On Contra $nIs_{\alpha}g-$ Continuity in Nano Ideal Topological Spaces

Dr.S.Pasunkilipandian [a], G.Baby Suganya [b] , Dr.M.Kalaiselvi [c]

[a] Associate Professor, Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur.
[b] Research Scholar, Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur.
[c] Associate Professor, Department of Mathematics, Govindammal Aditanar College for Women, Tiruchendur.

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Abstract: The purpose of this paper is to introduce the concept of contra $nIs_{\alpha}g-$ continuous and contra $nIs_{\alpha}\ g-$ irresolute funtions on nano ideal topological spaces and investigated their properties. Further we have discussed their characteristics under the composition of functions.
Keywords: $nIs_{\alpha}g-$ closed sets, contra $nIs_{\alpha}g-$continuous functions, contra $nIs_{\alpha}g-$irresolute functions

1. Introduction

M.Lellis Thivagar introduced the concept of nano topological space. Parimala et.al introduced and studied nano ideal generalized closed sets.Dontchev introduced and studied the notion of contra continuity in a topological space. Pasunkilipandian et.al introduced $nIs_{\alpha}g-$ closed sets and studied $nIs_{\alpha}g-$ interior, $nIs_{\alpha}g-$ closure, $nIs_{\alpha}g-$ closed map, $nIs_{\alpha}g-$ open map, $nIs_{\alpha}g-$ continuous map and $nIs_{\alpha}g-$ irresolute map in nano ideal topological spaces. In this paper, we introduce and investigated the concept of contra $nIs_{\alpha}g-$ continuity and contra $nIs_{\alpha}g-$ irresolute mappings. Further, we have discussed their relationship and its composition with some of the existing contra continous and contra irresolute mappings.

2.Preliminaries

We recall the following definitions, which will be used in sequel.

Definition 2.1 (M.Lellis Thivagar et.al) Let $\mathcal{U}$ be a nonempty finite set of objects called the universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as indiscernibility relation. Then $\mathcal{U}$ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U},\mathcal{R})$ is said to be an approximation space. Let $X\subseteq\mathcal{U}$. Then,

(i)The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X)=\cup_{x\in U}\{R(X): R(X)\subseteq X, x\in U\}$ where $R(X)$ denotes the equivalence class determined by $x\in U$.

(ii)The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is, $U_R(X)=\cup_{x\in U}\{R(X): R(X)\cap X\neq\emptyset, x\in U\}$ where $R(X)$ denotes the equivalence class determined by $x\in U$.

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not $-X$ with respect to R and is denoted by $B_R(X)$. That is, $B_R(X)=U_R(X)-L_R(X)$.

Definition 2.2 (M.Lellis Thivagar et.al) Let U be a universe, R be an equivalence relation on U and $\tau_R(X)=\{U,\emptyset,L_R(X),U_R(X),B_R(X)\}$, where $X\subseteq U$, satisfies the following axioms:

(i) $U,\emptyset\in\tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Therefore, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U,\tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets (briefly, n- open sets). The complement of a nano open set is called a nano closed set (briefly, n- closed set).

Definition 2.3 (Qays Hatem Imran) A subset $\mathcal{C}$ of a nano topological space $(U,\mathcal{M})$ is said to be nano semi $\alpha-$ open set (briefly, $NS_{\alpha}-$ O.S) if there exists a $n\alpha-$ open set $\mathcal{P}$ in U such that $\mathcal{P}\subseteq\mathcal{C}\subseteq n-cl(\mathcal{P})$ or equivalently if $\mathcal{C}\subseteq n-cl\big(n\alpha-int(\mathcal{P})\big)$.The family of all $NS_{\alpha}-$ O.S of U is denoted by $NS_{\alpha}O(U,\mathcal{M})$.

