Ethnomathematical connections in bricks making in Salamina-Magdalena, Colombia, and geometric treatment with GeoGebra

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Article History: *Do not touch during review process(xxxx)*

Abstract: Ethnomathematical connections in the elaboration of the mud brick in Salamina-Magdalena, Colombia were analyzed. Theoretically, the work was based on ethnomathematical connections, ethnogeometry and universal activities. The methodology was qualitative exploratory with an ethnographic approach, carried out in three stages: 1) selection of the participant-bricklayer, 2) data collection through semi-structured interviews in the production areas (mud pits and kilns), and finally, 3) a data analysis of the measurement processes was carried out, portions referred to the amount of clay for the elaboration of each brick, emphasizing its geometric shape. We conclude that geometric concepts (rectangle, prism, vertex, parallelism, perpendicularity, angles, etc.), on the stove the parable was identified, and universal activities (counting, measuring, designing, locating, explaining) are identified in the elaboration of the clay brick. Finally, we reflect on the importance of connecting the shape of the brick with geometric concepts such as the parallelepiped and its treatment with GeoGebra, which is a mathematical content addressed in the classroom and suggested by international curricula.
Keywords: Ethnomathematical connections, Ethnogeometry, Ethnomathematics, parallelepiped, Mathematics Education.

1. Introduction

This research is part of the Ethnomathematics program, concerned from its inception with valuing mathematics immersed in the activities carried out by diverse cultural groups (D'Ambrosio, 2001; D'Ambrosio & Knijnik, 2020). Likewise, it is in charge of showing "what mathematical ideas exist in all human cultures, in the experiences of all peoples, of all social groups and cultures, both men and women" (Gerdes, 2013, p. 150). Over the years, this line of research has grown and is not only inclined towards the mathematics of cultural groups such as masons, seamstresses, athletes, shopkeepers, farmers, carpenters, cooks, fishermen, among others, but also emphasizes in linking emerging mathematical knowledge in these practices with institutionalized mathematics developed in classrooms at different school levels (Busrah & Pathuddin, 2021; Rodríguez-Nieto, 2020; 2021; Rodríguez-Nieto & Alsina, 2022).

In particular, the present work has chosen Ethnogeometry as the main focus, focused on the analysis of spatial patterns in a specific environment focused on considering the immaterial and abstract forms of geometry and converting them into concrete or material when working in daily practices (De la Hoz et al., 2017), research recognizes that studies on ethnogeometry explore the implicit or explicit geometric forms in everyday objects such as artifacts, ceramics, wood or clay carvings, metals, plastics, among other raw materials, where they are understood the ways of thinking of artisans in the materialization and construction of objects based on their sociocultural environment (De la Hoz et al., 2017). In the works of a carpenter, Castro et al. (2020) evidenced conventional (meter, inch) and non-conventional (tolerance) measurements, as well as geometric notions in angles and symmetries in the construction of furniture and ships. Prahmana et al. (2021) explored the culture of Yogyakarta used in learning mathematics, including mathematical models to determine the seasons in the pranatamangsa system and the ceremonial birth-death, being potential to be used in learning mathematics. Math topics such as patterns, modulus, and number sense.

In the field of research in Ethnomathematics and Ethnogeometry, significant studies have been carried out that address mathematical knowledge in fishing, one of them is characterized by the two non-conventional measurement systems in artisanal kite fishing in *Bocas of Cenizas* (Rodríguez-Nieto et al., 2019a) where they identified units of measurement such as the quarter, the boss, the finger and the stroke. Mansilla-Scholer et al. (2022) investigated the mathematics used in fishing in southern Chile, evidencing non-conventional measures such as the fathom and measures in the creation of longlines. In addition, it was identified that in Rodríguez-Nieto et al. (2019b) and Rodríguez-Nieto (2020) the mathematics used in the preparation of cassava buns were identified, for example, the bollero uses the fathom, the jeme, the bulk, the lao, the time, among others. On the other hand, in the fabric designs there is an ethno-mathematical experience in the Colombian Amazon (Parra, 2006) where they determined a different counting system such as the hand, two feet, and also identified some terms associated with the distribution (wüi-chígü = for each) and one to doubling (tareepüküna = twice, double). In masonry, different mathematical processes could be found, such as the conversion of measurements, the construction of circumferences and the measurement of angles, which are given tacitly during the performance of work activities in a certain social group (Arias et al., 2010). Rey and Aroca (2011) studied the mathematics of a group of masons, who used some artisanal tools, such as the strapping machine, the clamp, the formwork and the level hose, among other conventional and non-conventional measures.

Sunzuma and Maharaj (2020) showed that in-service teachers consider ethnomathematical approaches related to ethnogeometric practices and cultural experiences that are important for geometry teaching, Mallqui and Chávez (2021) focused on interpreting the experiences of mathematics teachers from the Santa Clara de Uchunya community in Peru around ethnomathematical studie. Likewise, “the teaching and learning of geometry must reflect and include the social diversity that integrates the learning of geometry in the context of an increasingly connected world” (Sunzuma & Maharaj, 2020, p. 22). Along these same lines, sharing the idea of connection, Umbara et al. (2021) state that mathematics cannot be separated from everyday life, since the use of mathematical, geometric, statistical concepts, etc. in cultural activities can be studied through ethnomathematics. Paternina-Borja et al. (2020) investigated the symmetries in the elaboration of the bull mask and the potential of non-conventional measures; In addition, Janiola et.al (2021) explored ethnomathematical practices through the experiences lived within the Eskaya tribe and in this way studied their Eskaya numbering systems, such as Eskaya numbers and numerals, the Eskaya name of forms basic and the four fundamental operations, and the use of Eskaya numbers to measure time, days and months. Nugraha et al. (2020) studied the increase in students’ mathematical understanding ability in the experiment class (using the CTL model based on Sundanese ethnomathematics) and control class (using the conventional learning model), for which obtained as a result that learning based on Sundanese ethnomathematics could facilitate group discussion in problem-solving activities and the role of ethnomathematical context in CTL that was elevated to a sense of meaning for learning.

Other research have studied the production of bricks for economic purposes or discussions generated around its material, quality, marketing, such as creating the first company that produces and markets bricks with recycled plastics in the municipality of Girón, department of Santander, Colombia (Flórez et al., 2015). In addition, the creation of ecological alternatives that allow the elaboration of concrete blocks maintaining the geometry and the manufacturing process of a conventional block (Meza et al., 2017). In the context of chemical engineering, Chire-Salazar and Rondán-Gutierrez (2014) investigated the use of sludge generated in the decanters of a drinking water treatment plant as raw material for the production of construction bricks, emphasizing the percentages of the components and proportions to obtain the product, for example, binary mixtures, composed of 50% mud and 50% Clay-Loam.

