GENERALIZED COMMON FIXED POINT THEOREMS OF COMPATIBLE MAPS IN COMPLEX VALUED METRIC SPACES

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ABSTRACT

The aim of the present paper is to establish the concept of a complex valued metric space is a generalization of the classical metric space common fixed point theorem of compatibility mapping complex valued metric space. We use mappings satisfying certain condition to construct.

Subject Classification: 47H10, 54H25

Keywords: Common fixed point; Compatibility; Weakly compatibility; Complex valued metric spaces.

1. INTRODUCTION

An essential and fundamental result namely Banach contraction principle (BCP) was established by Banach [1]. He proved that a contraction map on a complete metric space always possess a unique fixed point. After this fascinating result and its various applications, a huge number of generalization of this result are available the literature by using different types of contractive conditions in various abstract spaces. By generalizing the Banach contraction principle, Jungck [2] set out convention of common fixed point of mappings for two commuting mappings on complete metric space. After the result of many authors introduced many concepts namely weak commutativity, compatibility, weak compatibility of maps and established results regarding common fixed point theory. In authenticity commutativity of maps weak commutativity of maps compatibility of maps weak compatibility of maps, but the converse of these implications is not true. In 2011, Azam et al. [7] by introducing the notion of complex valued metric space, gave sufficient condition for the existence of some common fixed point for a pair of maps satisfying rational inequality.

As a generalization of metric space the structure of -metric space was developed by Bakhtin [8]. An analogous to the structure of -metric space, Rao et al. [9] developed the structure of complex valued -metric space and initiated the study of common fixed point of maps. After that number of researchers has proved several results regarding fixed point in context of complex valued -metric space [10-13].

The aim of this paper is to prove some results regarding common fixed point of maps, by using the notion of compatibility and weak compatibility of maps in complex valued -metric space fulfilling contractive circumstances involving rational expression

2. Preliminaries

Some simple definition and outcomes which will be make the most of in our subsequent conversation.

Definition 1. [7] Let be the set of complex numbers and $z_{1,}z_2\mathbb{C}$. Define a partial order$\precsim$ $\mathbb{C}$. On as: $z_{1\precsim,}z_2$ if$\mathrm{Re}(z_{1,}) \leq \mathrm{Re}(z_2)$ $\mathrm{im}(z_{1,}) \leq \mathrm{im}(z_2)$ It follows that $z_{1\precsim,}z_2$if one of the following conditions holds:

1. $\mathrm{Re}(z_{1,}) = \mathrm{Re}(z_2)$ $\mathrm{im}(z_{1,}) = \mathrm{im}(z_2)$
2. $\mathrm{Re}(z_{1,}) < \mathrm{Re}(z_2)$ $\mathrm{im}(z_{1,}) = \mathrm{im}(z_2)$
3. $\mathrm{Re}(z_{1,}) = \mathrm{Re}(z_2)$ $\mathrm{im}(z_{1,}) < \mathrm{im}(z_2)$
4. $\mathrm{Re}(z_{1,}) < \mathrm{Re}(z_2)$ $\mathrm{im}(z_{1,}) < \mathrm{im}(z_2)$

We write $z_{1\precsim,}z_2$ if $z_{1\neq}z_2$and one of (2) and (3) is satisfied and we write $z_{1,<}z_2$ if only (4) is satisfied. Here we note the following holds trivially:

1. If $0\precsim$, $z_{1\precsim,}z_2$then $|z_1 \,||\, z_2\,|$;
2. If $z_{1<}z_2$ and $z_{2<,}z_3$ then $z_{1<}z_3$
3. If a,b,$\in \mathbb{R}$ and $a \leq b$ then $az \precsim, bz$ for all $z \in \mathbb{C}$
4. If a,b,$\in \mathbb{R}$ and $0\leq a \leq b$ and$z_{1\precsim,}z_2$ implies .a $z_{1\precsim,}bz_2$

Definition 2. [7] Let be a nonempty set. A function is called a complex valued metric on if for all the following conditions are satisfied.

