A GENERAL REVIEW ON THE CONVEXITY STRUCTURE OF FIXED-POINT THEOREM

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Abstract
In the current paper we discussed some applications of fixed-point theorem especially convexity structure of fixed-point theorem along with pinot's formation there are some conditions on which the convexity structure was developed through the Kirks theorem which depends on metric space of fixed-point theorem and also showing some type of compactness and the characteristics of normal space we focused on some theorem in which we saw that how fixed point follows the property of convexity and how it became normal and compact studied the property of expansive mapping has fixed point

Hence various steps in unmetrical analysis and its theory of approximations which is in successive form of fixed-point theorem and it mapping in this paper We give a proper frame work to the concept of convex of metric space through the way of convexity structure by using this application we develop new dimension in metric space.

Keywords - Convexity structure, metric space, existence, numerical analysis

Introduction

In the field of Advanced mathematics fixed point theory is considered as important and powerful and also trustful Resource and also it is supposed that an important part of nonlinear analysis in some last decades it was seen that many researchers studied on this topic and till research in increasingly going on. This theory was originated in 19th century and some part is developed later. Which include solutions of uniqueness and its existence, differential equation. This theory was also associate with different mathematician like Liouville, Peano, Cauchy and many others it is considered that for the development of formulation of branch space fined point theory play an important role but one limitation of this theory was that it unable to give some information about Felix Browder's assumptions. Which was used for the improvement of functional analysis of non-linear form. Which is an important & huge part of mathematics.

If we think wider form of a fixed-point theorem. We can make a statement which shows that mapping S of a space y into a space X enters some more forms of y including y such that sy = y there is a multiple of fined point theorem. Which is useful you the mapping and also for giving the prof in specific types of sequences and conditions among all these conditions some outcomes are considered as a iterates of sequences which forms because of some contradictory conditions. Which later become Cauchy sequence and having the limit function of fixed point with its mapping if we studied the common fixed-point theorem a combined sequence of recreation is fixed for this work.

In the field of abstract space which may include convex space, metric space and also its applications which is Usefully for the solutions of operator equation of nonlinear form. Which contain partial differential equation, ordinary differential equation functional differential equation, integral equation fractional differential equation the solution of above all equation is generally illustrated in the space of specific function the in case of complex situations because of availability of different choices of application of fixed-point theorem and the use of specific properties having space function shows the major impact on it. The theory of fixed point is also called as integrative theory which is useful for the discernment and used for solution of complex problem now in current days there is modernization in fixed point and also in the theory of fixed point

In the current paper we discussed convexity structure of fixed-point theorem. Which depends on the kirk's theorem based on the metric space of fixed-point theorem here we also discussed some theorem regarding to this topic.

The Structures of Convexity

The convexity structure was developed through the Kirks theorem whose abstract formulation was done by Penot in 1977. The function S is called convexity structure when it is an abstract set and having the family of summation (Σ) of subject of S and when it follows the following condition.

i) The empty set Ψ which belongs to summation.

ii) When S belongs to Summation.

iii) and summation should be closed in case of arbitrary intersection.

Let we consider S be a any subset which is convex and which have the element of summation if we consider S is metric space, then we will always consider that closed balls are convex. Let we assume that f (s) be closed ball having the smallest convexity structure.

A (s) is the family which accepts the subset of S if we recollect that P is an acceptable subset of s only. When it will an intersection of closed ball here, we assume this because of theorem of Kirk's which contain some types of compactness and the characteristics of normal structure. It was not new that the generalized attempts can be explain by these two concepts in the form of metric space according to Takahashi in his work studied a very restrictive compact metric spaces and as per pe not conversely explain the term compactness for the structure of convexity. Which was responsible for less compactness in linear form or we can say like weak compactness so here we can say that a convexity structure Σ is compact when every family of subject of Σ having the characteristics of finite intersection properties. Which should be non-empty intersection we can write it as

$(A_j)\ j \in \beta$ with $A_j \in (\Sigma)$ then

$$\cap_{j \varepsilon \beta} A_j \neq \Psi$$

Provided $\cap_{j \varepsilon \beta} A_j \neq \Psi$ for Bf which is finite subset

finite subset of B because as per the normal structure property is a nothing but metric space. That's why it is not showing any complexity for the structure of conversely

Practically. We can say that the ε conversely structure is normal only when there is non-empty and bounded set A εΣ which is not reduced to single point then theorist aεA such that

sup {d (a, l) : l €A} < sup {d (l , m): l. m €A} = diam (A)

Which becomes a pinot's formulations

1) Theorem

Let (S. P) is bounded metric space which is non empty and showing the structure of convexity which becomes normal and compact then only every no expansive mapping T: S – S has a fixed point.

