Some Common Fixed Point Theorems in *N*-Fuzzy Metric Spaces with Applications

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Abstract: In this paper, we prove some common fixed point theorems in newly defined space N-fuzzy metric space [8]. Our results generalize and extend the results of Mahmoud Bousselsal and Mohamed Laid Kadri [M. Bousselsal and M.L. Kadri, [1] A common fixed point theorem in fuzzy metric spaces; Thai J. Math. Vol. X(20XX) No. X:XX-XX [1]].

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1. Introduction

In 1975, Kramosil and Michalek [7] introduced fuzzy metric spaces. By a slight modification of the Kramosil-Michalek definition, George and Veeramani [6] introduced fuzzy metric spaces and topological spaces induced by fuzzy metric. In the literature there are several generalization of metric spaces [2],[3],[4],[5],[9],[10],[11] as well as fuzzy metric space [6],[7],[8]. Very recently Neeraj Malviya introduced the N-Fuzzy metric space induced by S-metric space [11], which is the generalization of Q(G) fuzzy metric space [13].

In present paper we extend and generalized the results of M. Bousselsal and M.L. Kadri [1]. We prove the existence and uniqueness of a common fixed point in *N*-fuzzy metric space for by using a ϕ and ψ functions with examples.

2. Preliminaries and the definition of *N*-fuzzy metric space

We assume that the function $\phi: [0,1] \to [0,1]$ satisfying the following properties:
(P_1) ϕ is strictly decreasing and left continuous.
(P_2) $\phi(m) = 0$ if and only if $m = 1$
obviously, we obtain that $\lim_{m\to 1-} \phi(m) = \phi(1) = 0$.

2.1 Definition

A triplet $(X, N, *)$ is an *N*-fuzzy metric space, if X is an arbitrary set, $*$ is a continuous *t*-norm and *N* is a fuzzy set on $X^3 \times (0, \infty)$ satisfying the following conditions for all $u, v, w \in X$ and $r, s, t > 0$
(1) $N(u, v, w, t) > 0$
(2) $N(u, v, w, t) = 1$ if and only if $u = v = w$
(3) $N(u, v, w, r + s + t) \geq N(u, u, a, r) * N(v, v, a, s) * N(w, w, a, t)$
(4) $N(u, v, w, \cdot): (0, \infty) \to (0,1)$ is a continuous function.

3. Main Results

Theorem 3.1. Let $(X, N, *)$ be a complete *N*-fuzzy metric space and assume that $\phi: [0,1] \to [0,1]$ satisfying the foregoing properties (P_1) and (P_2). Furthermore, let α be a function from $(0, \infty) \to (0,1)$. Let *S* and *T* be maps that satisfy the following condition.

(i) $T(X) \subseteq S(X)$
(ii) S is continuous.

$$\phi\, (N(T(x), T(x), T(y), t)) \leq \alpha(t)\phi\, (N(S(x), S(x), S(y), t)) \tag{3.1}$$

Where $x, y \in X$ and $t > 0$, then S and T have a unique fixed point provided S and T commute.

Proof. Let x_0 be a point in X. By hypothesis (i), we can fixed x_1 such that $Sx_1 = Tx_0$, by induction we can define a sequence $\{x_n\}$ in X such that $Sx_n = Tx_{n-1}$. By induction again and by (3.1) we have

$$\phi\,(N(Sx_n, Sx_n, Sx_{n+1}, t)) = \phi\,(N(Tx_{n-1}, Tx_{n-1}, Tx_n, t))$$

$$\leq \alpha(t)\phi\,(N(Sx_{n-1}, Sx_{n-1}, Sx_n, t))$$

$$< \phi(N(Sx_{n-1}, Sx_{n-1}, Sx_n, t)) \qquad (3.2)$$

Since ϕ is strictly decreasing, then

$$N\,(Sx_n, Sx_n, Sx_{n+1}, t) > N(Sx_{n-1}, Sx_{n-1}, Sx_n, t) \qquad (3.3)$$

Setting $y_n(t) = N(Sx_n, Sx_n, Sx_{n+1}, t)$. For (3.3), the sequence $\{y_n(t)\}$ is strictly increasing and bounded then $y_n(t)$ converges to $y(t)$ for all $t > 0$.