Definition 2.4 (S.Pasunkilipandian et.al) A subset $\mathcal{H}$ of a nano ideal topological space $(\mathcal{U},\mathcal{M},\mathcal{J})$ is said to be nano ideal semi α generalized closed set (briefly, $nIs_{\alpha}g-$ closed set) if $\mathcal{H}_n^*\subseteq\mathcal{K}$ whenever $\mathcal{H}\subseteq\mathcal{K}$ and $\mathcal{K}$ is nano semi $\alpha-$ open.

Definition 2.5A mapping $\eta: (\mathcal{U}, \mathcal{M}) \to (\mathcal{V}, \mathcal{N})$ is said to be:
(i) contra-continuous if $\eta^{-1}(\mathcal{V})$ is closed in $(\mathcal{U}, \mathcal{M})$ for every open set $\mathcal{V}$ in $(\mathcal{V}, \mathcal{N})$.
(ii) contra $B\delta g -$ irresolute if $\eta^{-1}(\mathcal{V})$ is $B\delta g -$ closed in $(\mathcal{U}, \mathcal{M})$ for every $B\delta g -$ open set $\mathcal{V}$ in $(\mathcal{V}, \mathcal{N})$.
Definition 2.5 (M.Rajamani et.al) Let $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ and $(\mathcal{V}, \mathcal{N}, \mathcal{I})$ be nano topological spaces. A mapping $\eta: (\mathcal{U}, \mathcal{M}) \to (\mathcal{V}, \mathcal{N})$ is said to be contra $Ig -$ continuous if the inverse image of every open set in $(\mathcal{V}, \mathcal{N})$ is $Ig -$ closed in $(\mathcal{U}, \mathcal{M})$.