(CVM 1) $0\precsim$, $d(z_{1,}z_2)$ and $d(z_{1,}z_2) = 0$ if and only if $z_{1=,}z_2$

(CVM 2) $d(z_{1,}z_2) = d(z_{2,}z_1) = 0$

(CVM3) $d(z_{1,}z_2) \precsim d(z_{1,}z_3) + d(z_{3,}z_2)$

Then the pair (X,d) is called a complex valued metric space.

Example 3. [16] Let X $= \mathbb{C}$ Define the mapping by d:X x X$\rightarrow \mathbb{C}$

1. $d(z_{1,}z_2) = 3i|z_1 - z_2| = 0\forall\ z_1z_2 \in X$

Then ( ) is a complex valued metric space.

Definition 4. [9] Let (X,d) be a complex valued b-metric space. Consider the following:

(i) A point $x \in X$ is called interior point of a set A subset X whenever there exists $0 < r \in \mathbb{C}$
such that B(x,r) ={ y$\in \mathbb{C}$: d(x,y) $< r\}\subseteq A$

(ii) A point is called a limit point of a set $A \subseteq X$ whenever, for every $0 < r \in \mathbb{C}, B(x, r) \cap (A - X) \neq \emptyset$

(iii) A subset $B \subseteq X$ is called open whenever each element B of is an interior point of B .

(iv) A subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B .

(v) The family $F = \{B(x,r) \quad x \in X \text{ and } 0 < r\}$ is a sub basis for a topology on . This topology is denoted by τ_c

Definition 5. [9] Let (X,d) be a complex valued metric space and $\{z_n\}$ a sequence in and Consider the following:

(i) If for every $c \in \mathbb{C}$ with $0 < r$ there is $N \in \mathbb{N}$ such that, for all $n \geq N, d(z_n, z) < c$ then $\{z_n\}$ is said to be convergent, $\{z_n\}$ converges to , and is the limit point of $\{z_n\}$ We denote this
$\lim_{n\to\infty} z_n = z$ and $\{z_n\} \to z$ as $n \to \infty$

(ii) If for every $c \in \mathbb{C}$ with $0 < c$ there is $N \in \mathbb{N}$ such that, for all $n \geq N, d(z_n, z_{n+m}) < c$ where $m \in \mathbb{N}$ then $\{z_n\}$ is said to be a Cauchy sequence.

(iii) If every Cauchy sequence is convergent in (X,d),then (X,d)is said to be a complete complex valued metric space.

Lemma 6. [9] Let (X,d) be a complex valued metric space and let $\{z_n\}$ be a sequence in X..

1. Then $\{z_n\}$ converges to zif and only if $|\, d(z_n, z)\, | \to 0$, as $n \to \infty$.
2. Then $\{z_n\}$ is a Cauchy Sequence if and only if $|\, d(z_n, z_{n+m})\, | \to 0$, as $n \to \infty$ where $m \in \mathbb{N}$

Definition 7. Two self maps S and T of a complex valued metric space (X,d) are

1. weakly commuting if $|\, d(STz_n, TSz_n)| \leq |\, d(Sz, Tz)|$,
2. compatible if $lim_{n\to\infty} |\, d(STz_n, TSz_n)| \to 0$ whenever $\{z_n\}$ is a sequence in X such that $|\, \lim_{n\to\infty} Sz_n, = \lim_{n\to\infty} Tz_n) = z$ for some $z \in X$
3. weakly compatible if Sz =Tz implies that TSz =STz

Definition 8. A function defined on a complex valued metric space (X,d) is called continuous at a point $z_n \in X$ f for every $\epsilon > 0$ there exist $\delta > 0$ such that $|\, d(Tz, Tz_0)| < \epsilon$ for all $z \in X$ with $|\, d(z, z_0)| < \delta$ i.e. $lim_{z\to z_0} |\, d(STz_n, TSz_n)| = 0$

Proposition 9: Let S and T be two self mappings defined on a complex valued metric space (X,d). Then the

1.commutativity of and implies weak commutativity but the converse is not always true.

2.weak commutativity of and implies compatibility but the converse is not always true.

