New Classes of Open Sets in Topological Spaces

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ABSTRACT

In this paper, we have introduce a new open sets in topological spaces named *ic*-open sets. We present the relation of *ic*-open sets with other classes of open sets namely : semi-open, $\alpha-\text{open}$, $i-\text{open sets}$ and $i\,\alpha$ -open sets. Furthermore, we introduce the notion of *ic*-continuous map on topological spaces and we prove some properties and characterizations. Also, we study some separation axioms of this new class. Finally, depending on *ic*-open sets we define a new class called *icc*-open set and we discuss the same results obtained by the first one, as each open (closed) set in a topological spaces of any kind (X,τ) is *icc*-open (*icc*-closed) set.

Keywords: $\alpha-\text{open set}$, $i-\text{open set}$, *ic*- open set, Continuous map and Separation Axioms.

1. Introduction and Preliminaries

In topology and its applications, the concept of open sets is fundamental. In this work, we presented *ic*-open sets for a topological space(X,τ) as follows: $A\subseteq X$ is said to be *ic*-open set assuming there is a closed set $F\neq X,\emptyset\epsilon\tau^c$ such that $F\cap A\subseteq \text{int}(A)$, where *int(A)* denotes the interior points of A and τ^c denotes the family of closed sets. To investigate the relationship among these classes and the new class of *ic*-open sets, we present the semi-open set introduced by Levine in[3], the α-open set intruded by Njastad in[6], the *i*-open set introduced by Askander and Mohammed in [1], and the $i\,\alpha$ -open sets introduced by Mohammed and Kahtab in[4] in the first part. In the second part, we have proved some important theorems to discuss the property of *ic*-continuous map . In part three, we look at a few different types of separating axioms spaces and discuss the relationship between them such as$T_o, T_1, T_2, T_{o\,ic}, T_{1\,ic}, and\ T_{2\,ic}$. Finally, we define *icc*-open sets, a new type of open set that is based on *ic*-open set:$A\subseteq X$ is said to be *icc*-

open set if $A \in \tau^{ic} \bigcap \tau^{int}$ where τ^{ic} denotes the family of *ic* open sets and τ^{int} denotes the family of *int*- open sets, and we got similar results as in the part 2 and part 3. Throughout this paper we denote to any topological space by *TS* and we denote open set, respectively closed set by (*os*), (*cs*).

Definition.1.1. A subset *A* of *TS (X, τ)* is called

1. Semi-open set denoted by $(s-os)$[3] if "$A \subseteq cl(int(A))$".
2. α −open set denoted by $(\alpha-os)$[6] if "$A \subseteq int\left(cl\left(int(A)\right)\right)$".
3. i-open set denoted by $(i-os)$[1] assuming there is an open set $G \in \tau(x)$ that way i) $G \neq X, \emptyset$. ii) "$A \subseteq cl(A \cap G)$".
4. $i\alpha$-open set denoted by $(i\alpha-os)$[3] assuming there is an open set $G \in \tau^{\alpha}$ that way i)" $G \neq X, \emptyset$ ". ii) " $A \subseteq cl(A \cap G)$".
5. *Int*-open set denoted by $(int-os)$[3] assuming there is an open set $G \in \tau$ that way i) "$G \neq X, \emptyset$". *ii) "* int (A)=G".

   The family of all (os) [resp.$(s-os)$,$(\alpha-os)$,$(i-os)$, $(i\alpha-os)$,$(int-os)$] sets are denoted by $\tau, \tau^{s}, \tau^{\alpha}, \tau^{i}, \tau^{i\alpha}, \tau^{int}$.

Definition1.2. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is named:

1. Continuous denoted by $(contm)$ [1] if $f^{-1}(U)$is (os) in X for each (os) U in Y.
2. α-continuous denoted by $(\alpha-contm)$ [5] if $f^{-1}(U)$ is $(\alpha-os)$ in X for each (os) U in Y.
3. Semi-continuous denoted by $(s-contm)$ [3] if $f^{-1}(U)$ is $(s-os)$ in X for each (os) U in Y.
4. *i*-continuous denoted by $(i-contm)$ [1] if $f^{-1}(U)$ is $(i-os)$ in X for each (os) U in Y.
5. *i* α–continuous denoted by $(i\alpha-contm)$ [4] if $f^{-1}(U)$ is$(i\alpha-os)$ in X for each (os) U in Y.