Assume that $y(t) \in]0,1[$. Since $y_n(t) > y_{n-1}(t)$ for all $t > 0$, then

$$\phi\,(y_n(t)) \leq \alpha(t)\,\phi\,(y_{n-1}(t))$$

for every $t > 0$. Letting $n \to \infty$, since ϕ is left continuous, we have

$$\phi\,(y\,(t)) \leq \alpha(t)\,\phi\,(y\,(t)) < \phi\,(y\,(t))$$

for every $t > 0$, which is a contradiction. Hence $y\,(t) = 1$, that is the sequence $\{y_n(t)\}$ converges to 1 for every $t > 0$. Next, we show that the sequence $\{S\{x_n\}\}$ is a Cauchy sequence. Assume that it is not, then there exist $0 < \varepsilon < 1$ and two sequences $\{p(n)\}$ and $\{q(n)\}$ such that

$$p(n) > q(n) \geq n$$

$$N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{q\,(n)}, t) \leq 1 - \varepsilon$$

$$N(Sx_{p\,(n)-1}, Sx_{p\,(n)-1}, Sx_{q\,(n)-1}, t) > 1 - \varepsilon$$

$$N(Sx_{p\,(n)-1}, Sx_{p\,(n)-1}, Sx_{q\,(n)}, t) > 1 - \varepsilon \qquad (3.4)$$

for each $n \in N \cup \{0\}$, we get $\delta_n(t) = N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{q\,(n)}, t)$ then we have

$$1 - \varepsilon \geq \delta_n(t) = N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{q\,(n)}, t)$$

$$\geq N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{p\,(n)-1}, \tfrac{t}{3}) * N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{p\,(n)-1}, \tfrac{t}{3})$$

$$* N(Sx_{q\,(n)}, Sx_{q\,(n)}, Sx_{p\,(n)-1}, \tfrac{t}{3})$$

$$\geq y_{p(n)-1}(\tfrac{t}{3}) * y_{p(n)-1}(\tfrac{t}{3}) * 1 - \varepsilon \quad \text{by 3.4} \qquad (3.5)$$

Since $y_{p(n)-1}(\frac{t}{3}) \to 1$ as $n \to \infty$ for every $t > 0$. Supposing that $n \to \infty$, we note that the sequence $\{\delta_n(t)\}$ converges to $1 - \varepsilon$ for every $t > 0$. Moreover by (3.1) we have

$$N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{q\,(n)}, t) \leq \alpha(t)\,\phi\,(N\,(Sx_{p\,(n)-1}, Sx_{p\,(n)-1}, Sx_{q\,(n)-1}, t))$$

$$< \phi\,(N\,(Sx_{p\,(n)-1}, Sx_{p\,(n)-1}, Sx_{q\,(n)-1}, t)) \qquad (3.6)$$

According to the monotonicity of ϕ, we know that

$$N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{q\,(n)}, t) > N (Sx_{p\,(n)-1}, Sx_{p\,(n)-1}, Sx_{q\,(n)-1}, t)$$

for each n. Thus, on the basis of formula (3.6) we can obtain.

$$1- \varepsilon \geq N(Sx_{p\,(n)}, Sx_{p\,(n)}, Sx_{q\,(n)}, t) > N (Sx_{p\,(n)-1}, Sx_{p\,(n)-1}, Sx_{q\,(n)-1}, t) > 1- \varepsilon \qquad (3.7)$$

Clearly, this leads to a contradiction. Hence $\{Sx_n\}$ is a Cauchy sequence. By the completeness of X, $\{Sx_n\}$ converges to y, so $Tx_{n-1} = Sx_n$ tends to y. It can be seen that from (3.1) and the left continuous of ϕ that the continuity of S implies the continuity of T. So $T(S(x_n)) \to T(y)$.

