ q\, max\,\big(\, d(z,Tz), 0,0, d(z,Tz), d(Tz,z)\big)\big\} \leq 0$$

$$\Rightarrow F\big\{d(z,Tz) - \ q\, max\,\big(\, d(z,Tz), 0,0, d(z,Tz), d(Tz,z)\big)\big\} \leq 0$$

$$\Rightarrow F\{d(z,Tz) - \ q\, d(z,Tz)\} \leq 0$$

$$\Rightarrow (1 - \ q) \leq 0 \text{ for all } d(z,Tz) \leq 0$$

$$\Rightarrow q \geq 1,$$

Which is contradiction of (P_3) if $d(z,Tz) \neq 0$. Hence, $Tz = z$ and $z = Sz = Tz$. Therefore z is a common fixed point of $S\ and\ T$.
Suppose that $Su = Tu = u\ and\ \ Sv = Tv = v\ \ for\ u \neq v$.Then, by (ii) we have successively:

$$F\big\{d(Tu,Tv) - \ q\, max\,\big(\, d(Su,Sv), d(Su,Tu), d(Sv,Tv), d(Su,Tv), d(Sv,Tu)\big)\big\} \leq 0$$

$\Rightarrow F\{d(u,v) - \; q \; max\,(d(u,v), d(u,u), d(v,v), d(u,v), d(v,u))\} \leq 0$
$\Rightarrow F\{d(u,v) - \; q\, d(u,v)\} \leq 0$
$\Rightarrow (1 - \; q) \leq 0$ for all $d(u,v) \leq 0$
$\Rightarrow \; q \geq 1,$

Which is contradiction to (P_3) if $d(u,v) \neq 0$. Hence $u = v$. this completes the proof.
More precisely, we have subsequent theorem.

Theorem 4.6. Let X is a (nonempty) set and s is a parameter given as a real number such that $s \geq 1$. A function $d: X \times X \to R_+$ be a b-metric space. Let $S \; and \; T$ be self-mappings such that:
(a) $T \; and \; S$ satisfy the (E.A) property,
(b) $F\{ d(Tx,Ty), d(Sx,Sy), d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx)\} = F\{d(Tx,Ty)^2 - \; bd(Sx,Sy)d(Sx,Tx) - cd(Sy,Ty)d(Sx,Ty) - ed(Sx,Ty)d(Sy,Tx)\} \leq 0$ for each $x, y \in X$,
Where $F \in \mathfrak{F}_s$, $b \geq 0$, $c < \frac{1}{s^{\,3}}$ and $e < 1$.
© $S(X)$is a closed subspace of X.
Then
(i) The pair (T,S) has a point of coincidence.
(ii) The pair (T,S) has a unique common fixed point provided it is weakly compatible.

Proof: Since $S(X)$ is closed, there exists a point $a \in X$ such that$t = \lim_{n\to\infty} Sx_n = Sa$.
Since $T \; and \; S$ satisfy the property (E.A), there exists in X a sequence $\{x_n\}$ satisfying
$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t, \; for \; some \; t \in X.$
Therefore, $t = \lim_{n\to\infty} Tx_n = Sa$.To get contradiction, suppose that $Sa \neq Ta$. Then by using (b) for $x = x_n$ and $y = a$, we obtain that:

$$F\{d(Tx_n,Ta)^2 - \; bd(Sx_n,Sa)d(Sx_n,Tx_n) - cd(Sa,Ta)d(Sx_n,Ta) - ed(Sx_n,Ta)d(Sa,Tx_n)\} \leq 0$$

Since$d$ is a b $-$metric with parameter s then we have:
$d(Sa,Ta) - s\,d(Sa,Tx_n) \leq sd(Tx_n,Ta)$ and F is nondecreasing in first variable, then we get

$$\Longrightarrow F\left\{\left(\frac{1}{s}d(Sa,Ta) - \; d(Sa,Tx_n)\right)^2 - \; bd(Sx_n,Sa)d(Sx_n,Tx_n) - cd(Sa,Ta)d(Sx_n,Ta) - ed(Sx_n,Ta)d(Sa,Tx_n)\right\} \leq 0$$

$$\Rightarrow F\left\{\left(\frac{1}{s}d(Sa,Ta) - d(Sa,Sa)\right)^2 - bd(Sa,Sa)d(Sx_n,Tx_n) - cd(Sa,Ta)d(Sx_n,Ta) - ed(Sx_n,Ta)d(Sa,Tx_n)\right\} \leq 0$$

Since d is a $b-$metric with parameter s, then we have
$d(Sx_n,Ta) \leq s\{d(Sx_n,Sa) + d(Sa,Ta)\}$
Since F is nonincreasing in fifth variable then we get

$$\Rightarrow F\left\{\left(\frac{1}{s}d(Sa,Ta) - d(Sa,Sa)\right)^2 - bd(Sa,Sa)d(Sx_n,Tx_n) - cd(Sa,Ta)s\{d(Sx_n,Sa) + d(Sa,Ta)\} - es\{d(Sx_n,Sa) + d(Sa,Ta)\}d(Sa,Tx_n)\right\} \leq 0$$