2. *ic*-Open Sets in Topological Spaces

Definition2.1. A subset *A* of *TS* (X,τ) is named *ic*-open set denoted by$(ic-os)$[2] assuming there is (*cs*) $F \neq X, \emptyset \in \tau^{c}$ such that $F \cap A \subseteq int(A)$. The opposite of the *ic*-open set is named *ic*-closed set denoted by $(ic-cs)$. We use τ^{ic} to represent the family of all *ic*-open sets of(X,τ).

Example2.2. If $X=\{1,3,5\}$, $\tau=\{\phi, X, \{3\}, \{1,3\}\}$

Then, $\tau^{ic}=\{\phi, X, \{1\}, \{3\}, \{1,3\}\}$.

Theorem 2.3. *Each (os) in TS (X,τ) is (ic- os) but not conversely.*

Proof: Suppose that (X,τ) be a TS and let $A \subseteq X$ be an (os). Then, $A= int(A)$ but $F \cap A \subseteq A$ for any (cs) $F \neq X, \emptyset$ therefore, $F \cap A \subseteq A = int(A)$. Hence *A* is $(ic-os)$. ■

Example2.4. If $X=\{2,4,6\}$and $\tau = \{X, \emptyset, \{2\}, \{4,6\}\}$Then {2,4} is $(ic-os)$ but it is not(os).

Corollary.2.5. Any (cs) in a*TS* $(X,\tau)$is $(ic-cs)$.

Remark2.6. There is no relationship among $(\alpha - os)$ [resp.$(s-os)$,$(i-os)$,$(i\alpha - os)$] and $(ic-os)$ in a TS (X,τ) as shown in the following examples.

Example2.7. Let $X=\{1,3,5\}$. Now,

(1) If $\tau=\{\phi, X, \{1,3\}\}$. Then $\{1\}$ is $(ic-os)$ but not $(\alpha - os)$ and not $(s-os)$.
(2) If $\tau=\{\phi, X, \{5\}\}$. Then $\{3,5\}$ is $(\alpha - os)$ and $(s-os)$ but it is not $(ic-os)$.
(3) If $\tau=\{\phi, X, \{1\}\}$. Then $\{1,3\}$ is $(i-os)$ but it is not $(ic-os)$ and $\{1,3\}$ is $(i\alpha - os)$.
(4) If $\tau=\{\phi, X, \{1\},\{3,5\}\}$. Then $\{1,3\}$is $(ic-os)$ but it is not $(i-os)$ and not $(i\alpha - os)$.

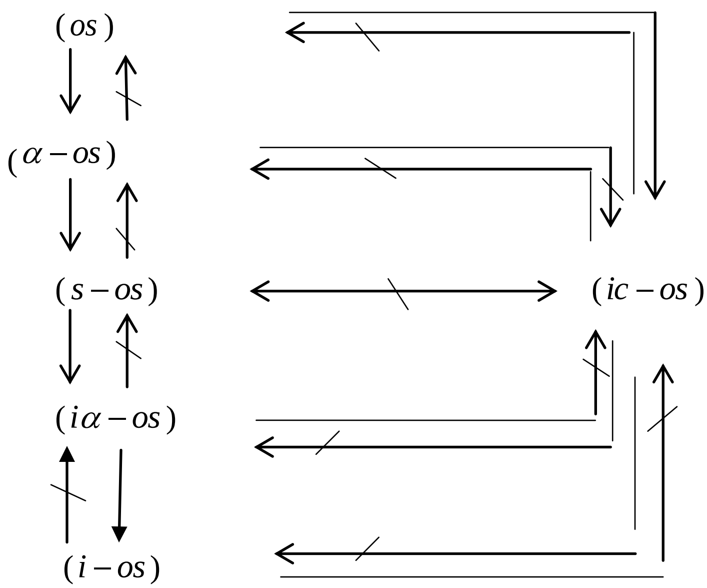

Figure (1)

The connections between ($ic-os$) and the other classes mentioned above.

3. *ic*-Continuous Mappings on Topological Spaces

Definition3.1. A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is named *ic*-continuous denoted by $(ic-contm)$ if $f^{-1}(U)$ is $(ic-os)$ in X for each (os) U in Y.

Theorem3.2. Each $(contm)$ is $(ic-contm)$ *but not conversely*.