However $TS\,(x_n) = ST\,(x_n)$ by the commutativity of S and T. So $S\,(T(x_n))$ converges to $S(y)$.

Because the limit is unique $S(y) = T(y)$ so $S(S(y)) = S(T(y))$ by commutativity and

$$\begin{aligned}\phi\,(N\,(T(y), T(y), T(T(y)), t)) &\leq \alpha(t)\,\phi\,(N\,(Sy, Sy, S\,(T(y)), t))\\ &\leq \alpha(t)\,\phi\,(\mathrm{N}\,(Ty, Ty, T(T(y)), t))\\ &< \phi\,(N\,(Ty, Ty, T\,(T(y)), t))\end{aligned}$$

then if $Ty \neq T\,(T(y))$, we have a contradiction hence, $T(y) = T\,(T(y))$. Then $T(y) = T\,(T(y)) = S\,(T(y))$. So $T(y)$ is a common fixed point of S and T. Now we prove the uniqueness of the common fixed point of S and T. If y and z are two common fixed points to S and T, and $y \neq z$, then

$$\begin{aligned}\phi\,(N\,(y, y, z, t)) &= (N\,(Ty, Ty, Tz, t))\\ &\leq \alpha(t)\,\phi\,(N\,(Sy, Sy, Sz, t))\\ &< \phi\,(N\,(Sy, Sy, Sz, t))\\ &= \phi\,(N\,(y, y, z, t))\end{aligned}$$

then $N\,(y, y, z, t) > N\,(y, y, z, t)$, which is a contradiction so $y = z$.

Remark 3.2. If we choose $S = I$ in theorem (3.1), we obtain the following corollary which is the main result of [12] and it will generalize in the setting of N-fuzzy metric space.

Corollary 3.3. Let $(X, N, *)$ be an complete N-fuzzy metric space and T a self map of X and assume that ϕ: $[0,1] \to [0,1]$ satisfy the foregoing properties (P_1) and (P_2). Furthermore, let α be a function from $(0,\infty) \longrightarrow (0,1)$. Let T be a continuous map that satisfies the following conditions :

$$\phi\,(N\,(T(x), T(x), T(y), t)) \leq \alpha(t)\,\phi\,N\,(x, x, y, t)$$

where $x, y \in X$ and $t > 0$, then T has a unique fixed point.

We now give an example that illustrate our main result.

Example 3.4: Let $X = \{ \frac{1}{n}, n \in N \} \cup \{0\}$ with the N-fuzzy metric space N is defined by

$$N\,(x, y, z, \mathrm{t}) = \begin{cases} 0 & if\ t = 0 \\ \dfrac{t}{t+|x-y|+|y-z|} & if\ t > 0, x, y, z \in X. \end{cases}$$

Clearly $N(x, y, z, *)$ is complete N-fuzzy metric space on X. Where is defined by a∗b∗c= abc. Define $T(x) = \frac{x}{6}$ and $S(x) = \frac{x}{2}$ on X. It is evident that $T(X) \subset S(X)$. Also, define the function α :$(0,\infty) \to (0,1)$ by α (t) = $\frac{2+\frac{1}{t}}{6+\frac{1}{t}}$ for

t > 0, the function ϕ: $[0,1] \to [0,1]$ defined by $\phi(t) = \frac{1-t}{1+t}$ satisfies the properties (P_1) and (P_2).

$$N(Tx, Tx, Ty, t) = \frac{3t}{3t+|x-y|} \quad t > 0, \ x, y \in X$$

$$\phi\,(N(Tx, Tx, Ty, t)) = \frac{|x-y|}{6t+|x-y|} \quad t > 0, \ x, y \in X$$

$$\phi\,(N(Sx, Sx, Sy, t)) = \frac{|x-y|}{2t+|x-y|} \quad t > 0, \ x, y \in X$$

Since $|x\text{-}y| \leq 1$ for $x, y \in X$, then it is easy to see that

$$N((Tx, Tx, T(y), t)) \leq \alpha\,(t)\, \phi\,(N(S(x), S(x), Sy, t))$$

All the hypothesis of theorem (3.1) are satisfied and thus S and T have a unique common fixed point $x = 0$.