In Ecuador there have been at least two studies related to brick from engineering points of view. For example, in civil engineering, Cachago and Caguano (2016) explored the use of sludge from the wastewater treatment plant of the company Franz Viegener F.V. - Andean area S.A. for the production of handcrafted bricks. Likewise, Camargo and Yambay (2020) investigated the possibility of taking advantage of the sludge resulting from the purification process of the Quitumbe Wastewater Treatment Plant (PTAR-Q) as a material for the production of artisanal bricks. These researchers used different percentages of residual sludge, both in wet sludge conditions (10%, 15% and 20%) and dry sludge (5%, 10% and 15%). On the other hand, Afanador et al. (2013) characterized the base material to elaborate designs of clay mixtures to improve the paste and the resistance of the bricks in Ocaña, Norte of Santander, in order to avoid cracking of the brick walls. the houses of strata 1 and 2.

The literature review shows that Ethnomathematics and Ethnogeometry have been widely studied in different daily practices and their link in the classroom. Also, on the elaboration of the brick that is the object of interest of this research, we assure that it has been worked in engineering contexts, production factors and chemical compositions, as well as wastewater and sludge (mud). Especially, it is recognized that on the elaboration of the brick, no study has yet been carried out committed to improving the teaching and learning of geometry from a sociocultural context. Therefore, the objective of this research is *to analyze the ethnomathematical connections in the elaboration of the clay brick in Salamina-Magdalena, Colombia, and its treatment with the GeoGebra software*.

1.1. Conceptual framework

1.1.1. Ethnomathematics

In various investigations it has been considered that Ethnomathematics is “the mathematics practiced by cultural groups, such as urban or rural communities, groups of workers, professional classes, children of a certain age, indigenous societies and other groups that are identified by common objectives and traditions. to the groups" (D'Ambrosio, 2001, p. 9). Furthermore, etymologically, from the perspective of D'Ambrosio (2014) he defines Ethnomathematics as a division of three roots (see Figure 1).

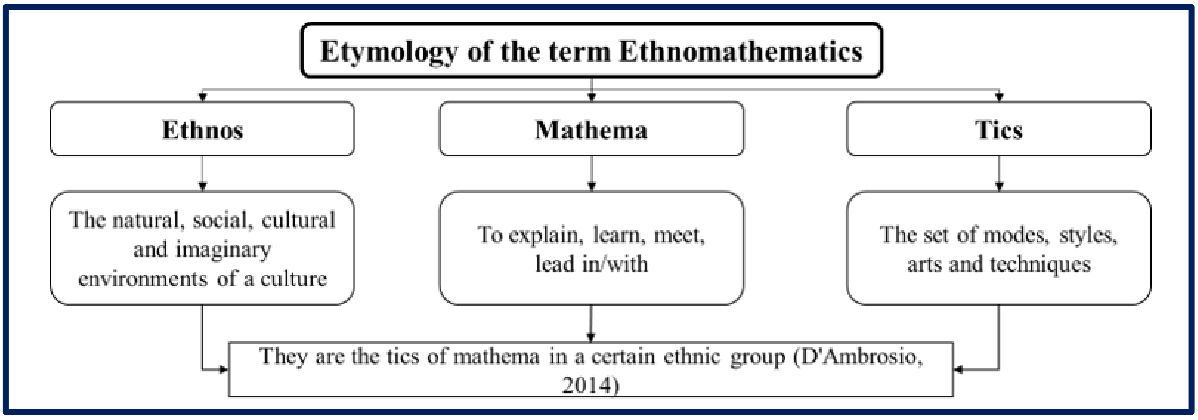

Figure 1. Etymology of the term Ethnomathematics (D'Ambrosio, 2014).

This concept of "ethno" includes all groups with their jargons, codes, symbols, myths and their specific processes of reasoning and inference. Acts and studies the way in which cultural groups elaborate, understand and use concepts, structures or meanings, which the researcher considers mathematical, in the development of their culture (D'Ambrosio, 1985).

1.1.2. Ethnomathematical connections

The term connection in mathematics education research has been defined as the relationship between two mathematical ideas A and B (Businskas, 2008). In addition, other definitions are recognized in the literature that point to the development of works focused on Calculus, especially with concepts related to the derivative, the rate of change, functions, and integrals (Rodríguez-Nieto et al., 2021a; Rodríguez-Nieto et al., 2021b; Rodríguez-Nieto et al., 2022a). In this context, connection has been defined as a cognitive process that a person performs when solving mathematical tasks and relating concepts, procedures, meanings, properties, graphs, to each other and to everyday life (García-García & Dolores-Flores, 2018). For the purposes of this research focused on the study of brick making, the definition of *ethnomathematical connection* is used, which is understood as the relationship between mathematics practiced by cultural groups and universally recognized institutionalized or public mathematics which found in the curricular materials as textbook (Rodríguez-Nieto, 2021).

It should be noted that mathematical connections are important for the creation of contextualized tasks (e.g., see research on the ethnomathematical connections in the elaboration of tops and tacos (Rodríguez-Nieto et al., 2022b) and the ethnomathematical connections in the elaboration of cheeses and drums and their potential for geometry classes (Rodríguez-Nieto et al., 2022c)) and favor the teaching and learning of mathematics from three aspects:

> 1) Ethnomathematical connections are relevant because first mathematics is valued in the daily practice carried out by a person and then the researcher identifies the connection and links it with institutionalized mathematics. 2) The ethnomathematical connections can favor the understanding of mathematical concepts considering that the student solves mathematical problems based on real life and, in turn, the suggestions on connections of the curricular organisms are shared (...). 3) Ethnomathematical connections can not only be recognized in a single daily practice, but in several, from the same sociocultural context or from different peoples, regions or countries, avoiding the local aspect of ethnomathematics when it is emphasized in a single daily practice (Rodríguez- Nieto & Escobar-Ramírez, 2022).

1.1.3. Ethnogeometry

Ethnogeometry focuses on the detailed analysis of spatial patterns in a specific environment focused on considering the immaterial and abstract forms of geometry and converting them into concrete or material when working in daily practices (De la Hoz et al., 2017). It should be noted that, in each society or cultural group, the way to communicate everyday objects or artifacts is sought through languages based on mathematical expressions, comparisons and qualities of products that are developed in that place. In the context of Ethnogeometry, Ferreira et al. (2017) states that they are all those methodological procedures that aim to recognize parts of geometric thinking that are hidden or frozen in the different daily activities carried out by the human being and that in turn have a long history.

Based on the articulation of ethnogeometry with Ethnomathematics, this work proceeds to analyze the concepts immersed in the elaboration of the brick, which is motivated from tangible realities to later extract concepts, identify theorems to work polyhedrons, identify figures diverse geometric shapes, modes of use to take them to the classroom. In addition, ethnomathematics is related to the universal activities that are carried out in daily life by people in their different daily activities.

1.1.4. Universal activities

According to Bishop (1999) there are six types of activities immersed in different ways in each of the activities carried out by the different communities in their daily lives and where the culture that surrounds the person influences, they correspond to the activities of counting, explaining, locating, measuring, design, playing.

Table 1. Universal activities' description.