3. Contra $nIs_{\alpha}g -$ Continuity and Contra $nIs_{\alpha}g -$ Irresolute

Definition 3.1 A function $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \to (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ is said to be contra $nIs_{\alpha}g -$ continuous if $\eta^{-1}(\mathcal{H})$ is $nIs_{\alpha}g -$ closed set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ for every $n -$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$.
Definition 3.2 A function $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \to (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ is said to be contra $nIs_{\alpha}g -$ irresolute if $\eta^{-1}(\mathcal{H})$ is $nIs_{\alpha}g -$ closed set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ for every $nIs_{\alpha}g -$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$.
Example 3.3 Let $\mathcal{U} = \{u_1, u_2, u_3, u_4\}$; $\mathcal{U}/\mathcal{R} = \{\{u_1\}, \{u_2, u_3\}, \{u_4\}\}$; $\mathcal{X} = \{u_1, u_3\}$; $\mathcal{J} = \{\emptyset, \{u_3\}\}$. $\mathcal{M} = \emptyset, \mathcal{U}, \{u_1\}, \{u_2, u_3\}, \{u_1, u_2, u_3\}$. $nIs_{\alpha}g -$ closed sets are
$\emptyset, \mathcal{U}, \{u_3\}, \{u_4\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_1, u_3, u_4\}, \{u_1, u_2, u_4\}, \{u_2, u_3, u_4\}$.
Let $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$; $\mathcal{V}/\mathcal{R} = \{\{v_1, v_3\}, \{v_2\}, \{v_4\}\}$; $\mathcal{Y} = \{v_2, v_3\}$; $\mathcal{J}' = \{\emptyset, \{v_1\}\}$.
$\mathcal{M}' = \emptyset, \mathcal{V}, \{v_2\}, \{v_1, v_3\}, \{v_1, v_2, v_3\}$. $nIs_{\alpha}g -$ closed sets are
$\emptyset, \mathcal{V}, \{v_1\}, \{v_4\}, \{v_, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4$.
Let $\mathcal{W} = \{w_1, w_2, w_3, w_4\}$; $\mathcal{W}/\mathcal{R} = \{\{w_1\}, \{w_2, w_3\}, \{w_4\}\}$; $\mathcal{Z} = \{w_1, w_4\}$; $\mathcal{I}' = \{\emptyset, \{w_1\}\}$.
$\mathcal{N}' = \emptyset, \mathcal{W}, \{w_2, w_4\}$. $nIs_{\alpha}g -$ closed sets are $\emptyset, \mathcal{W}, \{w_1\}, \{w_2, w_3\}, \{w_1, w_2, w_3\}, \{w_2, w_3, w_4\}$.
Define $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \to (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ by $\eta(u_1) = v_1; \eta(u_2) = v_4; \eta(u_3) = v_2; \eta(u_4) = v_3$ which is contra $nIs_{\alpha}g -$ continuous. Define $\zeta: (\mathcal{V}, \mathcal{M}', \mathcal{J}') \to (\mathcal{W}, \mathcal{N}', \mathcal{I}')$ by $\zeta(v_1) = w_4; \zeta(v_2) = w_2; \zeta(v_3) = w_3; \zeta(v_4) = w_1$ which is contra $nIs_{\alpha}g -$ irresolute.
Remark 3.4 Both $nIs_{\alpha}g -$ continuity and contra $nIs_{\alpha}g -$ continuity is independent of each other.
Example 3.5 Consider $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ and $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$ in the example 3.3.
The function η defined in is contra $nIs_{\alpha}g -$ continuous but not $nIs_{\alpha}g -$ continuous because for the $n -$ open sets $\{v_1, v_3\}$ and $\{v_1, v_2, v_3\}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$, $\eta^{-1}(\{v_1, v_3\}) = \{u_1, u_4\}$ and $\eta^{-1}(\{v_1, v_2, v_3\}) = \{u_1, u_3, u_4\}$. Both are $nIs_{\alpha}g -$ closed but not $nIs_{\alpha}g -$ open in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.
Now, define $\zeta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \to (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ by $\zeta(u_1) = v_3; \zeta(u_2) = v_1; \zeta(u_3) = v_2; \zeta(u_4) = v_4$ which is $nIs_{\alpha}g -$ continuous but not contra $nIs_{\alpha}g -$ continuous, because for the $n -$ open sets $\{v_1, v_3\}$ and $\{v_1, v_2, v_3\}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$, $\zeta^{-1}(\{v_1, v_3\}) = \{u_1, u_2\}$ and $\zeta^{-1}(\{v_1, v_2, v_3\}) = \{u_1, u_2, u_3\}$. Both are $nIs_{\alpha}g -$ open but not $nIs_{\alpha}g -$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.Hence, both $nIs_{\alpha}g -$ continuity and contra $nIs_{\alpha}g -$ continuity are independent of each other.
Theorem 3.6 Let $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \to (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ be a function. Then the following axioms are equivalent.

1. η is contra $nIs_{\alpha}g -$ continuous.
2. The inverse image of each $n -$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$ is $nIs_{\alpha}g -$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.
3. $\eta^{-1}(\mathcal{H})$ is $nIs_{\alpha}g -$ open in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ for every $n -$ closed set $\mathcal{H}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$.
4. For each point u in $\mathcal{U}$ and each $n -$ closed set $\mathcal{G}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$ with $\eta(u) \in \mathcal{G}$, there is a $nIs_{\alpha}g -$ open set $\mathcal{K}$ in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ containing u such that $\eta(\mathcal{U}) \subset \mathcal{G}$.