Proof: Assume that $f:(X,\tau)\rightarrow(Y,\sigma)$be $(contm)$ and let U be an (os) in Y. Since f is $(contm)$, then $f^{-1}(U)$is (os) in X. Since each (os) is $(ic-os)$, then $f^{-1}(U)$is $(ic-os)$. Hence f is $(ic-contm)$. ■

Example3.3. Let $X=Y=\{2,4,6\}$, $\tau=\{\phi, X, \{2\},\{2,4\}\}$, $\sigma=\{\emptyset, Y, \{2\}, \{4\}, \{2,4\}\}$and let $f:(X,\tau)\rightarrow(Y,\sigma)$be an identity map. Then f is not $(contm)$, because $\{4\}$ is (os) in Y but $f^{-1}(\{4\})=\{4\}$ is not (os) in X. But f is $(ic-contm)$.

Remark3.4. There is no relationship among ($\alpha - contm$) [resp.($s - contm$), ($i - contm$), ($i\alpha - contm$)] mapping and ($ic - contm$).

Example3.5. Let $X = Y = \{1,3,5\}$, $f : (X,\tau) \to (Y,\sigma)$ is an identity map. Now,

(1) If $\tau = \{\emptyset, X, \{1,3\}\}, \sigma = \{\emptyset, Y, \{1\}\}$. Then f is ($ic - contm$), but it is not ($\alpha - contm$) and it is not ($s - contm$) because {1} is (os) in Y but $f^{-1}(\{1\}) = \{1\}$ is not ($\alpha - os$) and it is not ($s - os$).

(2) If $\tau = \{\emptyset, X, \{5\}\}, \sigma = \{\emptyset, Y, \{3,5\}\}$. Then f ($\alpha - contm$) and ($s - contm$) but it is not ($ic - contm$) because $f^{-1}(\{3,5\}) = \{3,5\}$ is not ($ic - os$).

(3) If $\tau = \{\emptyset, X, \{1\}\}, \sigma = \{Y, \emptyset, \{1,3\}\}$. Then f is ($i - contm$) and ($i\alpha - contm$) but it is not ($ic - contm$) because $f^{-1}(\{1,3\}) = \{1,3\}$ is not ($ic - os$).

(4) If $\tau = \{\emptyset, X, \{1\}, \{3,5\}\}$ $\sigma = \{Y, \emptyset, \{1\}, \{3\}, \{1,3\}\}$. Then f is ($ic - contm$) but it is not ($i - contm$) and it is not ($i\alpha - contm$) because $f^{-1}(\{1,3\}) = (\{1,3\})$ is not ($i - os$) and it is not ($i\alpha - os$).

Remark3.6. The connections of ($ic - contm$) and ($contm$) can be explaining through the following diagram:

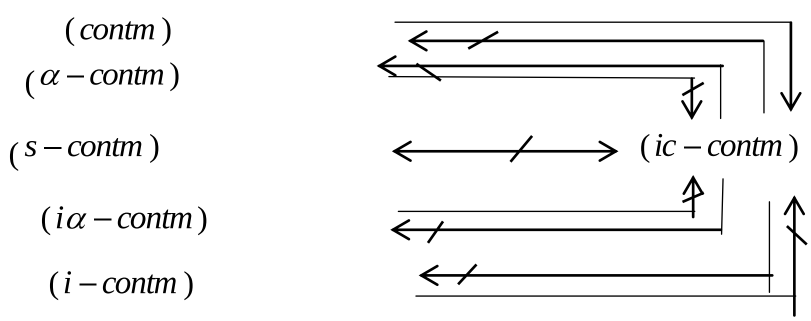

Figure (2)

4. *ic* -Open Sets and Separating Axioms

Definition4.1. A TS (X,τ) is named

1. T_{0ic} space if for any $m, n \in X$ with $m \neq n$ assuming there is ($ic - os$), L such that either, $m \in L$ and $n \notin L$ or $n \in L$ and $m \notin L$.
2. T_{1ic} space if for any $m, n \in X$ with $m \neq n$ assuming there is ($ic - os$) L, M containing m, n respectively that is either, $n \notin L$ and $m \notin M$.
3. T_{2ic} space if for any $m, n \in X$ with $m \neq n$ assuming there is disjoint ($ic - os$), L, M containing m, n respectively.

Theorem4.2. Each T_0- space is T_{0ic}- space *but not conversely*.