It is to show that $\lim_{n\to\infty} d(Sx_n,Tx_n) = 0$, so by letting $n$tends to infinity and using the continuity of F, we get:

$$\Rightarrow F\left\{\left(\frac{1}{s}d(Sa,Ta) - 0\right)^2 - b\,(0) - cd(Sa,Ta)s\{0 + d(Sa,Ta)\} - es\{0 + d(Sa,Ta)\}(0)\right\} \leq 0$$

$$\Rightarrow F\left\{\left(\frac{1}{s}d(Sa,Ta)\right)^2 - 0 - cd(Sa,Ta)s\{d(Sa,Ta)\} - 0\right\} \leq 0$$

$$\Rightarrow F\left\{\left(\frac{1}{s}d(Sa,Ta)\right)^2 - 0 - cd(Sa,Ta)s\{d(Sa,Ta)\} - 0\right\} \leq 0$$

$$\Rightarrow \left\{\left(d(Sa,Ta)\right)^2\left(\frac{1}{s^2} - cs\right)\right\} \leq 0$$

Which is a contradiction of $P_1(s)$. Hence, $Sa = Ta$. That is a is a coincidence point of the pair $\{S,T\}$. We set $z = Sa = Ta$. So, z is a point of coincidence of the pair $\{S,T\}$.

Let, $S\ and\ T$ are weakly compatible. Then $S\ and\ T$ compute at the point $z = Sa = Ta$. Next, we show that z is a common fixed point of $T\ and\ S$. we have $Tz = TSa = STa = Sz$.

By (b) for $x = a\ and\ y = z$ we have successively:

$$F\{d(Ta,Tz)^2 - bd(Sa,Sz)d(Sa,Ta) - cd(Sz,Tz)d(Sa,Tz) - ed(Sa,Tz)d(Sz,Ta)\} \leq 0$$

$$\Rightarrow F\{d(z,Tz)^2 - bd(z,Tz)d(z,z) - cd(Tz,Tz)d(z,Tz) - ed(z,Tz)d(Tz,z)\} \leq 0$$

$$\Rightarrow F\{d(z,Tz)^2 - 0 - 0 - ed(z,Tz)d(Tz,z)\} \leq 0$$

$$\Rightarrow F\{d(z,Tz)^2(1-e)\} \leq 0.$$

Which is contradiction of (P_3) if $d(z,Tz) \neq 0$. Hence, $z = Tz$ and $Sz = Tz = z$. Therefore z is a common fixed point of $S\ and\ T$.

Suppose that $Su = Tu = u$ and $Sv = Tv = v \, for \, u \neq v$. Then, by (b) we have successively

$$F\{d(Tu,Tv)^2 - \; bd(Su,Sv)d(Su,Tu) - cd(Sv,Tv)d(Su,Tv) \\ - ed(Su,Tv)d(Sv,Tu)\} \leq 0$$

$$\Rightarrow F\{d(u,v)^2 - \; bd(u,v)d(u,u) - cd(v,v)d(u,v) - ed(u,v)d(v,u)\} \leq 0$$

$$\Rightarrow F\{d(u,v)^2(1-e)\} \leq 0.$$

Which is a contradiction to (P_3) $if \;\; d(u,v) \neq 0$. Hence, $u = v$. This completes the prove.

Corollary 4.7: Let $s \geq 1$ and let d be a b −metric space on a set X with parameter s.let S and T two weakly compatible self-mappings of X such that:

$$F\{d(Tx,Ty)^3 - \; bd(Tx,Ty)^2 d(Sx,Sy) - cd(Tx,Ty)d(Sy,Ty)d(Sx,Ty) \\ - ed(Sx,Tx)d(Sx,Ty)d(Sy,Tx)\} \leq 0,$$

for each $(x,y) \in X^2$ and $F \in \mathfrak{F}_s$,Where $b < 1 \; and \; c < \frac{1}{s^3}$.

If $S(X)$ is a closed subspace of X, then $S \; and \; T$ have a common fixed point.

Corollary 4.8: Let (X,d) be a b −metric space with parameter s. Let $S \; and \; T$ be two self-mapping of X such that:

$$F\big\{d(Tx,Ty) \\ - \; q\, s^m \, max\left(d(Sx,Sy), d(Sx,Tx), d(Sy,Ty), d(Sx,Ty), d(Sy,Tx)\right)\big\} \leq 0$$

Where m is any nonnegative integer and $q < \frac{1}{s^{m+2}}$.

For each $x, y \in X$, where $F \in \mathfrak{F}_s$,

If $T \; and \; S$ satisfy the (E.A) property and $S(X)$is a closed subspace of X. Then (T,S) is weakly compatible with a unique common fixed point and has a point of coincidence.