Universal activities	Description
Counting	It refers to the systematic way of confronting and organizing differentiated objects, which implies body or digital counting, with marks, use of ropes or other objects for registration, which will be carried out according to the context of the people and the community where this activity is developed (Bishop, 1999).
Measuring	It is "important for the development of mathematical ideas, and deals with: comparing, ordering and quantifying qualities that have value and importance" (Bishop, 1999). It is important to highlight that the human body was the first measuring instrument that the human being used to carry out this activity, the units of measurement were: the fourth, the step, the fathom, among others (Bishop, 1999).
Explaining	Increases the cognition of the human being to express demonstrations that are above the level associated with explanations based on experience, interpretation of figures, their mathematical structure, mathematical models; and in turn giving answers to questions: How many? Where? How? And mainly the explanation is more abstract when answering the question Why? (Bishop, 1999).
Playing	It allows the fluidity of mathematical ideas, through games mathematical connections are born, closely related to the sociocultural contexts in which they are developed associated with rules, strategies, order, ingenuity, values, repetitions and imagination
Designing	It refers to the transformation of a form of a natural and cultural environment. That is, apply a specific structure or modify a part of nature for another thing or object, for example: clay, wood or soil, and turn it into a brick, furniture, among others.
Locating	It establishes the difference between the individual and the space that surrounds him, it is related to the knowledge of the environmental space that arises from the need to give meaning to the environment that surrounds the members of a community. Bishop considers the socio-geographical space to be the most relevant.

1.1.5. What is a brick?

For the purposes of this work, bricks are small ceramic units that have a parallelepiped shape, made up of clayey earth, molded, compressed and subjected to burning in artisan ovens, used in all kinds of construction due to their regular shape and easy handling (Moreno, 1981), see Figure 2.

Figure 2. Mud brick's representation.

2. Methodology

To identify the ethnomathematical connections, geometric concepts, and universal activities in the practice of brick making in Salamina Magdalena, an exploratory qualitative methodology with an ethnographic approach is chosen (Cohen et al., 2018; Restrepo, 2016), carried out in three stages (see Figure 1).

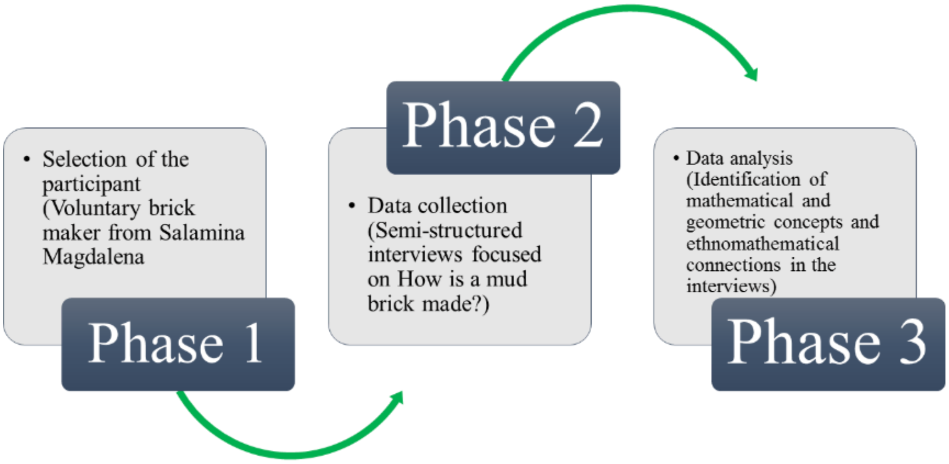

Figure 3. Methodological scheme of the research.

The ethnographic approach is convenient when seeking to describe the customs or activities that occur in a community, a trade, a workers' union, a classroom or a subject in a certain social context (Martínez, 2004).

2.1. Participants and context

The voluntary participation of a bricklayer from the municipality of Salamina Magdalena, Colombia, named Don Edgardo (E), was considered. The participant is 47 years old and has 22 years of experience in mud brick making. He has a bachelor's degree and also has a professionalization course which certifies him to teach, but he does not practice it because he had economic difficulties and stayed only in this job, which he does with much love and appreciation.

2.2. Data collection

Field work and semi-structured interviews (Longhurst, 2010) were developed for data collection, where verbal exchanges were carried out between the researchers (I1 and I2) and participant E. During the interview, the researchers asked initial questions, for example, "How is a brick made?", "What is the use of the mud brick?". Subsequently, in the middle of the dialogue, the participant mentioned key aspects of their practice, immediately a question was asked to delve into that aspect (e.g., infer a geometric concept), that is, the questions emerged naturally in the dialogue. In addition, video cameras and field notes were used.

2.3. Data analysis

A detailed qualitative analysis was carried out from the perspective of Hernández et al. (2014) with adaptations presented in Rodríguez-Nieto (2021), developed in the following steps based on the interviews conducted. In this context, 1) the interviews were transcribed and left in text form. 2) codes were identified that suggest universal activities considered in the conceptual framework. 3) the recognized universal activities were grouped taking into account similarities or points in common that make up a theme referring to a universal activity, for example, for the construction of brick, measures that are connected with its design or form are considered. 4) the ethnomathematical connections that emerge from the explanations given by the bricklayer in terms of universal activities and their relationship with institutionalized mathematics were recognized. In particular, he mentions that the large brick "is about 27 cm long * 19 cm wide * 8.5 cm high", which shows the connection with the parallelepiped (see connection example in Figure 4).

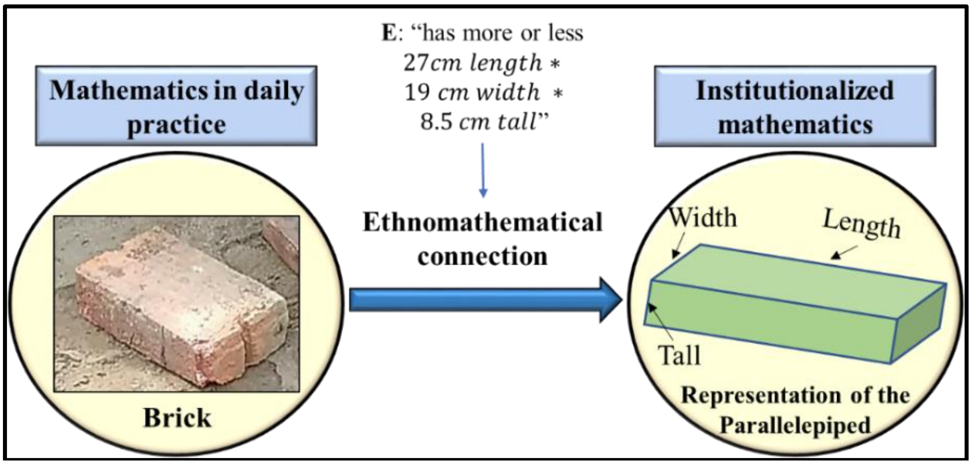

Figure 4. Example of ethnomathematical connection with brick.

3. Results and Discussion

The findings of this research are presented operationally using the conceptual framework, presenting the ethnomathematical connections that emerge in each phase of mud brick making and their link to institutional mathematics. In addition, the commercialization of the bricks is explained considering mathematical aspects and, finally, its treatment with GeoGebra.

