SUB CLASS OF HARMONIC UNIVALENT FUNCTIONS WITH INTEGRAL OPERATOR

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ABSTRACT

This paper introduced a new class of harmonic univalent function defined by an integral operator. Additionally study investigates some properties of this subclass such as essential as well as adequate coefficient bounds, extreme points, distortion bounds and hadamard product.

Keywords: Harmonic function, Integral operator, integral operator Extreme point, Distortion bounds, distortion inequalities, Convolution

1.1. INTRODUCTION:

Harmonic functions are famous for their use in the revision of minimal surfaces as well as also play vital roles in a variety of problems during applied mathematics. Harmonic functions have been studied in many areas such as differential geometers [5-9]; mathematical finance [10-14].Silverman [10] provided sufficient coefficient condition for normalized harmonic functions to map onto either starlike otherwise convex regions. These conditions be in addition shown to be necessary when the coefficients are negative. Ahuja [1] investigated harmonic analogs and formed certain harmonic functions which preserve close-to-convexity below convolution. Ahuja [13] determined representation theorems, distortion bounds, convolutions, convex combinations, as well as neighbourhoods for harmonic functions. Yalçin [14] defined and investigated a new division of harmonic univalent functions as well as obtained coefficient conditions, extreme points, distortion bounds, convex harmonic univalent functions Ang et al. [18] consequent several sufficient conditions of the linear combinations of harmonic univalent mappings to be univalent and convex in the direction of the real axis. Li and Ponnusamy [16] investigated the subject of disk of convexity of sections of univalent harmonic functions. Ho [17] established the mapping properties of integral operators on space of bounded signify oscillation and Campanato spaces. Berra et al. [18] provided the mapping properties of some integral operators on space of bounded. Li et al. [15] provided approximation of functions by linear integral operators on variable exponent spaces associated with a general exponent function on a domain of a Euclidean space.a subclass of harmonic univalent functions involving of complex-value functions and investigate some properties of this subclass.

1.2. REVIEW OF LITERATURE:

1. R. Parvatham and A. Swaminathan, On subclasses of uniformly convex functions and corresponding class of starlike functions,
2. N. E. Cho and S. Owa, Sufficient conditions for meromorphic starlikeness and close to-convexity of order α
3. M. Darus, S. Hussain, M. Raza and J. Sokol, On a subclass of starlike functions,
4. H. Aldweby and M. Darus, On Harmonic Meromorphic Functions Associated with Basic Hypergeometric Functions,
5. R. M. El-Ashwah, M. K. Aouf, A. A. Hassan and A. H. Hassan, Certain new classes of analytic functions with varying arguments,
6. E. A. Elrifai, H. E. Darwish and A. R. Ahmed, On certain subclasses of meromorphic functions associated with certain differential operators.
7. Aqueel ketab al-khafayi waggas galib atshan : some properties of a class of harmonic multivalent functions defined by an integral operator .

2.1 PRELIMINARIES:

Here, we tend to investigate some important concepts of Harmonic Functions .we start with introducing the following important concepts that are used throughout the paper. Hence, let H denote the class of functions which are complex-valued, harmonic, univalent, sense-preserving in $\Delta = \{z \in C : |z| < 1\}$ normalized by

$$f(0) = h(0) = f_z(0) - 1 = 0.$$

Definition 2.1: Each $f \in H$ can be expressed as $f = h + g \in H$, where h and g are analytic in Δ. Therefore if $f \in H$, then

$$h(z) = z^p + \sum_{n=2}^{\infty} a_{k+p-1}\, z^{k+p-1} \qquad g(z) = \sum_{n=1}^{\infty} b_{k+p-1}\, z^{k+p-1} |b_p| < 1. \qquad (1)$$

are the analytic and co-analytic part of f respectively.

With respect to the definition 1, we assume that $H_{\lambda,k}$ be the subfamily of H consisting harmonic functions $f = h + g$ where we define new operator $I^n f$ as

$$I^n f(z) = I^n h(z) + (-1)^n \overline{I^n g(z)} \quad p > n \,.\, Z \in U. \qquad (2)$$

In this case, if co-analytic part of $f = h + g$ is identically zero, then H reduces to the class of S of normalized analytic univalent functions.