3.compatibility of and implies weak compatibility but the converse is not always true..Suppose that $\lim_{n\to\infty} Sx_n, = x$ for some $x \in X$ and if is continuous. Then $\lim_{n\to\infty} TSx_n, = Sx$

Theorem 10. [10] Let (x,d) be a complete complex valued -metric space with the coefficient $S \geq 1$ and $S,T: X \to X$ be mapping satisfying

1. $d(Sz,Tw) \leq \lambda d(z,w) + \mu \frac{d(z,Sz)d(w,Tw)}{1+d(z,w)}$
2. $d(Sz,Tw) \leq \alpha \frac{d(z,Sz)d(z,Tw)+d(w,Sz)d(w,Tw)}{d(z,Sz)+d(w,Tw)}$ for all $z,w \in X$ where λ, μ are non negative real numbers with $s\lambda + \mu < 1$ then S,T have a unique common fixed point in X

3.MAIN RESULTS:

Now by using the notion of compatibility and weak compatibility maps, we generalize the above results by taking four maps as opposed to two maps.

Theorem 3.1. Let (x,d) be a complete complex valued metric space and mappings S and T satisfying.

1. $SX \subset Tx$
2. $d(Tx,Ty) \geq \alpha d(Sx,Tx) + \beta d(sy,Ty) + \gamma(Sx,Sy)$ (3.1)

where x,y in X where α, β, γ are non negative reals with $\alpha + \beta + \gamma < 1$

(iv) Suppose that is continuous, is compatible and weak compatible. OR

(v) (iv) is continuous, is weak compatible and compatible.

Then S and T are also wekly compatible then S and T have unique common fixed point in X.

Proof. For any given $x_0 \in X$, we can use the condition $SX \subset T X$ to construct two sequences $\{x_n\}$ and $\{y_n\}$ satisfying

$y_n = Sx_n = T x_{n+1}, \forall n = 0, 1, 2, \cdots$.

If there exists n satisfying $x_n = x_{n+1}$, then yn is the point of coincidence of S and T. Hence we assume that $x_n \neq x_{n+1}, \forall n = 1, 2, \cdots$.

Suppose that $\alpha < 1$. Take $x = x_n$, $y = x_{n+1}$, then by (3.1),

$d(T x_n, T x_{n+1}) \geq \alpha d(Sx_n, T x_n)+\beta d(Sx_{n+1}, T x_{n+1})+\gamma d(Sx_n, Sx_{n+1})$,

that is,

$d(y_{n-1}, y_n) \geq \alpha d(y_n, y_{n-1}) + \beta d(y_{n+1}, y_n) + \gamma d(y_n, y_{n+1})$, hence

(3.2) $d(y_n, y_{n+1}) \leq \frac{1-\alpha}{\beta+\gamma} d(y_{n-1}, y_n), \forall n = 1, 2, 3, \cdots$.

Suppose that $\beta < 1$. Take $x = x_{n+1}$, $y = x_n$, then by (3.1),

$d(T x_{n+1}, T x_n) \geq \alpha d(Sx_{n+1}, T x_{n+1})+\beta d(Sx_n, T x_n)+\gamma d(Sx_{n+1}, Sx_n)$,

that is $d(y_n, y_{n-1}) \geq \alpha d(y_{n+1}, y_n) + \beta d(y_n, y_{n-1}) + \gamma d(y_{n+1}, y_n)$,

hence

(3.3) $d(y_n, y_{n+1}) \leq \frac{1-\alpha}{\beta+\gamma} d(y_{n-1}, y_n), \forall n = 1, 2, 3, \cdots$..

Combining (3.2), (3.3) and (ii), we get

(3.4) $d(y_n, y_{n+1}) \leq hd(y_{n-1}, y_n), \forall n = 1, 2, 3, \cdots$,

where $h = \max\{ \frac{1-\alpha}{\alpha+\gamma} \frac{1-\alpha}{\beta+\gamma} \} < 1$. This shows that $\{y_n\}$ is a Cauchy sequence by Lemma 6.