3.1. Brick production phases

Three phases of brick making were recognized: a) mixture of materials (Mud and water); b) design and molding of the brick and c) organization and burning of the bricks in the artisan oven.

3.1.1. Phase 1: Mixing of materials (Layers of earth, water)

In this phase, the brick maker goes to the mud pits to prepare and mix it, using a shovel, hoe and water, establishing a classification between the mud used for the bricks and the other that is discarded. This process lasts approximately one day until the mud mixed with sand found in the layers settles (transcript extract and Figure 5).

> I2: How do you make mud brick?
> P: The brick is completely handmade.
> P: It consists of mixing layers of earth with water pointed from a hoe and shovel, also taking into account the type of mud, because all mud does not work. When everything is mixed, it is left to rest for a period of 18 to 24 hours (see Figure 5).
> In this the clay is being kneaded with the help of a hoe.

In this conversation between the researchers and the bricklayer, I2 was asked: What type of mud do you use? To which he replied:

> P: No, the mud that we use in this area is practically the same, but rather that all mud does not work, it is not the same, something is missing. The mud, just as all those layers are lowered one after the other, one finds sand and the sand also goes inside the mud, we do not remove it.


Figure 5. Kneading and mixing the mud with water in the well.

3.1.2. Phase 2: Design and molding of the brick

The importance of universal activity prevails in the design of the bricks since the raw material is transformed into an artifact or object. Likewise, the brick maker uses molds made of wood that function as patterns in order to make all the bricks with equal measurements, that is, a geometric pattern made up of measurements that are preserved is taken (see extract from the transcript and Figure 6).

P: After that time I take the clay and with the help of a mold I make the different figures of the brick, once the clay is in the mold we add grit so that the clay does not stick to the mold. It is waterproofed with sand, extracted from the river or the same surface of the layer of sand that the earth brings. Here we can see the mold, made of wood.

Figure 5. Prepared mud mix and molds.

Subsequently, the activity of counting and measuring is emphasized when the researchers referred to inquiring about the types of bricks (see transcript excerpt).

I2: How many types of bricks are there?
P: There is the big block (see Figure 7a) which is the commercial one and the other mold which is the one that gives you the smaller brick (see Figure 7b).

Considering the sizes of the bricks, the researchers delved into the measurements, where the activity of measuring and counting the sides of the brick emerged (Excerpt from the transcript and Figure 7).

I2: How big is this big block? (Points to the brick in Figure 7a)
P: This one is about 27 cm long x 19 cm wide x 8.5 cm high
I2: What are the dimensions of this type of brick? (Point to the brick in Figure 7b).
P: 10cm wide x 18cm long x 5cm high.

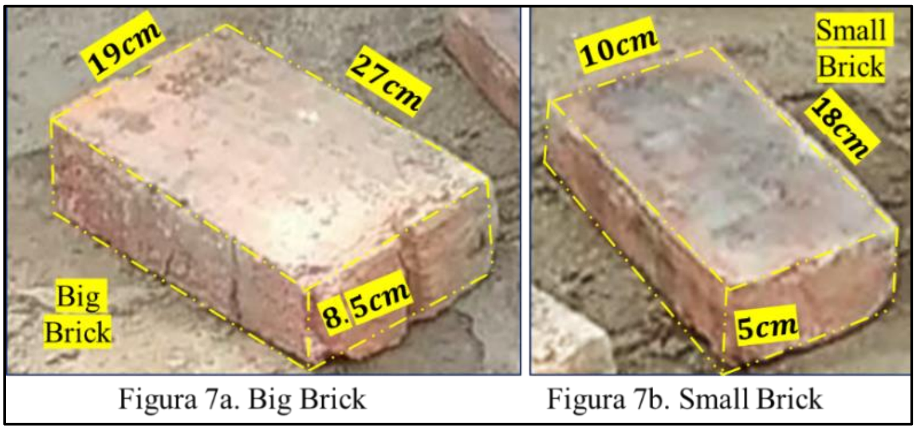

Figure 6. Dimensions of the types of bricks.

In addition, the bricklayer set out to explain and point out what those brick measurements are, ratifying once again about the notion of parallelepiped that, although it is not explicit on his part, with his explanation the mathematical richness connected with real life can be interpreted, that is, the ethnomathematical connection is evident (see extract of the transcript).

I2: What is the length, what is the width and what is the height?
P: This is the length (Figure 8a), this is the width (Figure 8b), this is the thickness of the block being the height (Figure 8c), [points with his fingers at Figure 8].

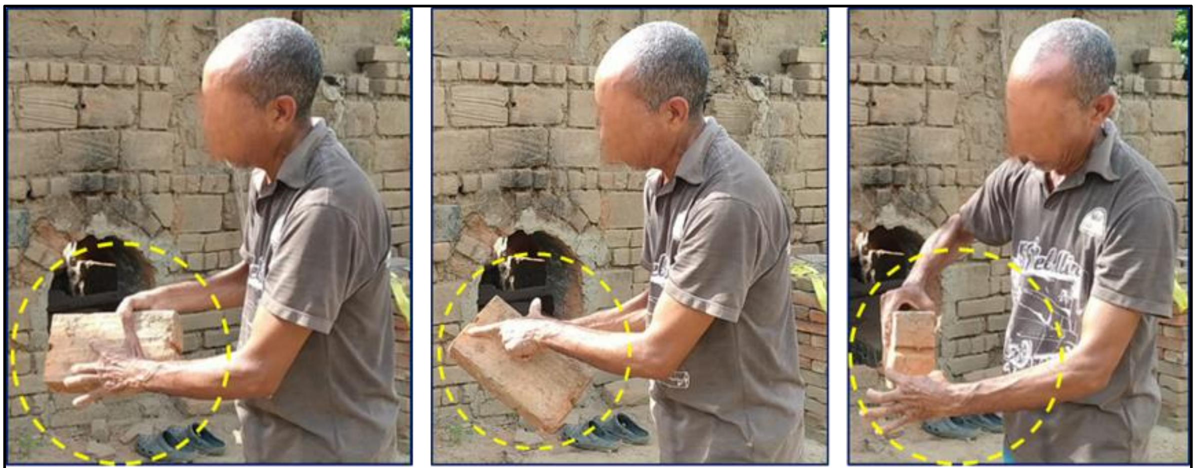

Figure 8a. Length Figure 8b. Width Figure 8c. Tall

Figure 7. Brick measurements.

Now, the bricklayer argues and clarifies that people think that the height of the brick is the width, but in reality, it refers to the thickness or thickness (see excerpt from the transcript and Figure 9).

> P: Many people believe that the height is this (Figure 9a) not because you put it that way, then the height is given to you because the block is made lying down (Figure 9b).

Figure 9a. Width and not height. Figure 9b. Adequate height (points with his fingers).

Figure 8. Argumented explanation about the height of the brick.

In phase 2, the mathematical potential is recognized in the explanations and gestures of the bricklayer, essential for the identification of the ethnomathematical connections (1, 2, 3 and 4) that are presented in Figure 10, where the mathematics used to design is connected. and elaborate the brick with different mathematical and geometric concepts.