For $0 < \lambda \leq 1$, $0 \leq \beta$, $r < 1$, $k \in N_0 = N \cup \{0\}$, $0 \leq t \leq 1, \alpha$, $\theta \in R$, we have the following useful definition.(2) gives

Definition 2.2: The class $H_{\lambda,k}(\alpha, \beta, t)$ is a set of all functions f ∈ H satisfying the relation

$$\mathrm{Re}\{\frac{I^n f(z)}{I^{n+1} f(z)}\} > \beta \left|\frac{I^n f(z)}{I^{n+1} f(z)} - 1\right| + \alpha \quad (3)$$

Let the subclass $H_{\lambda,k}(n+1,n,\alpha,\beta)$ consisting of functions f = h + g ∈ H and (3) holds true. The function f_n =h + $\overline{g_n}$ in $H_{\lambda,k}(n,\alpha,\beta)$ so that h and g are of the form

$$h(z) = z^p - \sum_{n=2}^{\infty} a_{k+p-1} z^{k+p-1} \qquad g_n(z) = (-1)^{n-1} \sum_{n=1}^{\infty} b_{k+p-1} z^{k+p-1} |b_p| < 1.$$

3 Main Results:

In the current subdivision, we examine to obtain coefficient bounds for functions in the subclasses $H_{\lambda,k}(n+1,n,\alpha,\beta,)$ and $\overline{H\lambda, k}(n+1,n,\alpha,\beta,)$. These properties consist of essential as well as enough coefficient bounds, extreme points, distortion bounds and Hadamard product. The subsequent theorem reveals an central property for a function to be harmonic univalent.

Theorem 3.1:

Let f = h + g ∈ $\overline{H}$ and also given by (1) . If

$$\sum_{k=1}^{\infty}\{\psi(n+1,n,p,\alpha,\beta\left|a_{k+p-1}\right| + \Theta(n+1,n,p,\alpha,\beta)\left|b_{k+p-1}\right| \leq 2, \quad (4)$$

Where $\Psi(n+1, n, p, \alpha, \beta) = \frac{\left(\frac{\beta}{k+p-1}\right)^n (1+\beta) - (\beta + \alpha\left(\frac{\beta}{k+p-1}\right)^{n+1}}{\lambda(1-\alpha)}$.

$$\Theta(n+1,n,p,\alpha,\beta) = \frac{\left(\frac{\beta}{k+p-1}\right)^n (1+\beta) - (\beta + \alpha\left(\frac{\beta}{k+p-1}\right)^{n+1}}{\lambda(1-\alpha)}.$$

$a_p = 1, 0 \leq \alpha < 1, \beta \geq 0$, n ∈ ℵ then f∈$H_{\lambda,k}(n+1, n, p, \alpha, \beta)$

Proof: According to (2) and (3) we only need to show that

$$\mathrm{Re}\left(\frac{I^n f(z) - \alpha I^{n+1} f(z) - \beta e^{i\Theta} \left|I^n f(z) - \alpha I^{n+1} f(z)\right|}{I^{n+1} f(z)}\right) \geq 0$$

The case r=0 is obvious . for $0< r < 1$ it follow that

$$\mathrm{Re}\left(\frac{I^n f(z) - \alpha I^{n+1} f(z) - \beta e^{i\Theta} \left|I^n f(z) - \alpha I^{n+1} f(z)\right|}{I^{n+1} f(z)}\right) =$$

$$\mathrm{Re}\{\frac{(1-\alpha)z^p + \sum_{k=2}^{\alpha} a_{k+p-1}[\Gamma^n - \alpha\Gamma^{n+1}]}{z^p + \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1} z^{k+p-1} + (-1)^{n+1} \sum_{k=1}^{\alpha} \Gamma^{n+1} \bar{b}_{k+p-1} \bar{z}^{-k+p-1}}$$

$$+\frac{(-1)^{n+1} \sum_{k=1}^{\alpha} \bar{b}_{k+p-1} \bar{z}^{-k+p-1} [\Gamma^n - \alpha\Gamma^{n+1}]}{z^p + \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1} z^{k+p-1} + (-1)^{n+1} \sum_{k=1}^{\alpha} \Gamma^{n+1} \bar{b}_{k+p-1} \bar{z}^{-k+p-1}} -$$

$$\frac{\beta e^{i\Theta} \left|\sum_{k=2}^{\alpha} a_{k+p-1}[\Gamma^n - \alpha\Gamma^{n+1}]\right|}{z^p + \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1} z^{k+p-1} + (-1)^{n+1} \sum_{k=1}^{\alpha} \Gamma^{n+1} \bar{b}_{k+p-1} \bar{z}^{-k+p-1}}$$