Application: Let $Y = \{\chi : [0,1[\longrightarrow[0,1[$, χ is a Lebesgue integrable mapping which is summable, nonnegative and satisfies $\int_{1-\varepsilon}^{1} \chi(t)\ dt > 0$ for each $0 < \varepsilon < 1\}$

Theorem 3.5. Let $(X, N, *)$ be a complete N-fuzzy metric space and T a self map of X and assume that ϕ: $[0,1] \to [0,1]$ satisfies the foregoing properties (P_1) and (P_2). If for any $t > 0$, S and T satisfy the following condition:

$$\int_{1-\phi(N(Tx,Tx,T(y),t))}^{1} \chi(s)ds \leq \alpha(t) \int_{1-\phi(N(Sx,Sx,Sy,t))}^{1} \chi(s)ds \quad \text{for } \chi \in Y. \tag{3.5}$$

where $x, y \in X$, then S and T have a unique common fixed point.
Proof. for $\chi \in Y$, we consider the function :

$$\wedge : [0,1] \to [0,1] \ \text{ by } \wedge(\epsilon) = \int_{1-\varepsilon}^{1} \chi(s)ds.$$

$\wedge$ is continuous, $\wedge(0) = 0$, $\wedge$ is strictly increasing (3.1) becomes.

$$\wedge\left(\phi\big(N(T(x), T(x), T(y), t)\big)\right) \leq \alpha(t) \wedge \left(\phi\big(N(Sx,\ Sx, Sy, t)\big)\right)$$

Setting $\phi_1 = \wedge \circ \phi$ and ϕ_1 is strictly decreasing, left continuous and satisfies the properties (P_1)and (P_2) for any $t > 0$, then by theorem (3.1), S and T have a unique common fixed point.

4 The Second Main Result

In this section, we assume that the functions ϕ, ψ : $[0,1] \to [0,1]$ satisfying the following properties:

(q_1) ϕ is strictly decreasing and left continuous,
(q_2) $\phi\,(m) = 0$ if and only if $m = 1$
(q_3) ψ is lower semi-continuous and $\psi\,(m) = 0$ if and only if $m = 1$.
Obviously, we obtain that $\lim_{m\to 1^-} \phi\,(m) = \phi\,(1) = 0$

Theorem 4.1: Let $(X, N, *)$ be a complete N-fuzzy metric space and assume that ϕ, ψ : $[0,1] \to [0,1]$ satisfies the foregoing properties (q_1), (q_2) and (q_3). Let S and T be maps that satisfy the following condition:
(i) $T(X) \subset S(X)$
(ii) S continuous

$$\phi\big(N(Tx, Tx, Ty, t)\big) \leq \phi\big(N(Sx, Sx, Sy, t)\big) - \psi\big(N(Sx, Sx, Sy, t)\big) \tag{4.1}$$

where $x, y \in X$ and $t > 0$, then S and T have a unique common fixed point provided S and T commute.