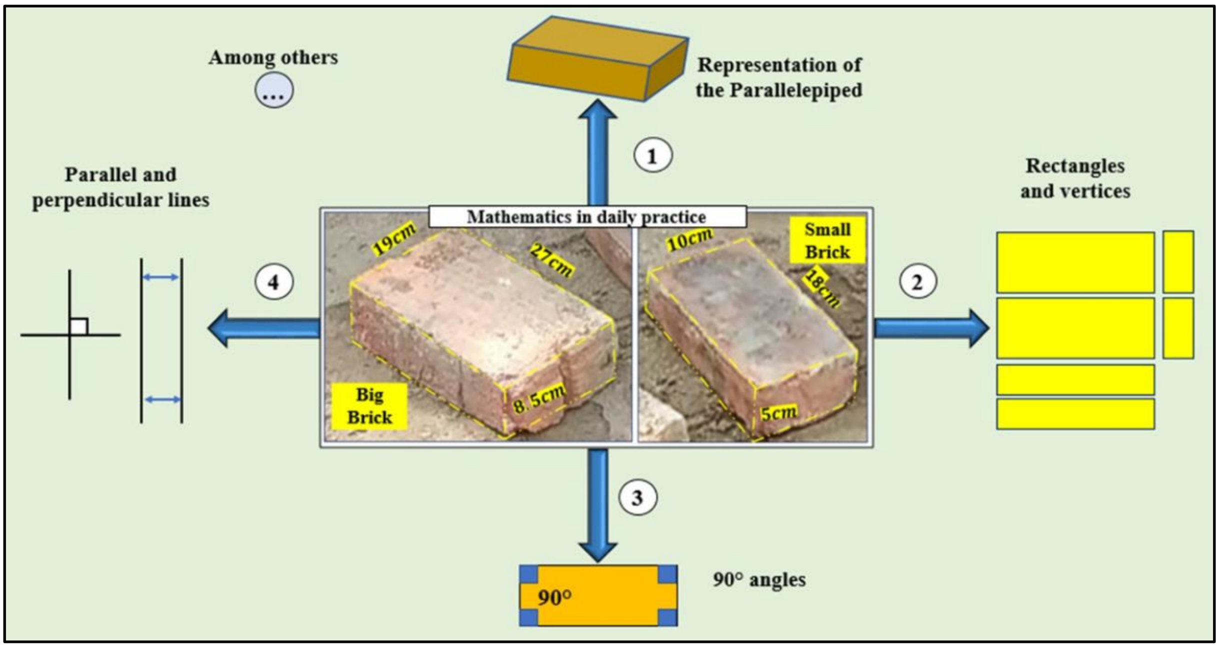

Figure 9. Ethnomathematical connections in the making and shape of the brick.

It should be noted that Figure 10 refers to "among others" because without a doubt in the brick there are other ethnomathematical connections that are not mentioned, for example, it was recognized that the large brick has a weight of 7 kilos equivalent to 14 pounds (see transcript excerpt). In addition, you could delve into the measurements of area, perimeter, volume, among others.

I2: Does it have any weight to have an exact measurement?
P: To weigh that yes, that one weighs 7 kilos, the big brick.

3.1.3. Phase 3: Organization and burning of bricks

In this section it is stated that the brickmaker goes to the artisanal kilns, in which he organizes the bricks, identifying the universal activities of locating and counting. Then, the brick maker sets out to explain the number of bricks that are put into each kiln, and, in turn, the amount of firewood used to burn them (see transcript excerpt).

I1: How many bricks come out in each burning?
P: In this furnace approximately 5500 blocks come out, it is not something that is precise.
I1: Of the 5500 are the blocks and the smallest?
P: I have never filled this oven with the smallest one, but my father-in-law tells me that approximately 45,000 of the small brick has to be put in. These are ovens that burn at the tip of pure wood.

Now, in relation to wood (firewood), the researchers proceed to ask:

I1: How much firewood is used in each burning?
P: The amount of wood for this oven, in my case I put 18 Wood pile.
I1: What is a Wood pile?
P: A Wood pile is an approximate measure that is 1 meter wide, 1 meter long by one and a half meters high (see Figure 11). That is determined by Wood pile here the custom.

Figure 11. Dimensions of "a Wood pile".

It is important to highlight that in Figure 11 there are ethnomathematical connections in the conformation of the Wood pile and the 18 Wood pile, for example, between the unconventional measure "Wood pile" and the parallelepiped with the measures: one meter long by one meter wide. By a meter and a half high that works as a measurement standard that is reproduced in the form of a "Wood pile" until forming the eighteen donkeys evidencing another parallelepiped (see Figure 12).

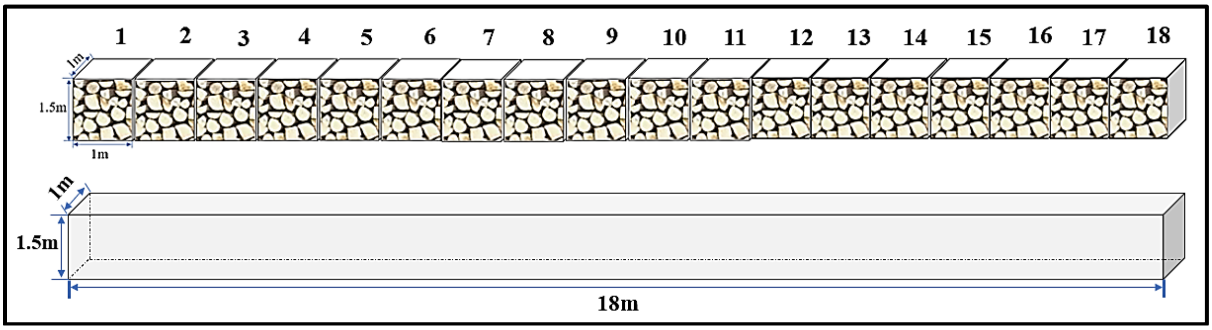

Figure 12. Conformation of the parallelepiped based on Wood pile.

Subsequently, the brick maker proceeds to burn the bricks with fire and at high temperatures. With this, they would be ready for commercialization (see extract of the transcript and Figure 13).

> P: After making the figure of the bricks comes the process of burning the bricks with the help of a fairly traditional oven, which are subjected to high temperatures.