$$= \mathrm{Re}\{\frac{(1-\alpha)z^p + \sum_{k=2}^{\alpha} a_{k+p-1}[\Gamma^n - \alpha\Gamma^{n+1}]}{1 + \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1} z^{k+p-1} + (-1)^{n+1} \sum_{k=1}^{\alpha} \Gamma^{n+1} \bar{b}_{k+p-1} \bar{z}^{-k+p-1}}$$

$$+\frac{(-1)^{n+1} \sum_{k=1}^{\alpha} \bar{b}_{k+p-1} \bar{z}^{-k+p-1} [\Gamma^n - \alpha\Gamma^{n+1}]}{1 + \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1} z^{k+p-1} + (-1)^{n+1} \sum_{k=1}^{\alpha} \Gamma^{n+1} \bar{b}_{k+p-1} \bar{z}^{-k+p-1}} -$$

$$\frac{\beta e^{i\Theta} \left|\sum_{k=2}^{\alpha} a_{k+p-1}[\Gamma^n - \alpha\Gamma^{n+1}]\right|}{1 + \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1} z^{k+p-1} + (-1)^{n+1} \sum_{k=1}^{\alpha} \Gamma^{n+1} \bar{b}_{k+p-1} \bar{z}^{-k+p-1}}$$

$$= \mathrm{Re} \frac{(1-\alpha) + A(z)}{1 + B(z)}$$

Where $\Gamma = \frac{p}{k+p-1}$

For $z = r\, e^{i\Theta}$ we have

$A(r\, e^{i\Theta}) = \sum_{k=2}^{\alpha} [\mathrm{A}^n - \alpha \mathrm{A}^{n+1}] a_{k+p-1}\ r^{k-1} e^{(k-1)i\Theta}$

$+(-1)^n \sum_{k=1}^{\alpha} [\mathrm{A}^n + \alpha \mathrm{A}^{n+1}] \bar{b}_{k+p-1}\ r^{k-1} e^{-(k+2p-1)i\Theta}$ - $\beta e^{-(2p-1)i\Theta}$ D(n+1,n,p,α)=
$\left|\sum_{k=2}^{\alpha} [\mathrm{A}^n - \mathrm{A}^{n+1}] a_{k+p-1}\ r^{k-1} e^{-(k+p-1)i\Theta} + \right.$
$\left.(-1)^n \sum_{k=1}^{\alpha} [\mathrm{A}^n + \mathrm{A}^{n+1}] \bar{b}_{k+p-1}\ r^{k-1} e^{-(k+p-1)i\Theta}\right|$

$B(r\, e^{i\Theta}) = \sum_{k=2}^{\alpha} \mathrm{A}^{n+1} a_{k+p-1}\ r^{k-1} e^{(k-1)i\Theta}$

$$+(-1)^n \sum_{k=1}^{\alpha} A^{n+1} \overline{b}_{k+p-1} \; r^{k-1} e^{-(k+2p-1)i\Theta}$$

Setting $\frac{1-\alpha+A(z)}{1+B(z)} = (1-\alpha)\frac{1+w(z)}{1-w(z)}$

The proof will be complete if we can show that

$|w(z)| \leq r < 1$ this is the case since , by the condition (4)(we can write)

$$|w(z)| \quad \left|\frac{A(z)-(1-\alpha)B(z)}{A(z)+(1-\alpha)B(z)+2(1-z)}\right| \leq$$

$$\frac{\sum_{k=1}^{\infty}\left|[(1+\beta)(A^n - A^{n+1})a_{k+p-1} + [(1+\beta)(A^n + A^{n+1})b_{k+p-1}\right| r^{k-1}}{4(1-\alpha)\{\left|[(1+\beta)A^n - \Lambda A^{n+1})]a_{k+p-1} + [(1+\beta)A^n + \Lambda A^{n+1})b_{k+p-1}\right| r^{k-1}}$$

$$< \frac{\sum_{k=1}^{\infty}\left|[(1+\beta)(A^n - A^{n+1})a_{k+p-1} + [(1+\beta)(A^n + A^{n+1})b_{k+p-1}\right| r^{k-1}}{4(1-\alpha)\{\left|[(1+\beta)A^n - \Lambda A^{n+1})]a_{k+p-1} + [(1+\beta)A^n + \Lambda A^{n+1})b_{k+p-1}\right| r^{k-1}}$$