Proof: Consider that X is a T_0- space and let m, n be two different X points. Since X is T_0 -space. Assuming there is os L in X that is $m \in L$ and $n \notin L$ or $n \in L$ and $m \notin L$. Since each(os)is($ic-os$). We get L is ($ic-os$) in X that is $m \in L$ and $n \notin L$ or $n \in L$ and $m \notin L$. Henceforth X is T_{0ic}- space. ■

Example4.3. Let $X=\{2,4,6\}$, $\tau=\{\emptyset, X, \{2,4\}\}$. Therefore, (X,τ) is not T_0- space, but (X,τ^{ic}) is T_{0ic} -space.

Theorem4.4. Each T_1- space is T_{1ic} – space *but not conversely*.

Proof: Consider m, n be two distinct points in X. Since X is T_1- space. Then there is two (os) L and M in X that is $m \in L$, $n \notin L$ and $n \in M$ and $m \notin M$. Since each (os) is ($ic-os$). Then L, M is ($ic-os$) in X, that is $m \in L$ and $n \notin L$ and $n \in M$ and $m \notin M$. Henceforth X is T_{1ic}- space. ■

Example4.5. Let $X=\{1,3,5\}$, $\tau=\{\emptyset, X, \{1\}, \{1,3\}, \{1,5\}\}$. Then (X,τ) is not T_1- space, but (X,τ^{ic}) is T_{1ic} - space.

Theorem4.6. Any T_2- space is T_{2ic} – space *but not conversely*.

Proof: Consider X as a T_2- space and m, n be two distinct points in X. Since X is T_2- space. Then there is disjoint (os), L, M containing m, n respectively. Since each (os) is ($ic-os$). Then L and M are disjoint ($ic-os$) containing m, n respectively. Hence X is T_{2ic}- space.

Example4.7. Let $X=\{2,4,6\}$, $\tau=\{\emptyset, X, \{2\}, \{4,6\}\}$. Then (X, τ) is not T_2 -space, but (X,τ^{ic}) is T_{2ic} - space.

Remark4.8. The connections among separation axioms.

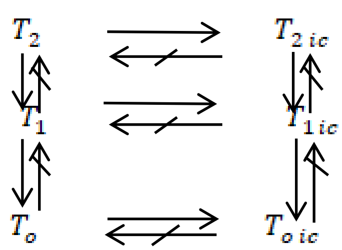

Figure (3)

Theorem 4.9. *A space (χ,τ) is $T2ic$ – space iff (χ,τ^{ic}) is Hausdorff -space.*

Proof: Assumes $n, m \in \chi$ with $n \neq m$. Since χ is T_{2ic} –space, there exists disjoint (ic-os) H and K in χ s.t. $n \in H$ and $m \in K$, $H \cap K=\emptyset$. Here, $H, K \in \tau^{ic}$, so, obviously (χ,τ^{ic}) ceases to be a T_{2ic} –space i.e. a Hausdorff space.

Conversely, whenever (χ,τ^{ic}) is a T_{2ic} –space , there exists a pair of members of τ^{ic}, say, p & Q for a pair of distinct points p &q of χ such that $p \in p$ & $q \in Q$ & $p \cap Q = \emptyset$. But $ico(\chi,\tau)= \tau^{ic}$. Combing all these facts (χ, τ) is T_{2ic} –space. ■

Theorem 2.10. *Each open subspace of a* T_{2ic} −space *is* T_{2ic}.
Proof: Suppose U be an open subspace of a T_{2ic} −space (χ,τ). Let k and p be any two distinct points of U. Since χ is T_{2ic} −space and $U\subset\chi$, there exists two disjoint (*ic-os*) G and H in χ such that $k\in G\ \&\ p\in H$. Let $A=U\cap G\ \&\ B=U\cap H$. Then $A\ \&\ B$ are (*ic-os*) in U containing k and p. Also, $A\cap B=\emptyset$. Hence (U,T_u) is T_{2ic}. ■

5. *icc*-Open Sets in Topological Spaces

Definition5.1. A subset A of $TS\ (X,\tau)$ is named *icc*- open set denoted by $(icc-os)$ if $A\in\ \tau^{ic}\bigcap\tau^{int}$ where τ^{ic} denotes the family of $(ic-os)$ and τ^{int} denotes the family of *int*-open sets. The opposite of the *icc*-open set is named *ic*c-closed set denoted by $(icc-cs)$. We denote the family of all $(icc-os)$ of topological space by τ^{icc}.