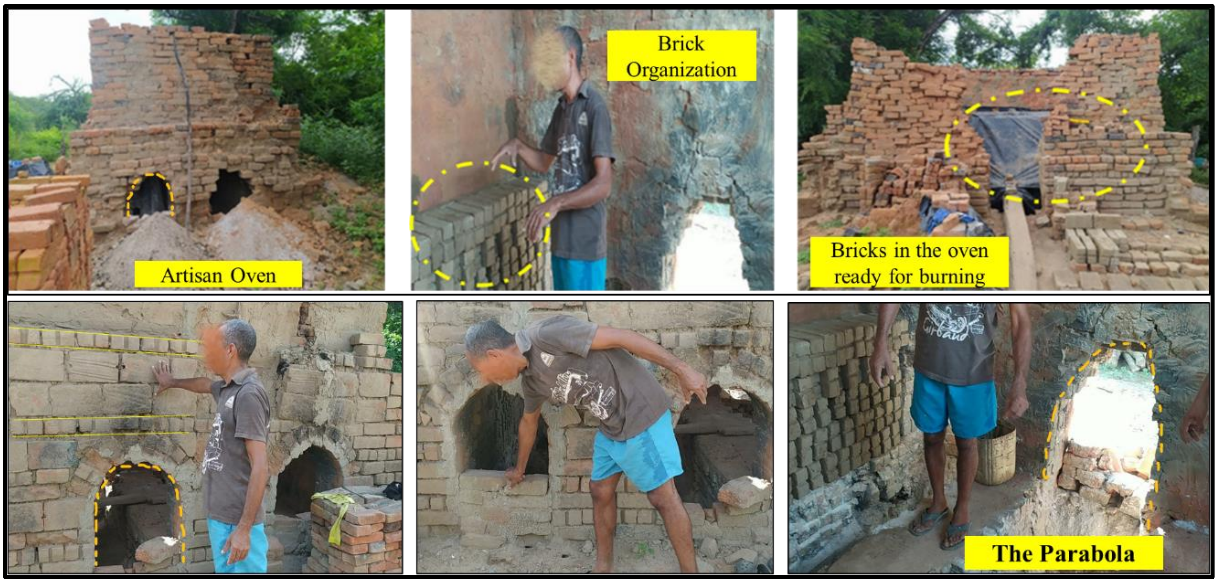

Figure 13. Location, burning of the bricks and representation of the parabola.

In addition to the elaboration of the brick, in the form of the artisan oven, other concepts are recognized, such as parallel lines in the rows of bricks and in the holes or pipes that the oven has as the entrance to the tunnels filled with wood, other ethnomathematical connections that relate the shape of the pipe to the graph of the parabola (Excerpt from the transcript and Figures 13 and 14).

> I2: Going back to the question of the instruments for making the brick, here we can see the oven.
> P: Of course this is the main tool, this is the oven, this is an oven made of pure brick, it can be said that it is crude and it is glued with pure clay, it has no cement, it is pure clay and brick, clay and brick.
> I2: And these holes?
> P: These holes are the cannons, which means cannons, we take and leave the tunnels, those tunnels close at this height more or less.
> P: This here is what I was telling you out there, they are the canyons, we reach two rows, to what reaches 8, here the bricks are going to be found because they are closing, and they form a tunnel. These tunnels are filled with thick wood, until you reach the top where it does not enter any more, it is sealed over there or over here (either of the sides where the tunnels are) on one of the two sides it must be sealed, then on the side the one that remains open is put on a candle.

It should be noted that, in the structure and design of the oven, the ethnomathematical connection with the parabola that opens downwards was recognized, which is addressed in all the curricula worldwide as a graph of the quadratic function that is essential to solve various

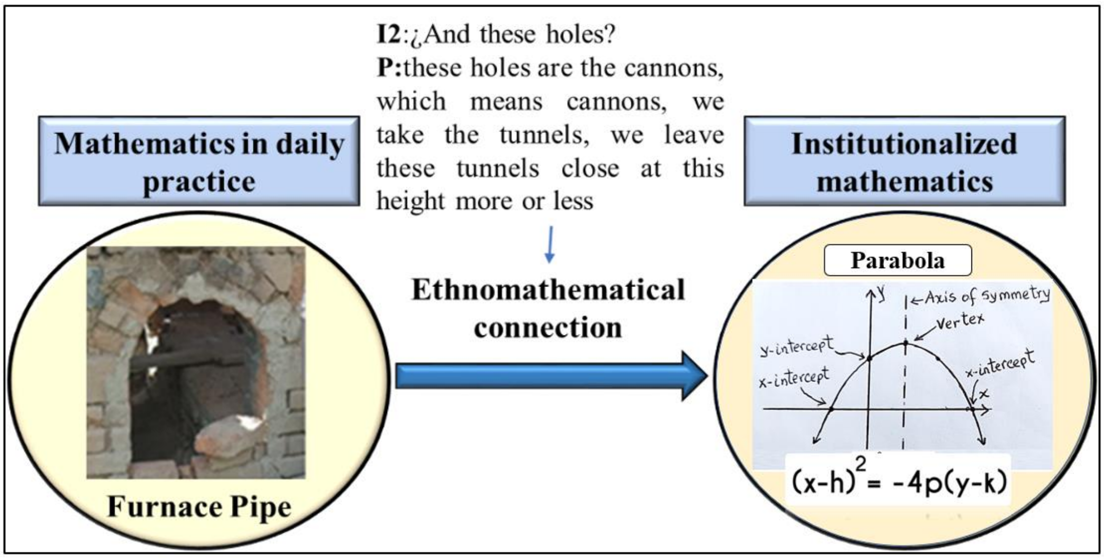

Figure 14. Ethnomathematical connection between the pipe and the Parabola.

4. Connections between the number of bricks and the price

In the information provided by the brickyard on brick prices, connections were identified between the number of bricks and the price at the time they were sold. Thus, considering the types of bricks in Figure 7, the sale of a large brick (block) Figure 7a is modeled as shown in Table 2 and in the transcript excerpt.

Table 2. Commercialization of large brick (block).

Large Brick Quantity (C)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**...**	
Price in Colombian pesos p(c)	550	1100	1650	2200	2750	3300	3850	…	$\boldsymbol{p(c) = 550n}$

I2: In the commercialization part, what value per unit does the brick have?
P: The block per unit costs 550 pesos, meaning that 1,000 blocks cost you 550,000 pesos.

Meanwhile, the commercialization of the small bricks Figure 7b is modeled as shown in Table 3 and in the transcript excerpt.

Table 3. Commercialization of small brick

Small brick quantity (*n*)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**...**	
Price in Colombian pesos p(*n*)	300	600	900	1200	1500	1800	2100	…	***p(n)=300n***

I1: And the smallest?
P: the smallest one is worth 300 pesos per unit, that is, the thousand of the smallest one comes out to 300,000 pesos.

It is considered that when modeling the situation regarding the number of bricks and their respective prices, a simple direct rule of three is mathematically evidenced, in which the main objective is the solution of problems of two directly proportional magnitudes.

On the other hand, to contribute to the teaching of mathematics, some implications for teaching are shown by connecting and modeling the practice through the GeoGebra software, especially the construction of the brick carried out considering the following steps.

4.1. Treatment of mud brick with GeoGebra

For the construction of the parallelepiped in GeoGebra, the measurements exposed by the bricklayer during the interviews were considered (Figure 15). Next, GeoGebra 6.0 was entered. and in the first instance "View" - 3D graphic view was selected, hiding the axes; To do this, right click on the axes and deactivate the option or selection "Axis". Then, proceed to select the "Grid" options (see Figure 15a) the Line tool, tracing it vertically to the plane from a point A to a point B, with a point outside the line using the Point tool and clicking on the plane (Figure 15b).