Suppose that T X is complete. Since $y_n = Sx_n = T x_{n+1} \in T X$, there exists $z \in X$ such that $y_n \to T z$ as $n \to \infty$. When $\alpha \neq 0$, we take $x = z$, $y = x_{n+1}$ and use (3.1) to obtain

$d(T z, T x_{n+1}) \geq \alpha d(Sz, T z) + \beta d(Sx_{n+1}, T x_{n+1}) + \gamma d(Sz, Sx_{n+1})$,

hence

$d(T z, y_n) \geq \alpha d(Sz, T z) + \beta d(y_{n+1}, y_n) + \gamma d(Sz, y_{n+1}) \geq \alpha d(Sz, T z)$,

so $|d(T z, yn)| \geq \alpha |d(Sz, T z)|$. Let $n \to \infty$, then $|d(Sz, T z)| = 0$ by Lemma 6, hence $Sz = T z$.

when $\beta \neq 0$, we take $x = x_{n+1}$, $y = z$ and use (3.1) to obtain

$d(y_n ,T z,) \geq \alpha d(yn+1, y_n) + \beta d(Sz, T z) + \gamma d(y_{n+1}, Sz) \geq \beta d(Sz, T z)$, hence $|d(y_n, T z)| \geq \beta |d(Sz, T z)|$. Let $n \to \infty$, then $|d(Sz, T z)| = 0$ by Lemma 4, hence $Sz = T z$.

When $\gamma \neq 0$, we take $x = x_{n+1}$, $y = z$ and use (3.1) to obtain

$d(y_n, T z) \geq \alpha\, d(y_{n+1}, y_n) + \beta\, d(Sz, T z) + \gamma\, d(y_{n+1}, Sz) \geq \gamma\, d(y_{n+1}, Sz)$,

hence $|d(y_n, T z)| \geq \gamma |d(y_{n+1}, Sz)|$. Let $n \to \infty$, then $|d(Sz, T z)| = 0$ by Lemma 4, hence $Sz = T z$. So in any case, $T z = Sz$ holds. Let $u = T z = Sz$, then u is a point of coincidence of S and T. Suppose that SX is complete.

Since $y_n = Sx_n \in SX \subset T X$, there exist $z_1, z_2 \in X$ satisfying $y_n \to Sz_1 = T z_2$. Hence we can similarly obtain that $Sz_2 = T z_2$, therefore S and T have a point of coincidence. If α, β, γ satisfy $\gamma > 1$ and $\alpha < 1$ or $\beta < 1$, then they also satisfy (ii) and (iii), hence S and T have a point of coincidence $u = Sz = T z$.

assume that $w = Sv = T v$ is in addition a point of coincidence of S and T.

Let $x = z$, $y = v$, then by (3.1),

$$d(u, w) = d(T z, T v) \geq \alpha\, d(Sz, T z) + \beta\, d(Sv, T v) + \gamma\, d(Sz, Sv) \geq \gamma\, d(Sz, Sv)$$
$$= \gamma\, d(u, w), \text{ hence } d(u, w) = 0$$

since $\lambda > 1$. So $u = w$, i.e., u is the unique point of coincidence of S and T. The last result follows from Lemma 6 .

Example 3.2. Consider the complex valued metric space (X, d) Let $X = \{a, b, c\}$. Define a mapping $d : X \times X \to C$ by $d(a, a) = d(b, b) = d(c, c) = 0$, $d(a, b) = d(b, a) = 3+4i$, $d(a, c) = d(c, a) = 2+3i$, $d(b, c) = d(c, b) = 4+5i$. Obviously, (X, d) is a complex valued metric space Define two mappinggs $T, S : X \to X$ by

$T a = a$, $T b = c$, $T c = b$, $Sa = a$, $Sb = a$, $Sc = c$.