According to Kirk's theorem a function whose all family of convex set is said to be normal and a normal structure are called as branch space. But there their family should be large and it includes some sets. Which is admissible let we take example, consider X^{∞} is branch space. Which showing the power of pinots formulation While X^{∞} not successes to have the properties
of the normal structure but A (X^{∞}) is become normal and compact which denotes that if A be subset of X^{∞} which is nonempty and admissible then only every non expansive napping T: A – A snowing fixed point. Which was evaluated by sine and sordid.

2) Theorem

Let S be admissible subject which is nonempty of X^{∞} then every non expansive mapping T: S – S has fixed point.

As we know that according to kirk's theorem for the proof of minimal invariant set through lemma of Zorn's we used weak compactness for the countable compactness we got some results from the study of Gillespie and Williams by giving some constructive proof so in other word we can say that the convexity structure Σ is consider to satisfy a countable intersection property means for any $(Am)_{1<m}$ with Am € Σ then

$$\square^{\infty}_{M=1} \quad Am \neq \Psi$$

Provided that

M = 1 for many n>1 this weakening is very significant in many feasible conditions by using topology we can't define compactness but it can define sequentially if the convexity structure Σ is countably compact then only it is uniformly normal this results we got in branch spaces.

Which is called as metric translation and it was given by Maluta. When we are dealing with the normal structure which is uniform at that time, we got positive answer for that is we can say that convexity structure is countably compactor a basically compact if A (s) is countably compact and showing its normal ness then only A (s) is compact.

through the way of convexity structure by using this application we develop new dimension in metric space.

3) Theorem -Let (S, d) be a complete metric space, $f : X \to S$ continuous function and (φ_i) sequence of continuous functions such that $\varphi_i : [0, \infty) \to [0, \infty)$ and for each p, q ∈ S, $d(f^i(p), f^i(q)) \leq \varphi_i(d(p, q,))$. Assume also that there exists function: $[0, \infty) \to [0, \infty)$ such that for any $r > 0$, $\varphi(r) < r$, $\varphi(0) = 0$ and $\varphi_i \to \varphi$ uniformly on the range of d. If there exists p ∈ S such that orbit f at p is bounded then f has a unique fixed-point q ∈ S and all sequences of by f converge to q.

Proof. From the statement of the theorem, it follows that φ is continuous, because the sequence

(φ_i) is uniformly convergent. For any $x, y \in X$, $x \neq y$, we have

$\lim d(f^n(x), f^n(y)) \leq \lim \varphi_n d(x, y) = \varphi d(x, y) < d(x, y)$1533

If there exist x, y ∈ X and ε > 0 such that $\lim d(f^n(x), f^n(y)) = \varepsilon$ then there exists k such that

$\Phi(d(f k(x), f k(y))) < \varepsilon$, because φ is continuous, and $\varphi(\varepsilon) < \varepsilon$. This implies that

$\lim d(f^n(x), f^n(y) = \lim d(f^n(f^k(x)), f^n \leq (f^k y))) \leq \lim \varphi_n (d(f^k(x), f^k(y))$

$= \varphi(d(f^k(x), f^k(y)) < \varepsilon$1533

which is a contradiction. So, we obtain that

$\lim d(f^n(x), f^n(y)) = 0,$

for any p, q ∈ X, which implies that all sequences defined by f, are equi-convergent and bounded

Now let $a \in S$ be arbitrary, (a_n) be a sequence of at point a, $Y = (a_n)$ and $F_n = \{p \in Y : d(p, f^k(p)) \leq 1/n, k = 1,.,n\}$. Y is bounded because (a_n) is bounded. From above it is following that F_n is nonempty and since f is continuous F_n is closed, for any n. Also, we have $F_{n+1} \subseteq F_n$. Let (p_n) and (q_n) be arbitrary sequences, such that $p_n, q_n \in F_n$. Let (nj) be a sequence of integers, such that

$Lim\, d(p_n j, q_n j) = \lim d(p_n, q_n)$.

Now we have

$Lim\, d(p_n j, q_n j) \leq \lim (d\, p_n j, f^n j(p_n j) + d(f^n j(p_n j), f^n j(q_n j) + (d(q_n j, f^n j(q_n j)$
$= \lim \varphi_n j\, d(p_n j, q_n j) = \varphi \lim d(p_n j, q_n j)$,1533

and so $\lim d(p_n j, q_n j) = \varphi(\lim d(p_n j, q_n j))$ which implies that $\lim d(p_n j, q_n j) = 0$, because Y is bounded. Thus $\lim d(p_n, q_n) = 0$ and so $\lim d(p_n, q_n) = 0$. This implies that $\lim diam\, F_n = 0$. By completeness of Y follows that there exists z ∈ S such that $\bigcap_{i=1}^{\infty} Fn = \{z\}$. Since $d(z, f(z)) \leq 1/n$ for any n, we have $f(z) = z$. From (1) follows that all sequences of defined by f converge to z.

4)Theorem:

Let P be the Banach space having self-mapping A, B & C of P following.