Example5.2. Let $X=\{1,3,5\}$ and $\tau=\{\emptyset,X,\{1,3\}\}$. Then τ^{ic}=$\{X,\emptyset,\{1\},\{3\},\{1,3\}\}$,

τ^{int}=$\{X,\emptyset,\{1,3\}\}$, $\tau^{icc}=\tau^{ic}\cap\tau^{int}$=$\{X,\emptyset,\{1,3\}\}$.

Theorem5.3. Each (os) in any $TS\ (X,\tau)$ is $(icc-os)$ *but not conversely*.
Proof: Clear.

Example5.4. Let $X=\{2,4,6\}$ and $\tau=\{\emptyset,X,\{2\},\{4,6\}\}$.

Then τ^{ic}=$\{X,\emptyset,\{2\},\{4\},\{6\},\{2,4\},\{2,6\},\{4,6\}\}$, τ^{int}=$\{X,\emptyset,\{2\},\{4,6\},\{2,4\},\{2,6\}\}$,

$\tau^{icc}=\tau^{ic}\cap\tau^{int}$=$\{X,\emptyset,\{2\},\{2,4\},\{2,6\},\{4,6\}\}$. Obviously, $\{2,6\}$ is $(icc-os)$ but it is not (os).

Corollary5.5. Any (cs) in any TS (X,τ) is $(icc-cs)$.

Remark5.6. Note for any $TS\ (X,\tau)$, each $(icc-os)$ is $(ic-os)$ and $(int-os)$. But the converses are not true as shown in Example 5.4, it is clear that $A=\{4\}$ is not $(icc-os)$ set but it is $(ic-os)$.

Remark5.7. There are no relationships among $(\alpha-os)$,$(s-os)$,$(i-os)$ and $(i\alpha-os)$ with $(icc-os)$ as shown in the following example:

Example5.8. Let $X=\{1,3,5\}$. Now:

1. If $\tau=\{\emptyset,X,\{1\},\{3,5\}\}$, Then $\{1,3\}$ is $(icc-os)$ but it is not $(\alpha-os)$ and it is not $(s-os)$.

2. If $\tau=\{\emptyset,X,\{5\}\}$. Then $\{3,5\}$ is $(\alpha-os)$ and $(s-os)$ but it is not $(icc-os)$.

3. If $\tau=\{\emptyset,X,\{1\}\}$. Then $\{1,3\}$ is *i*-open set but it is not $(icc-os)$, it is $(i\alpha-os)$.

4. If $\tau = \{\emptyset, X, \{1\}, \{3,5\}\}$ Then $\{1,3\}$ is $(icc-os)$ but it is not $(i-os)$ and it is not $(i\alpha-os)$.

Remark5.9. The connections among *icc*-open sets and other classes:

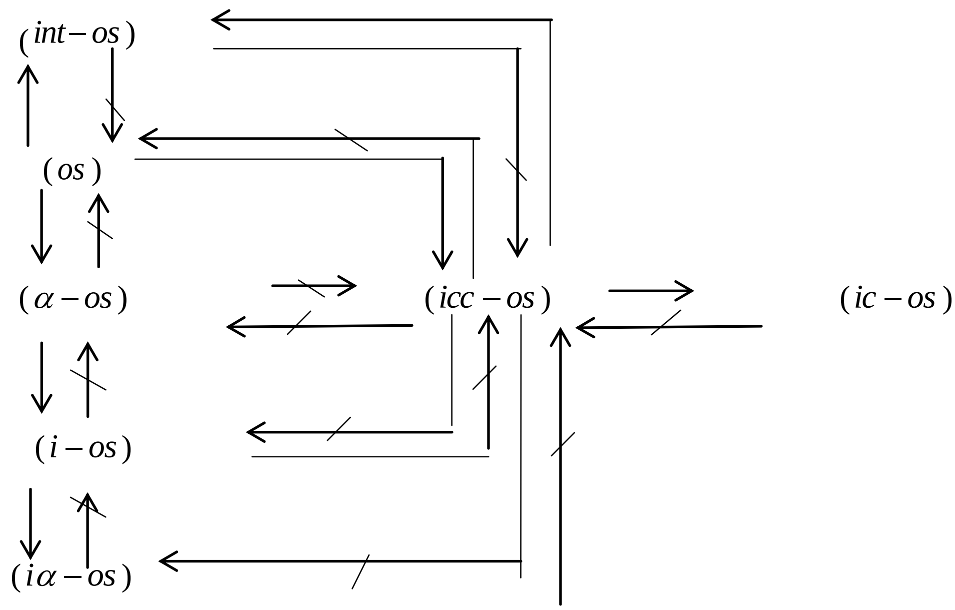

Figure (4)

Definition5.10. A mapping $f:(X,\tau)\to(Y,\sigma)$ is called *icc*-continuous denoted by $(icc-contm)$ if $f^{-1}(U)$ is $(icc-os)$ in X for each (os) U in Y.