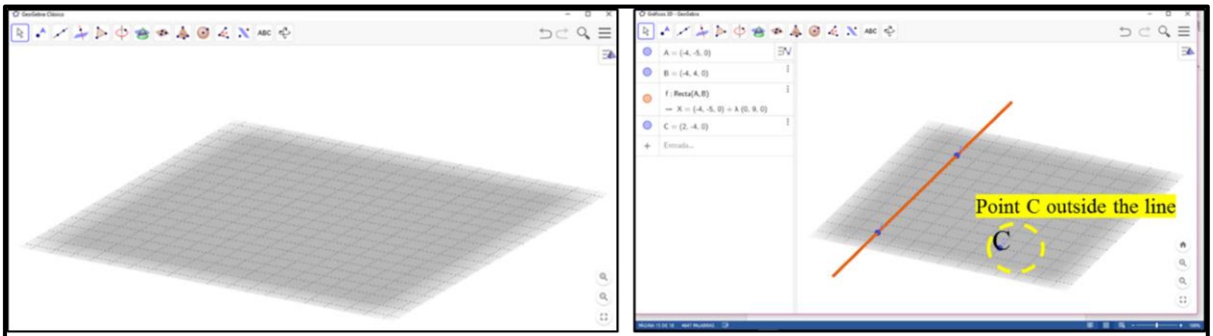

Figure 15a. Adequacy of the Plan　　Figure 15b. Line and point in the plane

Figure 15. Start of Parallelepiped Construction.

Continuing with the construction (Figure 16), proceed to draw a perpendicular line that passes through point C that is outside the line, using the "Perpendicular Line" tool, click on the previously drawn line and then on the point with which we obtain the new line and with it an intersection point is found by means of the "Intersection Point" tool (Figure 16a). Next, draw a line parallel to the first line AB that passes through the same point as the perpendicular line, using the "Parallel Line" option (Figure 16b).

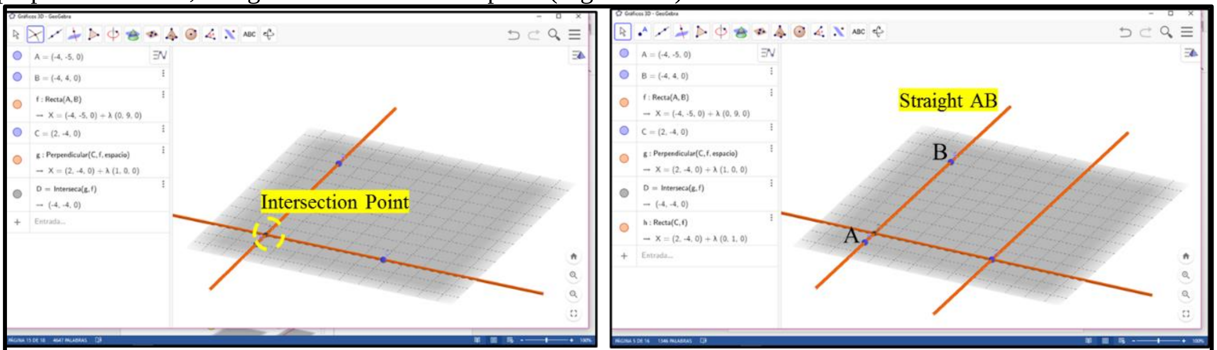

Figure 16a. Perpendicular Line and Point of Intersection　　Figure 16b. Parallel Line

Figure 16. Representation of parallel and perpendicular lines.

In addition, with the point tool (Figure 17), a point E is drawn on line AB and then a line parallel to the perpendicular that passes through said point E with the "Parallel Line" option (see Figure 17a). Likewise, with the "polygon" option, one the four points. Then, I hide the perpendicular and parallel lines by right clicking – "Visible object" (Figure 17b).

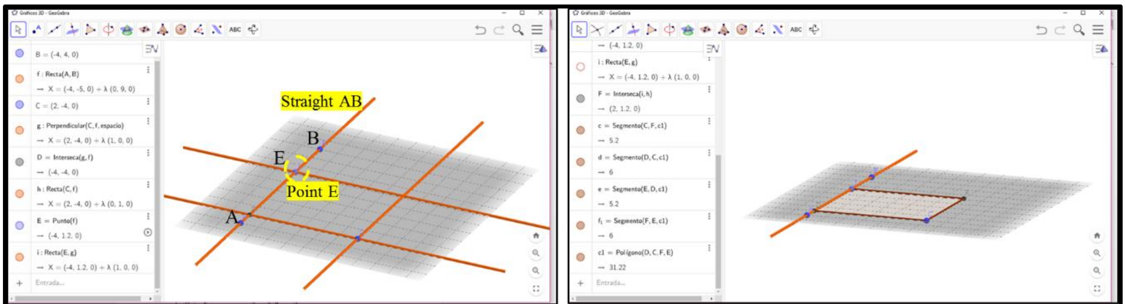

Figure 17a. Point E on the first line　　Figure 17b. Hidden parallel and perpendicular lines

Figure 17. Representation of Point and Polygon.

Lastly, the "Prism extrusion (Prism or Cylinder from its base)" tool is selected, giving a sustained click to the polygon which will allow us to force the image to surface and thus take it to a 3D view (see Figure 18).

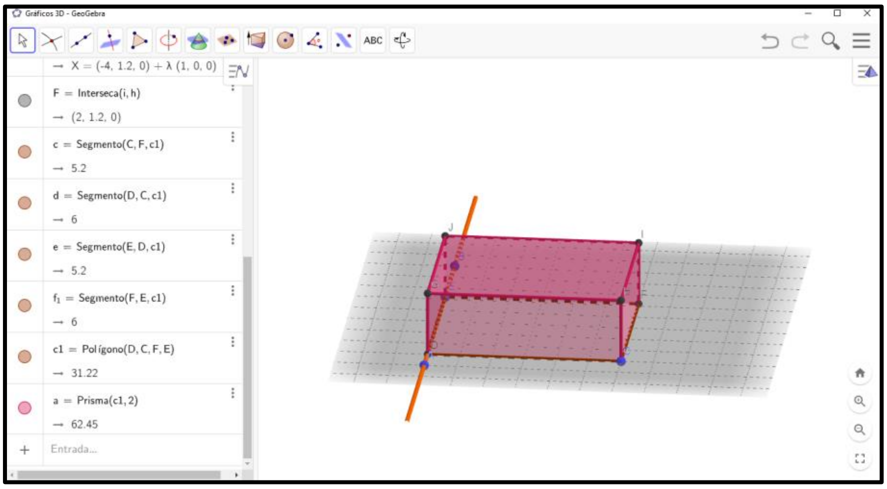

Figure 18. Representation of the parallelepiped.

In addition, through GeoGebra you can find the total area and volume of the brick, taking into account the measures suggested by the bricklayer.