Obviously, $T X = X$ is complete, $SX \subset T X$ and S and T are weakly compatible. Take $\alpha = \frac{1}{16}$, $\beta = \frac{2}{16}$,, $\gamma = \frac{17}{16}$,. It is easy to check that

$$d(T a, T b) = 2 + 3i \geq \frac{1}{16}\, 0 + \frac{2}{16}\,(2 + 3i) + \frac{17}{16} 0$$
$$= \alpha\, d(Sa, T a) + \beta\, d(Sb, T b) + \gamma\, d(Sa, Sb);$$
$$d(T a, T c) = 3 + 4i \geq \frac{1}{16}\, 0 + \frac{2}{16}\,(4 + 5i) + \frac{17}{16}\,(2 + 3i)$$
$$= \alpha\, d(Sa, T a) + \beta\, d(Sc, T c) + \gamma\, d(Sa, Sc);$$
$$d(T b, T a) = 2 + 3i \geq \frac{1}{16}\,(2 + 3i) + \frac{2}{16} 0 + \frac{17}{16} 0$$
$$= \alpha\, d(Sb, T b) + \beta\, d(Sa, T a) + \gamma\, d(Sb, Sa);$$
$$d(T b, T c) = 4 + 5i \geq \frac{1}{16}\,(2 + 3i) + \frac{2}{16}\,(4 + 5i) + \frac{17}{16}\,(2 + 3i)$$
$$= \alpha\, d(Sb, T b) + \beta\, d(Sc, T c) + \gamma\, d(Sb, Sc);$$
$$d(T c, T a) = 3 + 4i \geq \frac{1}{16}\,(4 + 5i) + \frac{2}{16} 0 + \frac{17}{16}\,(2 + 3i)$$
$$= \alpha\, d(Sc, T c) + \beta\, d(Sa, T a) + \gamma\, d(Sc, Sa);$$
$$d(T c, T b) = 4 + 5i \geq \frac{1}{16}\,(4 + 5i) + \frac{2}{16}\,(2 + 3i) + \frac{17}{16}\,(2 + 3i)$$
$$= \alpha\, d(Sc, T c) + \beta\, d(Sb, T b) + \gamma\, d(Sc, Sb).$$

Hence, T, S, α, β and γ satisfy all conditions of Theorem 3.1. So T and S have a unique common fixed point. In fact, a is the unique common fixed point of T and S.

Corollary 3.3. Let (X, d) be a complex valued metric space, $S : X \to X$ a mapping. If for each $x, y \in X$ with $x \neq y$,

$d(x, y) \geq \alpha\, d(Sx, x) + \beta\, d(Sy, y) + \gamma\, d(Sx, Sy)$,

where $\alpha, \beta, \gamma \geq 0$. Suppose that (i) SX is complete; (ii) $\alpha + \beta + \gamma > 1$; (iii) $\alpha < 1$ or $\beta < 1$. Then S has a fixed point. In particular, if $\gamma > 1$ and $\alpha < 1$ or $\beta < 1$, then S has a unique fixed point.

Corollary 3.4. Let (X, d) be a complex valued metric space, $T : X \to X$ a mapping. Suppose that for each $x, y \in X$ with $x \neq y$,

$d(Tx, Ty) \geq \alpha\, d(T^2x, Tx) + \beta\, d(T^2y, Ty) + \gamma\, d(T^2x, T^2y)$,

where $\alpha, \beta, \gamma \geq 0$.

If (i) T X is complete;

(ii) $\alpha + \beta + \gamma > 1$;

(vi) $\alpha < 1$ or $\beta < 1$. Then T has a fixed point. In particular, if $\gamma > 1$ and $\alpha < 1$ or $\beta < 1$, then T has a unique fixed point. Theorem 3.6. Let (X, d) be a complex valued metric space, $S, T : X \to X$ two mappings satisfying $SX \subset TX$. If for each $x, y \in X$ with $x \neq y$,

(3.5) $d(Tx, Ty) + \alpha\, d(Sx, Ty) + \beta\, d(Sy, Tx) \geq \gamma\, d(Sx, Sy)$, where $\alpha, \beta, \gamma \geq 0$. Suppose that

(i) T X or SX is complete;

(ii) $1+2\alpha < \gamma$ or $1 + 2\beta < \gamma$. Then S and T have a point of coincidence. Furthermore, if $1 + \alpha + \beta < \gamma$, then S and T have a unique point of coincidence. If S and T are also weakly compatible, then S and T have a unique common fixed point.