Theorem5.11. Any $(contm)$ is $(icc-contm)$.
Proof: Clear. ■
Example5.12. **Consider** $X=Y=\{2,4,6\}$, $\tau=\{\emptyset, X, \{2\},\{4,6\}\}$, $\sigma=\{\emptyset, Y, \{2,4\}\}$ and let $f:(X,\tau)\to(Y,\sigma)$ be an identity mapping. f is not $(contm)$, because $\{2,4\}$ is (os) in Y but $f^{-1}(\{2,4\})=\{2,4\}$ is not (os) in X. f is $(icc-contm)$.

Theorem5.13. Each $(icc-contm)$ is $(ic-contm)$ *but not conversely*.
Proof: Clear. ■
Example5.14. Consider $X=Y=\{1,3,5\}$, $\tau=\{\emptyset, X, \{1,3\}\}$, $\sigma=\{\emptyset, Y, \{1\}\}$ and let $f:(X,\tau)\to(Y,\sigma)$ be an identity mapping. Then f is not $(icc-contm)$, because $\{1\}$ is (os) in Y but $f^{-1}(\{1\})=\{1\}$ is not $(icc-os)$ in X. f is $(ic-contm)$.

Remark5.15. There are no relationships among $(\alpha-contm)$ [resp.,$(s-contm)$,$(i-contm)$ and $(i\alpha-contm)$] with $(icc-contm)$ as indicated in the example below:

Example5.16. Consider $X=Y=\{2,4,6\}$ and let $f:(X,\tau)\to(Y,\sigma)$ be an identity map. Now,

1. If τ ={Ø, X, {2}, {4,6}}, σ ={Ø, Y,{2,4}}. Then f is ($icc-contm$) but it is not ($\alpha-contm$), it is not ($s-contm$) because {2,4} is (os) in Y but $f^{-1}(\{2,4\}) = \{2,4\}$ is not ($\alpha-os$) and it is not ($s-os$) in X.

2. If τ ={Ø, X, {6}}, σ ={Ø, Y,{4,6}}. Then f is ($\alpha-contm$) and ($s-contm$) but it is not ($icc-contm$), because $f^{-1}(\{4,6\}) = \{4,6\}$ is not ($icc-os$) in X.

3. If τ ={Ø, X, {2}}, σ ={Ø, Y,{2,4}}. Then f is ($i\alpha-contm$) and ($i-contm$) but it is not ($icc-contm$), because $f^{-1}(\{2,4\}) = \{2,4\}$ is not ($icc-os$) in X.

4. If τ ={Ø, X, {2},{4,6}}, σ ={Ø, Y,{2},{2,4}}. Then f is ($icc-contm$) but it is not ($i-contm$) and it is not ($i\alpha-contm$), because $f^{-1}(\{2,4\}) = \{2,4\}$ is not ($i-os$) and it is not ($i\alpha-os$) in X.

Remark5.17. The connections among ($icc-contm$) and some other classes which are mentioned above.

