# OVERVIEW OF FIXED-POINT THEORY AND ITS APPLICATIONS: A REVIEW

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**Abstract**

In the present paper, a review on fixed point theory is given. Fixed point theory plays an important role in pure and applied mathematics. The contribution by various authors for said topic is discussed.

**Key words: Fixed point, Common fixed point**

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## Introduction

The theorems concerning the existence of fixed points and their properties are called fixed point theorems. Informally speaking, fixed point theory is a branch of mathematics that attempts to identify all self maps (or self correspondence) under which at least one element is left invariant. Topological fixed point theory, which came from Brouwer's fixed point theorem in 1912. Discrete fixed point theory, it came from Tarski's fixed point theorem in 1955.

Metric fixed point theory, although the basic idea about the metric fixed point theory was known to others earlier, but the credit of making it useful and popular goes to polish mathematician Stefan Banach.

In mathematics, Banach fixed point theorem (also known as the contraction mapping theorem or contraction mapping principle) is an important tool in the theory of metric space. It guarantees the existence and uniqueness of fixed points of certain self maps of metric space, and provides a contractive method to find those fixed points. The theorem is named after Stefan Banach (1892-1945), and was first stated by him in 1922.

In 2012, Shyam lal singh *et al.*,[12] presented general common fixed point theorems and applications. Later, Farshid khojasteh *et al.*, [11] defined it $\theta$-Metric Space: A Generalization in 2013 and in 2014, N.Hussain *et al.*, [10] extended fixed point theory in α-complete metric spaces with applications. In addition, Rajendra pant *et al.*, [8] studied on some new fixed point theorems in partial metric spaces with applications in 2017, after that Amelia BUCUR [9] obtained about applications of the fixed point theory. In 2018, Duran turkoglu *et al.*, [5] found fixed point theorems in a new type of modular metric spaces. Common fixed point theorems in metric

spaces with applications was introduced in 2018 by Puspendra semwal and Komal [6] which was more general than the well known partial metric spaces that were introduced in 2018 by Kanayo stella eke and Jimevwo godwin oghonyon [7] which proved some fixed point theorems in ordered partial metric spaces with applications and in 2019, Khaled berrah *et al.,* [4] presented applications and theorem on common fixed point theorem in complex valued b-metric space. In 2020, Lili chen *et al.,* [1] proved several fixed point theorems in convex-b metric space and applications. After that Eliyas zinab *et al.*,[2] introduced a fixed point theorem for generalized weakly contractive mappings in b-metric spaces in 2020. Later, Karim chaira *et al.,* [3] defined an fisher fixed point results in generalized metric spaces with a graph.

**Literature Review**

**Lili Chen [1] *et al.,* (2020)** studied on several fixed point theorems in convex b-metric spaces and applications. He said that the indicating a way of generalizing Mann's iteration algorithm and a series of fixed point results in the frame work of b-metric spaces. First, the concept of a convex b- metric space by means of a convex structure is introduced and Mann's iteration algorithm is extended to this space. Then the help of Mann's iteration scheme, strong convergence theorems for two types of contraction mappings in convex b-metric spaces are obtained. Moreover, the problems of T-stability of Mann's iteration procedure for the above mappings in complete convex b-metric spaces are obtained.

**Eliyas Zinab [2] *et al.,* (2020)** worked on a fixed-point theorem for generalized weakly contractive mappings in b-metric spaces. They established a fixed-point theorem for generalized weakly contractive mappings in the setting of b-metric spaces and concluded the existence and uniqueness of a fixed point for self- mappings satisfying the established theorem. Their result extends and generalizes the result of choudhary.

**Karim Chaira [3] *et al.,* (2020)** proved that fisher fixed point results in generalized metric spaces with a graph. Discussed about the fisher's fixed point theorem for mappings defined on a generalized metric space endowed with a graph. This work should be seen as a generalization of the classical fisher fixed point theorem. It extends some recent works on the enlargement of Banach contraction principle to generalize metric space with graph. Currently, fixed point theory is a very active area of research because of its applications in multiple fields. It concerns the results which indicate that, under certain conditions, self-mapping on a set admits a fixed point. Among all the results in metric fixed point theory, the Banach Contraction Principle is the most celebrated one due to its simplicity and ease of application in major areas of mathematics.