Proof Let x_0 be a point in X. By hypothesis (i), we can fixed $x_1 \in X$ such that $Sx_1=Tx_0$, by induction, we can define a sequence $\{x_n\}$ in X such that $Sx_n=Tx_{n-1}$. By induction again and by (4.1) we have

$$\phi\,(N\,(Sx_n, Sx_n, Sx_{n+1}, t)) = \phi\,(N\,(Tx_{n-1}, Tx_{n-1}, Tx_n, t))$$

$$\leq \phi\,(N\,(Sx_{n-1}, Sx_{n-1}, Sx_n, t)) - \psi\,(N\,(Sx_{n-1}, Sx_{n-1}, Sx_n, t)) \quad (4.2)$$

Setting $\theta_n(t) = N\,(Sx_n, Sx_n, Sx_{n+1}, t)$ then,
$\phi\,(\theta_n(t)) \leq \phi\,(\theta_{n-1}(t)) - \psi\,(\theta_{n-1}(t))$

Since ϕ is strictly decreasing, it is easy to show that $\{\theta_n(t)\}$ is an increasing sequence for every $t > 0$ with respect to n. That is $\theta_n(t) \geq \theta_{n-1}(t)$ for all $n \geq 1$. We put $\lim_{n\to\infty} \theta n(t) = \theta(t)$ and assume that $0 < \theta(t) < 1$. From (4.2), we have

$$\phi\,(\theta_n(t)) \leq \phi\,(\theta_{n-1}(t)) - \psi\,(\theta_{n-1}(t)) \quad (4.3)$$

for every t, by supposing that $n\to\infty$, Since ϕ is left continuous, we have

$$\phi\,(\theta(t)) \leq \phi\,(\theta(t)) - \psi\,(\theta(t)) \quad (4.4)$$

which implies that $\psi\,(\theta(t)) = 0$. Hence $\theta(t) = 1$. That is the sequence $\{\theta_n(t)\}$ converges to1 for any $t > 0$. Next, we show that the sequence $\{Sx_n\}$ is a Cauchy sequence. Assume that it is not, then there exist $0 < \varepsilon < 1$ and two sequences {p(n)} and {q(n)} such that for every $n \in N \cup \{0\}$ and $t > 0$, we obtain:

$$p(n) > q(n) \geq n$$

$$N(x_{p(n)}, x_{p(n)}, x_{q(n)}, t) \leq 1-\varepsilon \quad (4.5)$$

$$N(Sx_{p(n)-1}, Sx_{p(n)-1}, Sx_{q(n)-1}, t) > 1-\varepsilon$$

$$N(Sx_{p(n)-1}, Sx_{p(n)-1}, Sx_{q(n)}, t) > 1-\varepsilon$$

for each $n \in N \cup \{0\}$, we assume that $\delta n(t) = N(Sx_{p(n)}, Sx_{p(n)}, Sx_{q(n)}, t)$, then we have

$$1-\varepsilon \geq \delta n(t) = N(Sx_{p(n)}, Sx_{p(n)}, Sx_{q(n)}, t)$$

$$> N(Sx_{p(n)}, Sx_{p(n)}, Sx_{p(n)-1}, \tfrac{t}{3}) * N(Sx_{p(n)}, Sx_{p(n)}, Sx_{p(n)-1}, \tfrac{t}{3})$$

$$* N(Sx_{q(n)}, Sx_{q(n)}, Sx_{p(n)-1}, \tfrac{t}{3})$$

$$> \theta_{p(n)-1}(\tfrac{t}{3}) * \theta_{p(n)-1}(\tfrac{t}{3}) * 1-\varepsilon \quad \text{by (4.5)} \quad (4.6)$$

Since $\theta_{p(n)-1}(\frac{t}{3}) \to 1$ as $n \to \infty$ for every t. We note that $\{\delta n(t)\}$ converges to $1-\varepsilon$ as $n\to\infty$ for any $t > 0$, moreover by (4.1), we have

$$\phi\,(\delta_n(t)) \leq \phi\,(\delta_{n-1}(t)) - \psi\,(\delta_{n-1}(t)) \quad (4.7)$$

Going to the limit in (4.7) as $n\to\infty$, for every $t > 0$, we obtain:

$$\phi\,(1-\varepsilon) \leq \phi\,(1-\varepsilon) - \psi\,(1-\varepsilon)$$

Clearly, this leads to $1-\varepsilon = 1$, which is a contradiction. Hence $\{Sx_n\}$ is Cauchy sequence in the complete N-fuzzy metric space X. Therefore we conclude that there exists a point $y \in X$ such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_{n-1} = y\,.$$