Where $\Lambda = \beta + 2\alpha - 1$

The harmonic univalent function

$$f(z) = z^p + \sum_{k=2}^{\infty} \frac{1}{\Psi(n+1,n,p,\alpha,\beta)} x_p z^{k+p-1} + \sum_{k=1}^{\infty} \frac{1}{\Theta(n+1,n,p,\alpha,\beta)} \overline{y_p z^{k+p-1}} \quad (5)$$

where n belong to $\mathbb{N}$, $0 \leq \alpha < 1, \beta \geq 0$

$\sum_{k=2}^{\infty}\left|x_p\right| + \sum_{k=1}^{\infty}|y_k| = 1$

Show that the coefficient bound given by (4) is sharp

The function of the form (6) are

$H_p\,[\Psi(n+1,n,p,\alpha,\beta)\,\left|a_{k+p-1}\right| + \Theta\Psi(n+1,n,p,\alpha,\beta)\,\left|b_{k+p-1}\right|] =$

$$1 + \sum_{k=2}^{\infty}\left|x_p\right| + \sum_{k=1}^{\infty}|y_k| = 2$$

In the following theorem it is show that the condition (5) is also necessary for the function $f_n = h + g_n$ where h and g_n are of the form (4).

Theorem 3.2: Let $f_n = h + \overline{g_n}$ and also be given by (4). It hence $f_n \in H_{\overline{\lambda,k}}$ (n + 1,n, p, α, β) if and only if

$$\sum_{k=1}^{\infty}\{\psi(n+1,n,p,\alpha,\beta\left|a_{k+p-1}\right| + \Theta(n+1,n,p,\alpha,\beta)\left|b_{k+p-1}\right| \leq 2, \quad (6)$$

$a_p = 1, 0 \leq \alpha < 1, \beta \geq 0$, n ∈ ℵ then $f \in H_{\lambda,k}$(n + 1, n, p, α, β)

Proof: since $H_{\overline{\lambda,k}}$ (n+1,n,p,α,β) ⊂ $H_{\lambda,k}$(n + 1, n, p, α, β) we only need to prove the only if part of the theorem for function f_n *of the form* (4), we note that the condition

$$\text{Re}\left\{\frac{I^n f(z)}{I^{n+1} f(z)}\right\} > \beta\left|\left\{\frac{I^n f(z)}{I^{n+1} f(z)}\right\} - 1\right| + \alpha$$

$$\text{Re}\{\frac{(1-\alpha)z^p + \sum_{k=2}^{\alpha} a_{k+p-1}[\Gamma^n - \alpha\Gamma^{n+1}]}{z^p - \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1}z^{k+p-1} + (-1)^{n+1}\sum_{k=1}^{\alpha} \Gamma^{n+1}\bar{b}_{k+p-1}\bar{z}^{-k+p-1}}$$

$$+\frac{(-1)^{n+1}\sum_{k=1}^{\alpha} \bar{b}_{k+p-1}\bar{z}^{-k+p-1}[\Gamma^n - \alpha\Gamma^{n+1}]}{z^p - \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1}z^{k+p-1} + (-1)^{n+1}\sum_{k=1}^{\alpha} \Gamma^{n+1}\bar{b}_{k+p-1}\bar{z}^{-k+p-1}} -$$

$$\frac{\beta e^{i\Theta}\left|\sum_{k=2}^{\alpha} a_{k+p-1}[\Gamma^n - \alpha\Gamma^{n+1}]\right|}{z^p - \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1}z^{k+p-1} + (-1)^{n+1}\sum_{k=1}^{\alpha} \Gamma^{n+1}\bar{b}_{k+p-1}\bar{z}^{-k+p-1}} \geq 0 \quad (7)$$

$$\text{Where } \Gamma = \frac{p}{k+p-1}$$

the above required condition (8) must hold for all values of z ∈ U. Upon choosing the values of z on the positive real axis where 0 ≤ z = r < 1, we must have

$$\frac{(1-\alpha) - \sum_{k=2}^{\alpha} a_{k+p-1}[\Gamma^n(1+\beta) + (\beta+\alpha)\Gamma^{n+1}]\, r^{k-1}}{1 - \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1}r^{k-1} + \sum_{k=1}^{\alpha} \Gamma^{n+1} b_{k+p-1}r^{ik+p-1}} \quad (8)$$

$$\frac{-\sum_{k=1}^{\alpha}[\Gamma^n(1+\beta) + (\beta+\alpha)\Gamma^{n+1}]\;\Gamma^{n+1} b_{k+p-1}r^{k-1}}{1 - \sum_{k=2}^{\alpha} \Gamma^{n+1} a_{k+p-1}r^{k-1} + \sum_{k=1}^{\alpha} \Gamma^{n+1} b_{k+p-1}r^{k-1}} \geq 0$$

If the condition (7) does not hold, then the expression in (8) is negative for r sufficiently close to 1.This contradicts the required condition for $f_n \in H_{\overline{\lambda,k}}$ (n+1,n,p,α,β). And so the proof is complete.