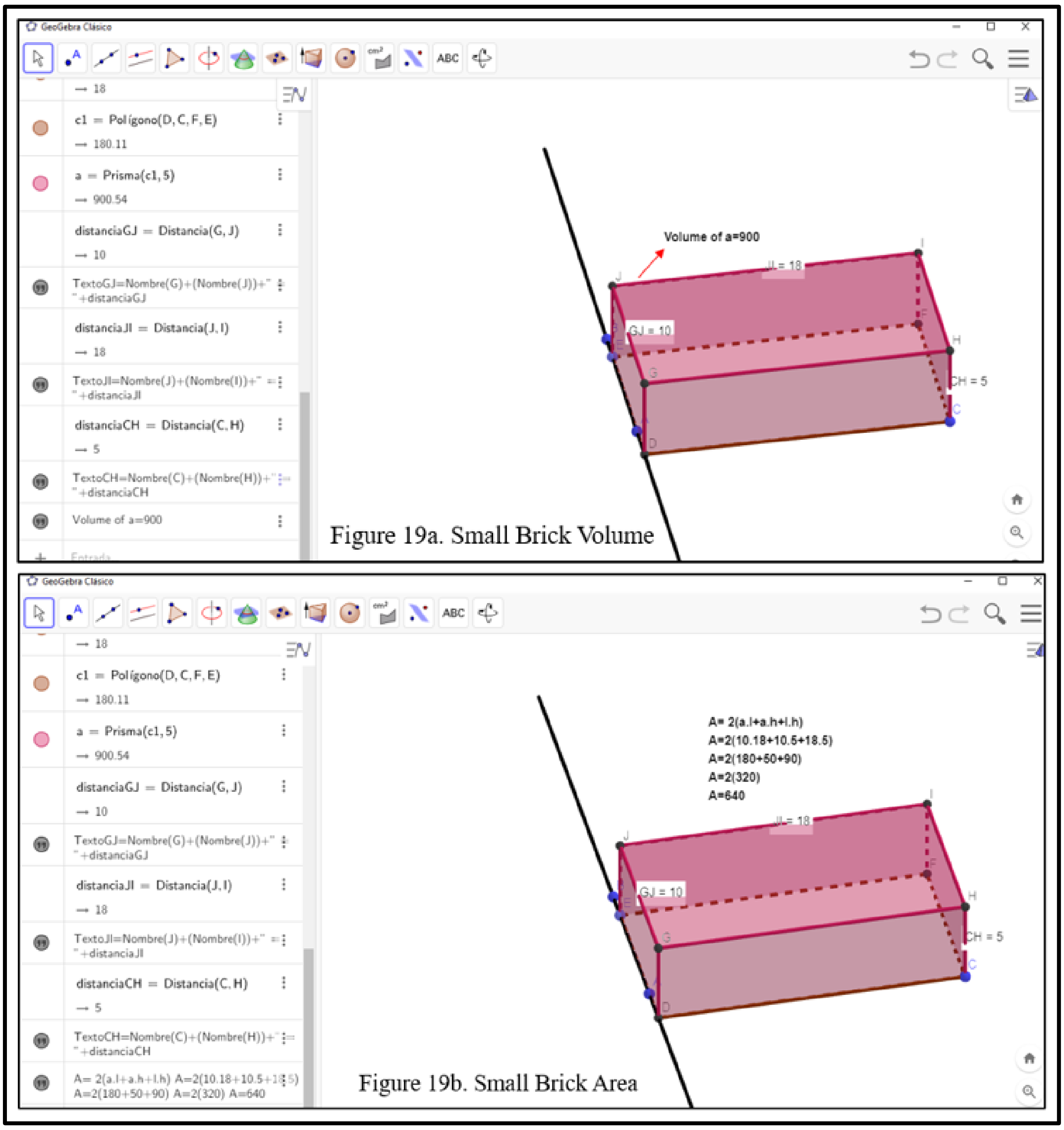

Figure 19. Volume and area of the small brick from GeoGebra.

Figure 19a shows the volume of the small brick ($900cm^3$), found with the area tool found at the top of the software, clicking there and then clicking on a point in the constructed figure. In this way, the volume is found, as is commonly explained in math classrooms, and the same result is obtained, in this context, $V = l * a * h$ where "l" is the length of the figure, "a" is the width and "h" is the height, replacing each of the measurements obtained in the interview would be as follows: $V = 18cm * 10cm * 5cm = 900cm^3$. Next in Figure 19b we have the area of the small brick with its parallelepiped shape, we proceed to use the formula used by students and teachers in classrooms: $A = 2\,(a * l + a * h + h * l)$ where "a" is the width of the figure, "l" is the length and "h" is the height, taking into account the measurements obtained we have: $A = 2\ (10cm * 18cm + 10cm * 5cm + 5cm * 18cm) = 2(180cm^2 + 50cm^2 + 90cm^2) = 2(320cm^2) = 640\ cm^2$.

In this research, the ethnomathematical connections in the elaboration of the brick and its treatment with GeoGebra were evidenced, which shows novelty and contributes to the teaching of mathematics, especially geometric concepts such as the rectangle, parallelepiped, angles, lines, etc. Likewise, this research was motivated to make a special treatment with dynamic software to make use of the results collected from ethnomathematics. Unlike other studies in ethnomathematics and ethnogeometry (De la Hoz et al., 2017; García-García & Bernardino-Silverio, 2019; Rodríguez-Nieto, 2020) that explore everyday practices, this work emphasizes a different and useful practice at the that would allow external connections to be made with other practices, for example, carpentry, welding, pharmaceutical activities in the form of medicine boxes, among others. In addition, this research is novel in relation to other works focused on brick making that did not have educational purposes, but rather commercial ones (Cachago & Caguano, 2016; Camargo & Yambay, 2020; Chire-Salazar & Rondán-Gutierrez, 2014; Cruz, 2015; Lengua, 2016).

5. Conclusions

The results of this research are also connected with the curriculum in favor of favoring the teaching of parallelepipeds and flat figures in the mathematics classroom, for example, in relation to working with mathematical objects in two and three dimensions (movements and transformations). , the Ministry of National Education [MEN] (2006) states that, "it allows integrating notions about volume, area and perimeter, which in turn enables connections with metric or measurement systems and with the notions of symmetry, similarity and congruence , among others" (p. 62).

This study can be used to follow curricular guidelines by the teacher, assuming that the Basic Learning Rights [DBA] (2016) consider it essential that primary school students relate objects in their environment, classify and represent two-dimensional and three-dimensional shapes. based on their common geometric characteristics, in accordance with the MEN (2006) where it is stated that for the understanding, appropriation of the students of the physical-daily and geometric-mathematical space "requires the study of different spatial relationships of solid bodies and gaps among themselves and with respect to the same students; of each solid or hollow body with its forms and with its faces, edges and vertices" (p. 62). Finally, for future research, it is recommended to design tasks considering this daily practice where the student establishes ethnomathematical connections related to daily life with institutional mathematics and performs their treatment with GeoGebra or other software.

Acknowledgment

We thank the merchant who produces mud bricks, resident in the municipality of Salamina-Magdalena, who provided us with valuable information for the development of this article, which is part of the Research hotbed: Ethnomathematical, Theoretical and Methodological Connections in Mathematics Education (CETMEM) of the Universidad del Atlántico, Suan campus, Colombia.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Declaration of interest: No conflict of interest is declared by authors.

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