5. References

1. Mohamed Akkouchi, Common fixed point theorems for two self mappings of a b-metric space under an implicit relation, Hacettepe Journal of Mathematics and Statistics Volume 40 (6) (2011), 805 – 810.

2. Banach, S. 1922. Sur les ope´rations dans les ensembles abstraits et leur application aux equation sintegrales. Fundamenta Mathematicae 3: 133–181.

3. Bakhtin, I.A.: The contraction principle in quasimetric spaces. Funct. Anal. 30, 26–37 (1989)

4. Czerwik, S. Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Ostraviensis 1, 5–11, 1993.

5. Popa, V., A Fixed point theorems for implicit contractive mappings. Studiisi Cercetari Stiintifice. Serious: Mathematics. Universitatea din Bacau 7 (1997): 127–133, 1997.

6. Berinde, V,. Approximating fixed points of implicit almost contractions. Hacettepe Journal of Mathematics and Statistics 40: 93–102., 2012.

7. Abbas, M., and D. Ilic. 2010. Common fixed points of generalized almost non-expansive mappings. Filomat 24 (3): 11–18.

8. Akkouchi, M., and V. Popa. 2010. Well-posedness of fixed point problem for mappings satisfying an implicit relation. Demonstration Mathematica XLII I (4): 923–930.

9. Ali, J., and M. Imdad. 2009. Unifying a multitude of common fixed point theorems employing an implicit relation. Communications of the Korean Mathematical Society 24:41–55.

10. Aliouche, A., and A. Djoudi. 2007. Common fixed point theorems for mappings satisfying an implicit relation without decreasing assumption. Hacettepe Journal of Mathematics and Statistics 36: 11–18.

11. Aydi, H., M. Jellali, and E. Karapinar. 2016. On fixed point results for a-implicit contractions in quasi-metric spaces and consequences. Nonlinear Analysis: Modelling and Control 21 (1): 40–56.

12. Imdad, M., M. Asim, and R. Gubran. 2018. Common fixed point theorems for g-generalized contractive mappings in b-metric spaces. Indian Journal of Mathematics 60 (1): 85–105.

13. Imdad, M., S. Kumar, and M.S. Khan. 2002. Remarks on some fixed point theorems satisfying implicit relations. Dedicated to the memory of Prof. Dr. Naza Tanovic´-Miller. RadoviMathematicki11: 135–143.123

14. Jleli, M., E. Karapinar, and B. Samet. 2013. Further remarks on fixed-point theorems in the context of partial metric spaces. Abstract and Applied Analysis 2013: 715456. https://doi.org/10.1155/2013/715456.

15. Petriciu, A., and V. Popa. 2020. A general fixed point theorem of C ´ iric´ type in quasi-partial metric spaces. Novi Sad Journal of Mathematics 50 (2): 1–6.

16. Popa, V. 1999b. Some fixed point theorems for compatible mappings satisfying an implicit relation. Demonstration Mathematica 32 (1): 157–163.

17. Popa, V. 2001. A general fixed point theorem for weakly compatible mappings in compact metric spaces. Turkish Journal of Mathematics 25: 465–474.

18. Popa, V. 2002. Fixed points for non surjective expansion mappings satisfying an implicit relation. Buletinuls¸tiint¸ ific Al Universitatii Baia Mare, Seria B, Fascicola Matematicaˇ-informaticaˇ18:105–108.

19. Vetro, C., and F. Vetro. 2013. Common fixed points of mappings satisfying implicit relations in partial-metric spaces. Journal of Nonlinear Sciences and Applications 6 (3): 152–161.

20. Jungck, G. Common fixed points for noncontinuous nonself mappings on nonmetric spaces, Far. East, J. Math. Sci. 4, 199–215, 1996.

21. Popa, V. Some fixed point theorems for compatible mappings satisfying an implicit relation Demonstratio Math. 32, 157–163, 1999.

22. Imdad, M. and Ali, J. Jungck's common fixed point theorem and E.A Property, Acta Mathematica Sinica, English Series 24 (1), 87–94, 2008.

23. Popa, V. General fixed point theorems for occasionally weakly compatible mappings, Stud. Cerc. St. Ser. Mat. Univ. Bacˇau 17, Supplement, 193–200, 2007.

24. Czerwik, S. Nonlinear set-valued contraction mappings in b-metric spaces, Atti Sem. Mat. Fis. Univ. Modena, 46 (2), 263–276, 1998.

25. Ali, J. and Imdad, M. An implicit function implies several contraction conditions Sarajevo J. Math. 4 (17), 269–285, 2008.

26. Jungck, G. and Rhoades, B.E. Fixed point theorems for occasionally weakly compatible mappings, Fixed Point Theory 7 (2), 287–296, 2006.