Proof: (1) $\Longrightarrow$ (2): Let $\mathcal{H}$ be $n -$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. Then its complement, $\mathcal{H}^c$ is $n -$ closed set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. Since η is contra $nIs_{\alpha}g -$ continuous, $\eta - 1(\mathcal{H}^c)$ is $nIs_{\alpha}g -$ open in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$. But $\eta^{-1}(\mathcal{V} - \mathcal{H}) = \mathcal{U} - \eta^{-1}(\mathcal{H})$ which implies that $\eta^{-1}(\mathcal{H})$ is $nIs_{\alpha}g -$ closed set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.
(2) $\Longrightarrow$ (3): Let $\mathcal{H}$ be $n -$ closed set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. Then its complement, $\mathcal{H}^c$ is $n -$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. From the hypothesis of (2), $\eta - 1(\mathcal{H}^c)$ is $nIs_{\alpha}g -$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$. But $\eta^{-1}(\mathcal{V} - \mathcal{H}) = \mathcal{U} - \eta^{-1}(\mathcal{H})$ which implies that $\eta^{-1}(\mathcal{H})$ is $nIs_{\alpha}g -$ open set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.
(3) $\Longrightarrow$ (4): Let $u \in \mathcal{U}$ and $\mathcal{H}$ be any $n -$ closed set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. From (3), $\eta^{-1}(\mathcal{H})$ is $nIs_{\alpha}g -$ open set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$. Set $\mathcal{K} = \eta^{-1}(\mathcal{H})$. Then there is a $nIs_{\alpha}g -$ open set $\mathcal{K}$ in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ containing u such that $\eta(\mathcal{K}) \subset \mathcal{G}$.
(4) $\Longrightarrow$ (1): Let $u \in \mathcal{U}$ and $\mathcal{H}$ be any $n -$ closed set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$ with $\eta(u) \in \mathcal{H}$. By (4), there is a $nIs_{\alpha}g -$ open set $\mathcal{K}$ in $\mathcal{U}$ containing u such that $\eta(\mathcal{K}) \subset \mathcal{G}$. This implies $\mathcal{K} = \eta^{-1}(\mathcal{H})$. Therefore, $\mathcal{U} - \mathcal{H} = \mathcal{U} - \eta^{-1}(\mathcal{H}) = \eta^{-1}(\mathcal{V} - \mathcal{H})$ which is $nIs_{\alpha}g -$ closed set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.
Theorem 3.7 Let $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \to (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ be a function. Then the following axioms are equivalent.

1. η is contra $nIs_{\alpha}g -$ irresolute.
2. The inverse image of each $nIs_{\alpha}g -$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$ is $nIs_{\alpha}g -$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.
3. $\eta^{-1}(\mathcal{H})$ is $nIs_{\alpha}g -$ open in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ for every $nIs_{\alpha}g -$ closed set $\mathcal{H}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$.
4. For each point u in $\mathcal{U}$ and each $nIs_{\alpha}g -$ closed set $\mathcal{G}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$ with $\eta(u) \in \mathcal{G}$, there is a $nIs_{\alpha}g -$ open set $\mathcal{K}$ in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$ containing u such that $\eta(\mathcal{U}) \subset \mathcal{G}$.

Proof: The proof is similar to Theorem 3.6.
Proposition 3.8 Every contra $nIs_{\alpha}g -$ continuity is contra $nIg -$ continuity.

Proof: Let $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ be contra $nIs_\alpha g-$ continuous mapping. Let $\mathcal{H}$ be $n-$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. Since η is contra $nIs_\alpha g-$ continuous, $\eta^{-1}(\mathcal{H})$ is $nIs_\alpha g-$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$. Since every $nIs_\alpha g-$ closed set is $nIg-$ closed, the result follows.

Remark 3.9 The reverse implication of the preceding proposition is not valid as shown in the successive example.

Example 3.10 Let $\mathcal{U} = \{u_1, u_2, u_3, u_4\}$; $\mathcal{U}/\mathcal{R} = \{\{u_1\}, \{u_2, u_3\}, \{u_4\}\}$; $\mathcal{X} = \{u_1, u_4\}$; $\mathcal{J} = \{\emptyset, \{u_1\}\}$. $\mathcal{M} = \emptyset, \mathcal{U}, \{u_1, u_4\}$. $nIg-$ closed sets are $\emptyset, \mathcal{U}, \{u_1\}, \{u_2\}, \{u_3\}, \{u_1, u_2\}, \{u_1, u_3\}$, $\{u_2, u_3\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_1, u_2, u_3\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}$. Let $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$; $\mathcal{V}/\mathcal{R} = \{\{v_1, v_3\}, \{v_2\}, \{v_4\}\}$; $\mathcal{Y} = \{v_2, v_3\}$; $\mathcal{J}' = \{\emptyset, \{v_1\}\}$.
$\mathcal{M}' = \emptyset, \mathcal{V}, \{v_2\}, \{v_1, v_3\}, \{v_1, v_2, v_3\}$. $nIs_\alpha g-$ closed sets are $\emptyset, \mathcal{V}, \{v_1\}, \{v_4\}, \{v_, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4$.
Define $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ by $\eta(u_1) = v_4; \eta(u_2) = v_1; \eta(u_3) = v_2; \eta(u_4) = v_3$ which is contra $nIg-$ continuous but not contra $nIs_\alpha g-$ continuous because for the $n-$ open set $\{v_2\}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$, $\eta^{-1}(\{v_2\}) = \{u_3\}$ is not $nIs_\alpha g-$ closed but $nIg-$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.