$\| By - Cx \| \leq \max\{\beta \| Ay - Ax \|, \|Ay - By\|, \|Ay - Cy\|, \| Ax - Cx \|, \|Ax - Cy\|\}$

for ∀ x, y ε y & α , β > 0, 1> α

Let ∃ $x_0 \in y$ such that the sequence {Ayn} can be defined as followed.

1) $Ay_{n+1} = (1 - k_n) Ay_n + Kn\, By_n, \forall\, n>0$

2) $Ay_{n+1} = (1 - k_n) Ay_n + knCy_n\ \forall\, n>o$

Proof -let the sequence $\{Ay_n\}$ can be defend as above then

$\| Aq - Bq \| \leq \| Aq - Ay_{n+1} \| + \|Ay_{n+1} - B_q \|$

$\leq \|Aq - Gy_{n+1}\| + (1 - kn) \|Ay_n - Bq\| + kn \| Cyn - Bq \|$

$\leq \|Aq - Ay_{n+1}\| (1 - kn) \| Ay_{n+1} - Cq \| + Kn\propto \max\{ \beta \|Ayn - Aq\|,$

$\|Ayn - Cyn\|, \|Aq - Bq\|, \|Ayn - Bq\|, \|A q - Byn\|\}$

It was noted that Ayn →q which stated the convergence of Cyn – q

Now if we take n →∞ , then

$\| Aq - Bq\| \leq (1 - f + f\alpha) \|Aq - Bq\|.$

Hence Aq = Bq, so we can say that Bq is the common fixed point of A and B

Similarly, we can find Bq = Cq, so we get Bq is common fixed point of A and B

$\therefore$ Aq = Bq = Cq

By doing same procedure of A at q we obtain Ayn = A^2 yn → Aq and hence Aq = q as for every Banach space is Hausdorff therefore q is common fixed point of A, B and C.

Conclusion

In the current research paper, we focus on one of the applications of fixed-point theorem that is also called as the convexity structure which is related to metric space of fixed-point theorem in our study it was seen that the convexity structure is based on kirik's theorem which was nothing but one significant form of fixed-point theorem it is useful for showing the compactness of the normal space in the field of linear analysis here we also discussed some theorem related to it.

Reference

1. A. Aghajani, K. Nourouzi Convex set in probabilistic formed space chaos, solitons & fractals vol (36) 2008 p.p. 322-328.
2. S.M. Brodskill, P.D. Milman on the center of convex set Dokl. Akkad, SSSR. Vol 59, 1948, p.p.251-258.
3. E.F. Browder non expansive nonlinear operator in branch space proc Nat Accad Sci USA vol 54, 1968 p.p.1041-1044
4. L. Dric, common fixed point there for family on non-set mapping in convex metric space and common fixed-point theorem, fixed point theory and application vol 3, 2005, pp 365-375
5. I.V. Istratescu, Strict convexity and complex strict convexity. Marcel Docketing New York and Basel, 1984
6. N.S.Jeisic, convex structure, normal structure and fixed point theorem in intuitionistic fuzzy metric space chose, vol. 42 2009 pp 290-231
7. B. Schweizer, tactical metric space pacific J. Math vol. 10. 1962. Pp 415-417
8. A.W. Kirik, A fixed point theorem for mapping which do not increase distance. Anner Math Monthly 71 1968. P.p.1004-1007.
9. J.P. Penot fixed point theorem without convexity Analysis nonconvex, Bull Soc, Math, France, vol60. 1980 p.p.142-15.
10. J.W. Takhashi, A fixed point theorem for no expansive mapping in metric space kodai matt Sem Rep. 21. 1968. P.p.325-331.
11. L. Khan, some common fixed-point theorem for mapping family in metrically convex spaces J of Math 1978 p.p.324-331.

12. S.D. Jaggi. some unique fixed-point theorem in aim Journal of pure and applied mathematics vol. 8. Pp 223-230.
13. B.K.Dass, extension of branch contraction theorem through rational expression bull cal math p.p.261-265.
14. K.S. Chaterje fixed point theorem compels and aced Inst Math. 1973, vol 25 pp725-731
15. B. Fisher, A fixed point theorem for compact metric space publ. Inst. Math. Soc, 1976, Vol. 60 p.p.71-78
16. S. Reich some remark concerning contraction mapping Canada math 1970 bull vol 1, pp 121-125.
17. R. Kannan. Some results of fixed-point theorem – II Bulletin of Calcutta Math Soc. Vol. 60, pp.70-75
18. I. Beg: inequalities in metric space with application top meth nonlinear anal 2001, vl. 17. Pp 183-190
19. C.R. Swami Nathan, Normed linear spaces that are uniformly convert in every direction caned J Math 1971. Vol 23. Pp. 1050-1059
20. S. Henrich ultra-product in branch space theory Journal of Angew Math 1980 vol. 313. Pp.72-105