Proof. Just as Theorem 3.1, we construct $\{x_n\}$ and $\{y_n\}$ such that $y_n = Sx_n = Tx_{n+1}$, $x_n \neq x_{n+1}$, $\forall\, n = 0, 1, 2, \cdots$. Suppose that $1+ 2\alpha < \gamma$.

Taking $x = x_{n+2}$, $y = x_{n+1}$ and using (3.5), we obtain

$$d(y_{n+1}, y_n) + \alpha\, d(y_{n+2}, y_n) \geq \gamma\, d(y_{n+2}, y_{n+1}),$$

hence $d(y_{n+1}, y_n) + \alpha\, [d(y_{n+2}, y_{n+1}) + d(y_{n+1}, y_n)] \geq \gamma\, d(y_{n+2}, y_{n+1})$.

So (3.6) $d(y_{n+2}, y_{n+1}) \leq \frac{1-\alpha}{\gamma-\alpha} d(y_{n+1}, y_n)$, $\forall\, n = 0, 1, 2, \cdots$.

Suppose that $1+ 2\beta < \gamma$. Taking $x = x_{n+1}$, $y = x_{n+2}$ and using (3.5), we obtain $d(y_n, y_{n+1}) + \beta\, d(y_{n+2}, y_n) \geq \gamma\, d(y_{n+1}, y_{n+2})$,

hence $d(y_{n+1}, y_n) + \beta\, [d(y_{n+2}, y_{n+1}) + d(y_{n+1}, y_n)] \geq \gamma\, d(y_{n+2}, y_{n+1})$, and so (3.7) $d(y_{n+2}, y_{n+1}) \leq \frac{1-\beta}{\gamma-\beta} d(y_{n+1}, y_n)$, $\forall\, n = 0, 1, 2, \cdots$.

Let $h = \frac{1-\beta}{\gamma-\beta}$ or $h = \frac{1+\beta}{\gamma-\beta}$, then $0 < h < 1$. by (3.6) and (3.7),

(3.8) $d(y_{n+2}, y_{n+1}) \leq h\, d(y_{n+1}, y_n)$, $\forall\, n = 0, 1, 2, \cdots$. Hence $\{y_n\}$ is a Cauchy sequence by Lemma 2.6. Suppose that T X is complete. Since $y_n = Sx_n = Tx_{n+1} \in TX$, there exists $z \in X$ satisfying $y_n \to Tz$. Suppose that $1 + 2\beta < \gamma$. Taking $x = x_{n+1}$, $y = z$ and using (3.5), we obtain

$d(y_n, Tz) + \alpha\, d(y_{n+1}, Tz) + \beta\, d(Sz, y_n) \succeq \gamma\, d(y_{n+1}, Sz)$, $\forall\, n = 1, 2, \cdots$, hence $|d(y_n, Tz)| + \alpha\, |d(y_{n+1}, Tz)| + \beta\, |d(Sz, y_n)| \geq \gamma\, |d(y_{n+1}, Sz)|$, $\forall\, n = 1, 2, \cdots$. Let $n \to \infty$, then by Lemma 4,

we have

$\beta\, |d(Sz, Tz)| \geq \gamma\, |d(Tz, Sz)|$. Hence $Sz = Tz$ since $\beta < \gamma$. Similarly, we can also obtain that $Sz = Tz$ for the case $1 + 2\alpha < \gamma$. Denote $u = Tz = Sz$, then u is a point of coincidence of S and T. Suppose that SX is complete. Since $y_n = Sx_n \in SX \subset TX$, there exist $z_1, z_2 \in X$ such that $y_n \to Sz_1 = Tz_2$, then we can similarly prove that $Sz_2 = Tz_2$, so S and T have a point of coincidence.