Proposition 3.11 Every contra nano ideal continuous function is contra $nIs_\alpha g-$ continuous.

Proof: Let $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ be contra $nIs_\alpha g-$ continuous mapping. Let $\mathcal{H}$ be $n-$ open set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. Since η is contra $nI-$ continuous, $\eta^{-1}(\mathcal{H})$ is $n-$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$. Since every $nI-$ closed set is $nIs_\alpha g-$ closed, the result follows.

Corollary 3.12 Every contra nano continuous function is contra $nIs_\alpha g-$ continuous.

Proof: Since every $n-$ open set is $nIs_\alpha g-$ open, the result follows.

Remark 3.13 The reverse implication of the Proposition 3.11 and Corollary 3.12 are not valid as shown in the successive example.

Example 3.14 Let $\mathcal{U} = \{u_1, u_2, u_3, u_4\}$; $\mathcal{U}/\mathcal{R} = \{\{u_1\}, \{u_2, u_3\}, \{u_4\}\}$; $\mathcal{X} = \{u_1, u_4\}$; $\mathcal{M} = \{\emptyset, \{u_1\}\}$. $\mathcal{M} = \emptyset, \mathcal{U}, \{u_1, u_4\}$. $nIg-$ closed sets are $\emptyset, \mathcal{U}, \{u_1\}, \{u_2\}, \{u_3\}, \{u_1, u_2\}, \{u_1, u_3\}$, $\{u_2, u_3\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_1, u_2, u_3\}, \{u_1, u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}$.
$nIs_\alpha g-$ closed sets are $\emptyset, \mathcal{U}, \{u_1\}, \{u_2, u_3\}, \{u_1, u_2, u_3\}, \{u_2, u_3, u_4\}$. Let $\mathcal{V} = \{v_1, v_2, v_3, v_4\}$; $\mathcal{V}/\mathcal{R} = \{\{v_1\}, \{v_2, v_4\}, \{v_3\}\}$; $\mathcal{Y} = \{v_1, v_2\}$; $\mathcal{J}' = \{\emptyset, \{v_2\}, \{v_3\}, \{v_2, v_3\}\}$. $\mathcal{M}' = \emptyset, \mathcal{V}, \{v_1\}, \{v_2, v_4\}, \{v_1, v_2, v_4\}$. $nIg-$ closed and $nIs_\alpha g-$ closed sets are $\emptyset, \mathcal{V}, \{v_2\}, \{v_3\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_2, v_3, v_4\}, \{v_1, v_3, v_4\}$. Define $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ by $\eta(u_1) = v_2; \eta(u_2) = v_1; \eta(u_3) = v_4; \eta(u_4) = v_3$ which is contra $nIg-$ continuous but not contra $nIs_\alpha g-$ continuous because for the $n-$ open set $\{v_1, v_4\}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$, $\eta^{-1}(\{v_2, v_4\}) = \{u_1, u_3\}$ is $nIg-$ closed but not $nIs_\alpha g-$ closed in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$. Define $\zeta: (\mathcal{V}, \mathcal{M}', \mathcal{J}') \rightarrow (\mathcal{U}, \mathcal{M}, \mathcal{J})$ by $\zeta(v_1) = u_3; \zeta(v_2) = u_2; \zeta(v_3) = u_1; \zeta(v_4) = u_4$ is contra $nIs_\alpha g-$ continuous but not contra $n-$ continuous because for the $n-$ open set $\{u_1, u_4\}$ in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$, $\zeta^{-1}(\{u_1, u_4\}) = \{v_3, v_4\}$ is $nIs_\alpha g-$ closed but not $n-$ closed in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$.