It can be seen that from (4.1) and the properties of ϕ and ψ, that the continuity of S implies the continuity of T. So $T(Sx_n) \to T(y)$. However $T(Sx_n) = S(Tx_n)$ by the commutativity of S and T. So $S(Tx_n)$ converges to $S(y)$. Because the limit is unique $S(y) = T(y)$. So by commutativity, we have $S(S(y)) = S(T(y))$ and

$$\phi\big(N(Ty, Ty, T(Ty), t)\big) = \phi\big(N(Sy, Sy, S(Ty), t)\big)$$

$$\leq \phi\big(N(Ty,Ty,T(Ty),t)\big) - \psi\big(N(Ty,Ty,T(Ty),t)\big)$$

Hence, necessarily *Ty* =*T*(*T*(*y*)), thus *Ty* = *T*(*Ty*) = *S*(*Ty*). So *Ty* is a common fixed point of *S* and *T*. Now we prove the uniqueness of the common fixed point of *S* and *T*.

If *y* and *z* are two common fixed points to *S* and *T* with $y \neq z$, then

$$\phi\ \big(N(y,y,z,t)\big) = \phi\ \big(N(Ty,Ty,Tz,t)\big)$$

$$\leq \phi\ \big(N(Sy,Sy,Sz,t)\big) - \psi\big(N(Sy,Sy,Sz,t)\big)$$

$$\leq \phi\ \big(N(y,y,z,t)\big) - \psi\big(N(y,y,z,t)\big)$$

then $\psi\big(N(y,y,z,t)\big) \leq 0$, So $N(y,y,z,t) = 1$ contradiction.

Example 4.2: Let *X*= [0,∞), a ∗ b ∗ c = a.b.c for all a, b, c ∈ [0, 1]. Define *N* : X × X × X× [0, ∞)→[0,1] by

$$N(x,y,z,t) = \begin{cases} e^{\frac{-[\,|x-y|+|y-z|+|z-x|\,]}{2t}} & if\ t > 0, \\ 0 & if\ t = 0, for\ all\ x,y,z \in X. \end{cases}$$

We claim that (*X*, *N*, ∗) is an *N*-fuzzy metric space. In fact, it is enaugh to prove that for r, s ,t > 0, *x*, *y*, *z*, *a* ∈ *X*

$$N(x,x,a,r) * N(y,y,a,s) * N(z,z,a,t)\ \leq e^{\frac{-2|x-a|}{2r}} \cdot e^{\frac{-2|y-a|}{2s}} \cdot e^{\frac{-2|z-a|}{2t}}$$

$$\leq e^{\frac{-2|x-a|}{2(r+s+t)}} \cdot e^{\frac{-2|y-a|}{2(r+s+t)}} \cdot e^{\frac{-2|z-a|}{2(r+s+t)}}$$

$$= e^{\frac{-2[|x-a|+|y-a|+|z-a|]}{2(r+s+t)}}$$

$$\leq e^{\frac{-[|x-y|+|y-z|+|z-x|]}{2(r+s+t)}}$$

$$= N(x,y,z,r+s+t)$$

$\phi = 1 - \sqrt{t}$, $\psi\ (t) = 1 - \frac{\sqrt{t}}{2}$ for $t \in [0,1]$
$f(x) = \frac{x}{2}$ and $g(x) = \frac{x}{6}$ for $x \in X$.