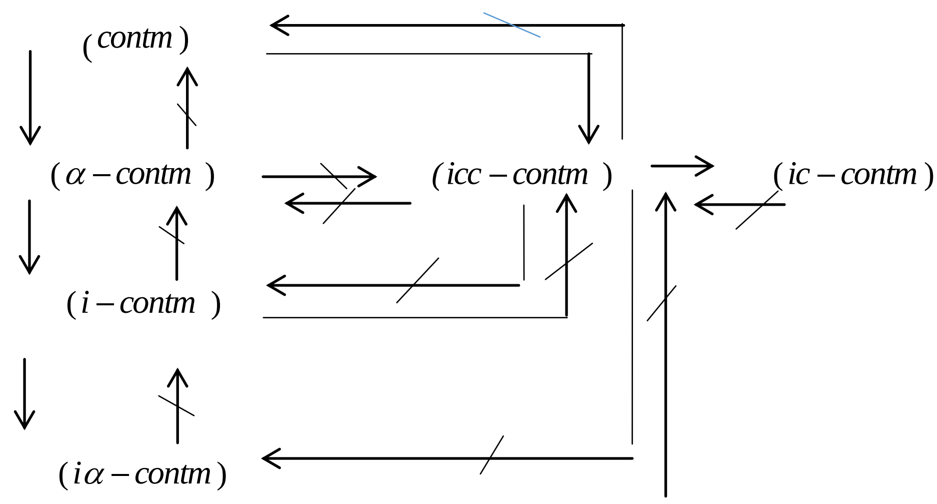

Figure (5)

Definition5.18. A TS (X, τ) is named

1. T_{0icc} space if for any $m, n \in X$ with $m \neq n$ assuming there is ($icc-os$), L, that is either, $m \in L$ and $n \notin L$ or $n \in L$ and $m \notin L$.
2. T_{1icc} space if for any $m, n \in X$ with $m \neq n$ assuming there is two ($icc-os$), L, M containing m, n respectively that is either, $n \notin L$ and $m \notin M$.
3. T_{2icc} space if for any $m, n \in X$ with $m \neq n$ assuming there is disjoint ($icc-os$), L, M containing m, n respectively.

Theorem5.19. Each T_{0-} space is T_{0icic-} space *but not conversely*.

Proof: Clear.

Example5.20. Consider $X = \{1,3,5\}$, $\tau = \{\emptyset, X, \{1\},\{3,5\}\}$. Then, (X, τ) is not T_{0}- space, but (X, τ^{icc}) is T_{0icc} -space.

Theorem5.21. Each T_{1}- space is T_{1icc} space *but not conversely*.

Proof: Clear.

Example5.22. Let $X = \{2,4,6\}$, $\tau = \{\emptyset, X, \{4\},\{2,4\}\}$. Then, (X, τ) is not T_{1}- space, but (X, τ^{icc}) is T_{1icc}-space.

Theorem5.23. Any T_{2}- space is T_{2icc} – space.

Proof: Consider X as a T_2- space and m, n be two distinct points in X. Since X is T_2- space. Then there is disjoint (*os*), L, M containing m, n respectively. Since each (os) is $(icc-os)$. Then L and M are disjoint $(icc-os)$ containing m, n respectively. Hence X is T_{2icc}- space.

Theorem 5.24.*Aspace (χ, τ) is $T2icc-$ space iff (χ, τ^{icc}) is Hausdorff -space.*

Proof: Assumes $n, m \in \chi$ with $n \neq m$. Since χ is $T2icc-$ space, there exists disjoint (*icc-os*) H and K in χ s.t. $n \in H$ and $m \in K$, $H \cap K = \emptyset$. Here, $H, K \in \tau^{icc}$, so, obviously (χ, τ^{icc}) ceases to be a T_{2icc} $-$space i.e. a Hausdorff space.

Conversely, whenever (χ, τ^{icc}) is a T_{2icc} $-$space , there exists a pair of members of τ^{icc}, say, p & Q for a pair of distinct points p &q of χ such that $p \in p$ & $q \in Q$ & $p \cap Q = \emptyset$. But $icco(\chi, \tau) = \tau^{icc}$. Combing all these facts (χ, τ) is T_{2icc} $-$space. ■

Theorem 2.10. *Each open subspace of a $T2icc-$ space is T_{2icc}.*

Proof: Suppose U be an open subspace of a T_{2icc} $-$space (χ, τ). Let k and p be any two distinct points of U. Since χ is T_{2icc} $-$space and $U \subset \chi$, there exists two disjoint (*icc-os*) G and H in χ such that $k \in G$ & $p \in H$. Let $A = U \cap G$ & $B = U \cap H$. Then A & B are (*icc-os*) in U containing k and p. Also, $A \cap B = \emptyset$. Hence (U, T_u) is T_{2icc}. ■

Remark5.23. The connections among *icc*-separation axioms.

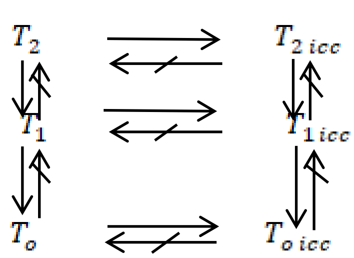

Figure (6)

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