Proposition 3.15 Every contra n^*- continuity is contra $nIs_\alpha g-$ continuity.

Proof: Since every n^*- closed set is $nIs_\alpha g-$ closed, the result follows.

Remark 3.16 The reverse implication of the preceding proposition is not valid as shown in the successive example.

Example 3.17 Let $\mathcal{U} = \{u_1, u_2, u_3\}$; $\mathcal{U}/\mathcal{R} = \{\{u_1\}, \{u_2, u_3\}\}$; $\mathcal{X} = \{u_2\}$; $\mathcal{J} = \{\emptyset, \{u_3\}\}$. $\mathcal{M} = \emptyset, \mathcal{U}, \{u_2, u_3\}$. $nIs_\alpha g-$ closed sets are $\emptyset, \mathcal{U}, \{u_1\}, \{u_3\}, \{u_1, u_3\}, \{u_1. u_2\}$. n^*- closed sets are $\emptyset, \mathcal{U}, \{u_1\}, \{u_3\}, \{u_1, u_3\}$.Let $\mathcal{V} = \{v_1, v_2, v_3\}$; $\mathcal{V}/\mathcal{R} = \{\{v_1, v_2\}, \{v_3\}\}$; $\mathcal{Y} = \{v_1, v_3\}$; $\mathcal{J}' = \{\emptyset, \{v_2\}\}$. $\mathcal{M}' = \emptyset, \mathcal{V}, \{v_3\}, \{v_1, v_2\}$. Define $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ by $\eta(u_1) = v_1; \eta(u_2) = v_2; \eta(u_3) = v_3$ which is contra $nIs_\alpha g-$ continuous but not n^*- continuous because for the $n-$ open set $\{v_1, v_2\}$ in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$, $\eta^{-1}(\{v_1, v_2\}) = \{u_1, u_2\}$ is $nIs_\alpha g-$ closed set but not n^*- closed set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$.

4. Composition of functions under contra $nIs_\alpha g-$ continuous and contra $nIs_\alpha g-$ irresolute

Theorem 4.1 Composition of two contra $nIs_\alpha g-$ irresolute function is $nIs_\alpha g-$ irresolute.

Proof: Let $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ and $\zeta: (\mathcal{V}, \mathcal{M}', \mathcal{J}') \rightarrow (\mathcal{W}, \mathcal{N}', \mathfrak{I}')$ be contra $nIs_\alpha g-$ irresolute functions. Then, $\zeta \circ \eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{W}, \mathcal{N}', \mathfrak{I}')$. Let $\mathcal{H}$ be $nIs_\alpha g-$ open set in $(\mathcal{W}, \mathcal{N}', \mathfrak{I}')$. Then $(\zeta \circ \eta)^{-1}(\mathcal{H}) = \eta^{-1}(\zeta^{-1}(\mathcal{H}))$. Since ζ is contra $nIs_\alpha g-$ irresolute, $\zeta^{-1}(\mathcal{H})$ is $nIs_\alpha g-$ closed set in $(\mathcal{V}, \mathcal{M}', \mathcal{J}')$. Since η is contra $nIs_\alpha g-$ irresolute, $\eta^{-1}(\zeta^{-1}(\mathcal{H}))$ is $nIs_\alpha g-$ open set in $(\mathcal{U}, \mathcal{M}, \mathcal{J})$. Therefore, $\zeta \circ \eta$ is $nIs_\alpha g-$ irresolute.

Theorem 4.2 Composition of a $nIs_\alpha g-$ irresolute and contra $nIs_\alpha g-$ irresolute function is contra $nIs_\alpha g-$ irresolute.

Proof: The proof is similar to Theorem 4.1.