Application: Let Y= { χ : [0,1[→[0,1[, χ is Lebesgue integrable mapping which is summable, nonnegative and satisfies $\int_{1-\varepsilon}^{1} \chi(t)\ dt > 0$ for each $0 < \varepsilon < 1$}

Theorem 4.3: Let $(X, N, *)$ be a complete *N*-fuzzy metric space and assume that $\phi, \psi : [0,1] \to [0,1]$ satisfy the foregoing properties (q_1), (q_2) and (q_3). Let *S* and *T* be maps that satisfy the following condition:

(i) *T*(*X*) ⊂ *S*(*X*)
(ii) *S* is continuous and

$$\int_{1-\phi(N(Tx,Tx,Ty,t))}^{1} \chi(s)ds \leq \int_{1-\phi(N(Sx,Sx,Sy,t))}^{1} \chi(s)ds - \int_{1-\psi(N(Sx,Sx,Sy,t))}^{1} \chi(s)ds \quad (4.3)$$

for $y \in \chi$,where *x*, *y* ∈ *X* and $x \neq y$. Then *S* and *T* have a unique common fixed point.

Proof. For χ ∈ *Y*, we consider the function ∧ : [0,1] → [0,1] by $\wedge(\varepsilon) = \int_{1-\varepsilon}^{1} \chi(s)ds$.

∧ is continuous, ∧ (0) = 0, ∧ is strictly increasing (4.3) becomes

$$\wedge\,(\phi(N(Tx,Tx,Ty,t))) = \wedge\left(\phi\big(N(Sx,\ Sx,Sy,t)\big)\right) - \wedge\left(\psi\big(N(Sx,\ Sx,Sy,t)\big)\right)$$

Setting $\phi_1 = \wedge \text{ o } \phi$ and $\psi_1 = \wedge \text{ o } \psi_1$. ϕ_1 is strictly decreasing, continuous and satisfies the foregoing properties (q_1) and (q_2) for any $t > 0$, and ψ_1 satisfies the property (q_3) then by theorem (4.1) S and T have a unique common fixed point.

References

Bousselsal M. and Kadri , (20XX) . A common fixed point theorem in fuzzy metric spaces; Thai J. Math. Vol. X No. X:XX-X.

Dhage B.C, (1992). Generalized Metric Space and Mapping with Fixed Point, Bull. Cal. Math. Soc. 84, 329-336.

Dhage B.C., (2000).Generalized Metric Spaces and Topological Structure. I, Analele stiintifice Universitatii Al. I. Cuzadin Iasi. Serie Noua. Matematica, Vol. 46, No. 1, pp.3-24.

Gahlers S., (1963). 2-metrische Raume ihre topologische structure, Math. Nachr, 26, 115-148.

Gahlers S., (1966). Zur Geometric 2-Metrische Raume, Revue Roumaine de Mathematiques Pures et Appliquees 40, 664-669.

George A., Veeramani P, (1994). On Some Results in Fuzzy Metric S paces. Fuzzy Sets Syst. 64, 395-399 , https://doi.org/10.1016/0165-0114(94)90162-7.

Kramosil O., Michalek J, (1975). Fuzzy Metrics and Statistical Metric Spaces. Kybernetika 11, 326-334 .

Malviya N. The N-Fuzzy Metric Spaces and Mappings with Application, Fasciculi Mathe-matici, https://doi.org/10.1515/fascmath-2015-001.

Malviya, N., Fisher, B.: N-Cone Metric Space and Fixed Points of Asymptotically Regular Maps. Accepted in Filomat J. Math. (Preprint).

Mustafa Z., Sims B., 2 (2006). A New Approach to Generalized Metric Spaces. Journal of Nonlinear and convex Analysis 7, 289-297.

Sedghi S., Shobe N., Aliouche A.,3 (2012). A Generalization of Fixed Point Theorems in S-Metric Spaces Mathematicki Vesnik 64, 258-266.

Shen Y., Qiu D. and Chenc W., (2012). Fixed point theorems in fuzzy metric spaces, Applied Mathematics Letters, 25, 138-141.

Sun G. and Yang K., (2010). Generalized Fuzzy Metric Spaces with Properties, Research journal of Applied Sciences, Engineering and Technology, vol. 2, no. 7, pp. 673-678.