**Khaled Berrah [4] *et al.,* (2019)** studied on applications and theorem on common fixed point in complex valued b-metric space. They provided a common fixed point theorem for four self mappings satisfying rational contraction has been proved in complex valued b-metric space. Results of this study proving both the existence and the uniqueness of a common solution of the system of Urysohn integral equations and the existence of a unique solution for linear equations system. The main purpose of their paper was to present common fixed point results of four self-mappings to satisfy a rational inequality on complex valued b- metric spaces.

**Duran Turkoglu [5] *et al.*, (2018)** worked on fixed point theorems in a new type of modular metric spaces. He said that the, considering both a modular metric space in the sense of jleli and samet, they introduce a new concept of generalized modular metric space. Then he presents some examples showing that the generalized modular metric space includes some kind of metric structure. They provided some fixed-point results for both contraction and quasi contraction type mappings on generalized modular metric spaces.

**Puspendra Semwal and Komal [6], (2018)** studied on common fixed point theorems in metric spaces with applications. He investigates the existence and uniqueness of common fixed point theorems for certain contractive type of mappings. As an application the existence and uniqueness of common solutions for a system of functional equations arising in dynamic programming are discuss by using their results.

**Kanayo Stella Eke and Jimevwo Godwin Oghonyon [7], (2018)** worked on some fixed point theorems in partial metric spaces with applications. This study said that the class of generalized weakly C-contractive mappings in partial metric space and he proved some fixed point results for such maps in ordered partial metric spaces without exploiting the continuity of any of the functions, they also established fixed point theorem for the integral type of these maps. There result generalizes the results of Chen and Zhu and others in the literature.

**Rajendra Pant [8] *et al.*, (2017)** studied on some new fixed point theorems in partial metric space with applications. Introduced the authors are able to present some general fixed point results for a wider class of mappings in partial metric spaces with illustrative examples and an application. Their results presented herein cannot be directly obtained from the corresponding metric space versions.

**Amelia BUCUR [9], (2017)** worked on about applications of the fixed point theory. They concluded that the fixed point theory has had many applications in the last decades. Its applications are very useful and interesting to the optimization theory, to the game theory, to conflict situations, but also to the mathematical modeling of quality and its management.

**N. Hussain [10] *et al.*, (2014)** studied on fixed point theory in α-complete metric spaces with applications. He concluded that the new concepts of α-η-complete metric space and α-η-continuous function and fixed point results for modified α-η-ψ-rational contraction mappings in α-η-complete metric spaces. They derived some Suzuki type fixed point theorems and new fixed point theorems for ψ-graphic-rational contractions.

**Farshid Khojasteh [11] *et al.*, (2013)** studied on $\theta$-Metric Space: A generalization. He introduces the notion of $\theta$-metric as a generalization of a metric by replacing the triangle inequality with a more generalized inequality. They investigate the topology of the spaces induced by a $\theta$-metric and present some essential properties of it. Further, they give characterization of well-known fixed point theorems, such as the Banach and Caristi types in the context of such spaces.

**Shyam Lal Singh [12] *et al.*, (2012)** worked on general common fixed point theorems and applications. The main purpose of their paper is a common fixed point theorem for a pair of multivalued map on a complete metric space extending a result of Doric and Lazovic (2011) for a multivalued map on a metric space satisfying Ciric-Suzuki-Type-generalized contraction. They obtained a generalization of an important common fixed point theorem of Ciric (1974). Existence of a common solution for a class of functional equations arising in dynamic programming is also discussed.

**Ramakant Bhardwaj et.al.[13] [2007],** worked on Application of fixed-point theory in metric space. In this manuscript the basic results on fixed point theory are reviewed and authors established some new results on metric space for rational expressions which satisfied previous well-known results.

**Thidaporn Seangwattana**, **et.al. [14,15]** "A New Random *S*-Contraction Mapping for Finding a Common Random Best Proximity Point" is discussed in this research paper. The results are in generalized form of well-known results. Results are also discussed for best proximity points.

**Sneha A Khandit et.al.**[16], authors discussed on the topic "Invariant Point theorems in partial ordered soft metric like spaces". The concept of fixed point and application were discussed in this paper.

**Sanath Kumar et.al [17] authors** discussed on **"**Fixed Point Theorems of Soft Metric Space Using Altering Distance Function". New contraction was defined at the basis of soft set property.

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