Theorem 4.3 Composition of a contra $nIs_\alpha g-$ irresolute function and contra $nIs_\alpha g-$ continuous function is $nIs_\alpha g-$ continuous.

Proof: Let $\eta: (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{V}, \mathcal{M}', \mathcal{J}')$ and $\zeta: (\mathcal{V}, \mathcal{M}', \mathcal{J}') \rightarrow (\mathcal{W}, \mathcal{N}', \mathfrak{I}')$ be $nIs_\alpha g-$ irresolute function and contra $nIs_\alpha g-$ continuous function respectively. Then,\ $\zeta \circ \eta : (\mathcal{U}, \mathcal{M}, \mathcal{J}) \rightarrow (\mathcal{W}, \mathcal{N}', \mathfrak{I}')$. Let $\mathcal{H}$ be $n-$ open set in $(\mathcal{W}, \mathcal{N}', \mathfrak{I}')$. Then $(\zeta \circ \eta)^{-1}(\mathcal{H}) = \eta^{-1}(\zeta^{-1}(\mathcal{H}))$. Since ζ is contra $nIs_\alpha g-$ continuous, $\zeta^{-1}(\mathcal{H})$ is $nIs_\alpha g-$

closed set in $(\mathcal{V},\mathcal{M}',\mathcal{J}')$. Since η is $nIs_\alpha g-$ irresolute, $\eta^{-1}(\zeta^{-1}(\mathcal{H}))$ is $nIs_\alpha g-$ open set in $(\mathcal{U},\mathcal{M},\mathcal{J})$. Therefore, $\zeta\circ\eta$ is $nIs_\alpha g-$ continuous.

Corollary 4.4 Composition of a $nIs_\alpha g-$ irresolute and contra $nIs_\alpha g-$ continuous function is contra $nIs_\alpha g-$ continuous.

Proof: The proof is similar to Theorem 4.3.

Theorem 4.5 Composition of a contra $nIs_\alpha g-$ continuous and $n-$ continuous function is contra $nIs_\alpha g-$ continuous.

Proof: Let $\eta:(\mathcal{U},\mathcal{M},\mathcal{J})\to(\mathcal{V},\mathcal{M}',\mathcal{J}')$ and $\zeta:(\mathcal{V},\mathcal{M}',\mathcal{J}')\to(\mathcal{W},\mathcal{N}',\mathcal{I}')$ be contra $nIs_\alpha g-$ continuous function and $n-$ continuous function respectively. Then, $\zeta\circ\eta:(\mathcal{U},\mathcal{M},\mathcal{J})\to(\mathcal{W},\mathcal{N}',\mathcal{I}')$. Let $\mathcal{H}$ be $n-$ open set in $(\mathcal{W},\mathcal{N}',\mathcal{I}')$. Then $(\zeta\circ\eta)^{-1}(\mathcal{H})=\eta^{-1}(\zeta^{-1}(\mathcal{H}))$. Since ζ is $n-$ continuous, $\zeta^{-1}(\mathcal{H})$ is $n-$ open set in $(\mathcal{V},\mathcal{M}',\mathcal{J}')$. Since η is $nIs_\alpha g-$ continuous, $\eta^{-1}(\zeta^{-1}(\mathcal{H}))$ is $nIs_\alpha g-$ closed set in $(\mathcal{U},\mathcal{M},\mathcal{J})$. Therefore, $\zeta\circ\eta$ is contra $nIs_\alpha g-$ continuous.

Corollary 4.6 (i) Composition of a $nIs_\alpha g-$ continuous and contra $n-$ continuous function is contra $nIs_\alpha g-$ continuous.

(ii) Composition of a contra $nIs_\alpha g-$ continuous and contra $n-$ continuous function is $nIs_\alpha g-$ continuous.

Proof: The proof is similar to Theorem 4.5.

Remark 4.7 Composition of two contra $nIs_\alpha g-$ continuous function need not be contra $nIs_\alpha g-$ continuous as shown in the succesive example.