If $1 + \alpha + \beta < \gamma$, then $1 + 2\alpha < \gamma$ or $1 + 2\beta < \gamma$, hence S and T have a point of coincidence. Suppose that $w = Sv = Tv$ is also a point of coincidence of S and T. Taking $x = z$, $y = v$ and using (3.5),

we obtain $d(Tz, Tv) + \alpha\, d(Sz, Tv) + \beta\, d(Sv, Tz) \succeq \gamma\, d(Sz, Sv)$,

i.e., $(1 + \alpha + \beta)d(u, w) \succeq \gamma d(u, w)$. Hence $d(u, w) = 0$, i.e., $u = w$. So u is the unique point of coincidence of S and T. The last result follows from Lemma 6

Corollary 3.5. Let (X, d) be a complete complex valued metric space, $T : X \to X$ an onto mapping. Suppose that for each $x, y \in X$ with $x \neq y$, $d(Tx, Ty) + \alpha\, d(x, Ty) + \beta\, d(y, Tx) \succeq \gamma\, d(x, y)$, where $\alpha, \beta, \gamma \geq 0$.

If $1 + \alpha + \beta < \gamma$, then T has a unique fixed point.

Corollary 3.6. Let (X, d) be a complex valued metric space, $S : X \to X$ a mapping. Suppose that for each $x, y \in X$ with $x \neq y$,

$d(x, y) + \alpha\, d(Sx, y) + \beta\, d(Sy, x) \succeq \gamma\, d(Sx, Sy)$, where $\alpha, \beta, \gamma \geq 0$. If (i) SX is complete; (ii) $1 + \alpha + \beta < \gamma$, then S has a unique fixed point. **Corollary 3.7.** Let (X, d) be a complex valued metric space, $T : X \to X$ a mapping. Suppose that for each $x, y \in X$ with $x \neq y$,

$d(Tx, Ty) + \alpha\, d(T^2x, Ty) + \beta\, d(T^2 y, Tx) \succeq \gamma\, d(T^2x, T^2 y)$, where $\alpha, \beta, \gamma \geq 0$. If

(i) TX is complete;

(ii) (ii) $1 + \alpha + \beta < \gamma$, then T has a unique fixed point.

Theorem 3.8. Let (X, d) be a complete complex valued metric space, $S, T : X \to X$ two onto mappings. Suppose that for each $x, y \in X$ with $x \neq y$,

(3.9) $d(Sx, Ty) + \alpha\, d(x, Ty) + \beta\, d(y, Sx) \succeq \gamma\, d(x, y)$, where $\alpha, \beta, \gamma \geq 0$, and $\gamma > 2\max\{\alpha, \beta\} + 1$, then S and T have a unique common fixed point.

Corollary 3.9. Let (X, d) be a complete complex valued metric space, $S, T : X \to X$ two onto mappings. If for each $x, y \in x$ with $x \neq y$, $d(Sx, Ty) \succeq hd(x, y)$, where $h > 1$. Then S and T have a unique common fixed point.

Corollary 3.10. Let (X, d) be a complete complex valued metric space, $T : X \to X$ an onto mapping. If for each $x, y \in X$ with $x \neq y$, $d(Tx, Ty) + \alpha\, d(x, Ty) + \beta\, d(y,$

T x) ≥ γ d(x, y), where α, β, γ ≥ 0 and γ > 2 max{α, β} + 1. Then T has a unique fixed point.

Theorem 3.11. Let (X, d) be a complete complex valued metric space, T : X → X an onto mapping. Suppose that d(y, T y) and d(x, y) are comparable for x, y ∈ X with x ≠y, and the following holds (3.11) d(T x, T y) ≥ h min{d(y, T y), d(x, y)}, where h > 1. If T is continuous, then T has a fixed point

4.Conclusion

In this article, we extended the study of fixed point theory by using the notions of compatibility and weakly compatibility of self mappings satisfying the new generalized rational type contractive conditions for four self mappings in the complete complex valued b-metric spaces. Our results generalized some earlier results exists in the literature. An illustrative example is also given to substantiate our newly proved results. Moreover we demonstrated an application in support of our main result. This idea is expected to bring wider applications of fixed point theorems which will be helpful for researchers to work in the development of fixed point theory.

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