The following theorem gives the distortion bounds for functions in $H_{\overline{\lambda,k}}$ (n+1,n,p,α,β) which yields a covering results for this class.

Theorem 3.3 Let $f_n \in H_{\overline{\lambda,k}}$ (n+1,n,p,α,β) then for $|z| = r < 1$ we have

$$|f_n(z)| \leq (1+b_p)r^p + +[\Phi(n+1,n,p,\alpha,\beta) - \Omega(n+1,n,p,\alpha,\beta)b_p]r^{n+1+p}$$

$$|f_n(z)| \geq (1-b_p)r^p + +[\Phi(n+1,n,p,\alpha,\beta) - \Omega(n+1,n,p,\alpha,\beta)b_p]r^{n+1+p}$$

Where Φ (n + 1, n, p, α, β) =.$\frac{\lambda(1-\alpha)}{\left(\frac{p}{p+1}\right)^n(1+\beta)-\left(\frac{p}{p+1}\right)^{n+1}(\beta+\alpha)}$

$$\Omega(n+1,n,p,\alpha,\beta) = \frac{\lambda(1-\alpha)+\lambda(\alpha+\beta)}{\left(\frac{p}{p+1}\right)^n(1+\beta)-\left(\frac{p}{p+1}\right)^{n+1}(\beta+\alpha)}.$$

Proof: We prove the right side inequality for |fn|. The proof for the left hand inequality can be done using similar arguments. Let $f_n \in H_{\bar{p}}$ (n + 1, n, α, β) Taking the absolute value of fn then by Theorem 2.2, we can obtain

$$|f_n(z)| = \left|z^p - \sum_{k=2}^{\infty}|a_{k+p-1}|z^{k+p-1} + (-1)^{n-1}\sum_{k=1}^{\infty}|b_{k+p-1}|\bar{z}^{kP-1}\right| \leq$$

$$\leq r^p - \sum_{k=2}^{\infty}|a_{k+p-1}|z^{k+p-1} + \sum_{k=1}^{\infty}|b_{k+p-1}|\bar{z}^{kP-1}$$

$$=\leq r^p + r^p b_p + \sum_{k=2}^{\infty}|a_{k+p-1}|z^{k+p-1} \leq$$

$$\leq r^p + r^p b_p + \sum_{k=2}^{\infty}|a_{k+p-1}|z^{k+p-1} =$$

$(1+b_p)r^p$ + Φ (n + 1, n, p, α, β)$\sum_{k=2}^{\infty}\frac{1}{\Phi\ (n+1,n,p,\alpha,\beta)}(a_{k+p-1} + b_{k+p-1}r^{p+1}$

$\leq(1+b_p)r^p$ + Φ (n + 1, n, p, α, β) r^{n+p+1}

$\leq(1+b_p)r^p$ + [Φ (n + 1, n, p, α, β)$-$ Ω (n + 1, n, p, α, β)] r^{n+p+1}

Corollary 3.4: Let $f_n \in H_{\bar{p}}$ (n + 1, n, α, β) then for $|z|$ = r<1 we have

$$\{w: L \leq 1 - b_p - [\Phi(n+1,n,p,\alpha,\beta) - \Omega(n+1,n,p,\alpha,\beta)b_p] \sqsubset f_n(U)\}$$

For β = 0 we obtain the results given in [4]. For β = 0, p = 1 and using the differential S˘al˘agean operator we obtain the results given [7]. The beautiful results, for harmonic functions, was obtained by P. T. Mocanu in [8].

4. Conclusion:

1. In this paper we carried out a new version of subclass of harmonic univalent functions that are useful in mathematical finance. Further, we investigated some properties of the proposed subclass such as necessary and sufficient bounds, extreme points, distortion bounds and hadamard product, salagean integral operator.
2. Certain new subclasses of meromorphic functions. We aim to study some important properties such as coefficient estimates, growth rate.
3. Fixed Coefficient Results for the subfamily of consisting of functions for which is fixed. It is easy to see that is a compact and convex family for which the extreme points.

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