Example 4.8

Let $\mathcal{U}=\{u_1,u_2,u_3,u_4\}$; $\mathcal{U}/\mathcal{R}=\{\{u_1\},\{u_2,u_3\},\{u_4\}\}$; $\mathcal{X}=\{u_1,u_3\}$; $\mathcal{J}=\{\emptyset,\{u_3\}\}$.
$\mathcal{M}=\emptyset,\mathcal{U},\{u_1\},\{u_2,u_3\},\{u_1,u_2,u_3\}$. $nIs_\alpha g-$ closed sets
$\emptyset,\mathcal{U},\{u_3\},\{u_4\},\{u_1,u_4\},\{u_2,u_4\},\{u_3,u_4\},\{u_1,u_2,u_4\},\{u_1,u_3,u_4\},\{u_2,u_3,u_4\}$.
Let $\mathcal{V}=\{v_1,v_2,v_3,v_4\}$; $\mathcal{V}/\mathcal{R}=\{\{v_1\},\{v_2,v_3\},\{v_4\}\}$; $\mathcal{Y}=\{v_1,v_4\}$; $\mathcal{J}'=\{\emptyset,\{v_1\}\}$. $\mathcal{M}'=\emptyset,\mathcal{V},\{v_1,v_4\}$.
$nIs_\alpha g-$ closed sets are, $\emptyset,\mathcal{V},\{v_1\},\{v_2,v_3\},\{v_1,v_2,v_3\},\{v_2,v_3,v_4\}$.
Let $\mathcal{W}=\{w_1,w_2,w_3,w_4\}$; $\mathcal{W}/\mathcal{R}=\{\{w_1,w_3\},\{w_2\},\{w_4\}\}$; $\mathcal{Z}=\{w_2,w_3\}$; $\mathcal{I}'=\{\emptyset,\{w_1\}\}$.
$\mathcal{N}'=\emptyset,\mathcal{W},\{w_2\},\{w_1,w_3\},\{w_1,w_2,w_3\}$.
Define $\eta:(\mathcal{U},\mathcal{M},\mathcal{J})\to(\mathcal{V},\mathcal{M}',\mathcal{J}')$ as $\eta(u_1)=v_1;\eta(u_2)=v_2;\eta(u_3)=v_3;\eta(u_4)=v_4$ which is contra $nIs_\alpha g-$ continuous.
Define $\zeta:(\mathcal{V},\mathcal{M}',\mathcal{J}')\to(\mathcal{W},\mathcal{N}',\mathcal{I}')$ as $\zeta(v_1)=w_2;\zeta(v_2)=w_1;\zeta(v_3)=w_3;\zeta(v_4)=w_4$ which is contra $nIs_\alpha g-$ continuous.
But $\boldsymbol{\zeta\circ\eta}$ is not contra $\boldsymbol{nIs_\alpha g}-$ continuous since for the $\boldsymbol{n}-$ open set $\{\boldsymbol{w_1},\boldsymbol{w_2},\boldsymbol{w_3}\}$ of $(\boldsymbol{\mathcal{W}},\boldsymbol{\mathcal{N}'},\boldsymbol{\mathcal{I}'})$, $(\boldsymbol{\zeta\circ\eta})^{-1}$ $(\{\boldsymbol{w_1},\boldsymbol{w_2},\boldsymbol{w_3}\})=\{\boldsymbol{u_1},\boldsymbol{u_2},\boldsymbol{u_3}\}$ which is not $\boldsymbol{nIs_\alpha g}-$ closed in $(\boldsymbol{\mathcal{U}},\boldsymbol{\mathcal{M}},\boldsymbol{\mathcal{J}})$.

5.Conclusion

In this paper, we have introduced and studied some characteristics of contra $nIs_\alpha g-$ continuous function. Also, we have discussed the necessary and sufficient conditions for a function to be contra $nIs_\alpha g-$ continuous and contra $nIs_\alpha g-$ irresolute. Further, we have investingated the composition of functions under contra $nIs_\alpha g-$ continuous and contra $nIs_\alpha g-$ irresolute function.

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