NEW SEPARATION AXIOMS IN ζ-NANO TOPOLOGICAL SPACES

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Abstract
In this paper, we represents few separation axioms are ζ-T_0, ζ-T_1 and ζ-T_2 in (V, τ_ζ). ζ-T_0 deals with the distinct points which not has same ζ-open set. ζ-T_1 gives that the distinct points are in different ζ-open set. ζ-T_2 approaches that the distinct points contain in disjoint ζ-open set. We discuss among with $\mathcal{N}$-T_0 (resp. $\mathcal{N}$-T_1, $\mathcal{N}$-T_2) and ζ-T_0 (resp. ζ-T_1, ζ-T_2). ζ-topologically distinguishable and ζ-symmentry space are associate with these separation axioms in ζ-nano Topology.

Keywords: *ζ-T_0, ζ-T_1 and ζ-T_2*

1 Introduction and Preliminaries

Thivagar and Richard [3] are introduced the term Nano topology. Some few separation axioms of nano topology are $\mathcal{N}$-T_0, $\mathcal{N}$-T_1 and $\mathcal{N}$-T_2 was developed by Sathishmohan et al.[5]. Jenavee et al.[1] extended the idea of nano topology into ζ-nano topological space. ζ-topologically distinguishable, ζ-separated and ζ-sierpinksi space are formed by Jenavee et al.[2].

Definition 1.1 *[3] Let $\mathcal{V}$ be a non-empty finite set of members are called the universe and $\mathcal{R}$ has an equivalence relation on $\mathcal{V}$ known as the indiscernibility relation. Members belonging to the same equivalence class are called to be indiscernible with each other. The pair $(\mathcal{V}, \mathcal{R})$ is called to be the approximation-space. Let $\mathcal{X} \subset \mathcal{V}$.*

1. The lower approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all members, which can be for certain classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and it is represented by $\mathcal{L}_\mathcal{R}(\mathcal{X})$. That is,
$$\mathcal{L}_\mathcal{R}(\mathcal{X}) = \cup_{x \in \mathcal{V}} \{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \subseteq \mathcal{X}\},$$
where $\mathcal{R}(\mathcal{X})$ denoted the equivalence class determined by $\mathcal{X}$.
2. The upper approximation of $\mathcal{X}$ with respect to $\mathcal{R}$ is the set of all members, which can be possibly classified as $\mathcal{X}$ with respect to $\mathcal{R}$ and it is represented by $\mathcal{U}_\mathcal{R}(\mathcal{X})$.
(i.e.), $\mathcal{U}_\mathcal{R}(\mathcal{X}) = \cup_{x \in \mathcal{V}} \{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \cap \mathcal{X} \neq \phi\}$
3. The boundary region of $\mathcal{X}$ wit respect to $\mathcal{R}$ is the set of all members, which can be neither in nor as not-$\mathcal{X}$ with respect to $\mathcal{R}$ and it is represented by $\mathcal{B}_\mathcal{R}(\mathcal{X})$.
(i.e.), $\mathcal{B}_\mathcal{R}(\mathcal{X}) = \mathcal{U}_\mathcal{R}(\mathcal{X}) - \mathcal{L}_\mathcal{R}(\mathcal{X})$.

Definition 1.2 *[3] Let $\mathcal{V}$ be the universe $\mathcal{R}$ be an equivalence relation on $\mathcal{V}$ and $\tau_\mathcal{R}(\mathcal{X}) = \{\mathcal{V}, \phi, \mathcal{U}_R(\mathcal{X}), \mathcal{L}_\mathcal{R}(\mathcal{X}), \mathcal{B}_\mathcal{R}(\mathcal{X})\}$, where $\mathcal{X} \subset \mathcal{V}$. Then $\tau_\mathcal{R}(\mathcal{X})$ satisfies the following axioms:*

1. $\mathcal{V}$ and $\phi \in \tau_\mathcal{R}(\mathcal{X})$.
2. The union of the members of any sub-collection of $\tau_\mathcal{R}(\mathcal{X})$ is in $\tau_\mathcal{R}(\mathcal{X})$.
3. The intersection of the members of finite sub-collection of $\tau_\mathcal{R}(\mathcal{X})$ is in $\tau_\mathcal{R}(\mathcal{X})$.

That is, $\tau_\mathcal{R}(\mathcal{X})$ is a topology on $\mathcal{V}$ is called the Nano topology on $\mathcal{V}$ with respect to $\mathcal{X}$. $(\mathcal{V}, \tau_\mathcal{R}(\mathcal{X}))$ is called the Nano topological space. Members of the Nano topology are called Nano open sets in $\mathcal{V}$. Members of $[\tau_\mathcal{R}(\mathcal{X})]^c$ are called Nano closed sets.

Definition 1.3 *In* $(V, \tau_{\mathcal{N}})$, *[5]*

1. V is said to be $\mathcal{N}$-T_0 for $u, w \in V$ and $u \neq w$, $\exists$ disjoint $\mathcal{N}$-open sets U such that $u \in U$ and $w \notin U$.
2. V is said to be $\mathcal{N}$-T_1 for $u, w \in V$ and $u \neq w$, $\exists$ disjoint $\mathcal{N}$-open sets U and V such that $u \in U$, $w \notin W$ and $w \in W$, $u \notin W$.
3. V is said to be $\mathcal{N}$-T_2 for $u, w \in V$ and $u \neq w$, $\exists$ disjoint $\mathcal{N}$-open sets U and V such that $u \in U$ and $w \in W$.

Definition 1.4 *[1] A subset J of a Nano topological space* $(V, \mathcal{N}_{\mathcal{R}})$ *is called* ζ*-Nano-open set if there exists a Nano open set* $Z \in \mathcal{N}_{\mathcal{R}}$-$O$, *such that*

1. $Z \neq \phi, V$.
2. $J \subseteq \mathcal{N}_{\mathcal{R}}$-$int(J) \cup Z$.

In $(V, \mathcal{N}_{\mathcal{R}})$, the member of the open set is said to be ζ-Nano-open and the complement is ζ-Nano-closed set. The collection of all ζ-Nano-open including ϕ, V is said to be ζ-Nano-topological space if satisfies topological space definition. So, this $(V, \mathcal{N}_{\mathcal{R}}, \zeta)$ or $\mathcal{N}$-$\tau_\zeta(J)$ can be rewritten in the form ζ-Nano-topological space on V.

Definition 1.5 *[1] Let E be a subset of a* ζ*-Nano-Topology.*

1. The union of all Nano-ζ sets contained in E is represent in the form of ζ-Nano-int(E). We can rewrite in the form $\zeta_i(E)$.
2. The intersection of all Nano-ζ sets containing in E is represent in the form of ζ-Nano-cl(E). Also we write in the form $\zeta_c(E)$.
3. The exterior of ζ-Nano-Topology in E is defined by $\zeta_e(E) = \zeta_i(V - E)$.
4. The frontier of ζ-Nano-Topology in E is defined by $\zeta_f(E) = \zeta_c(E) \cap \zeta_c(V - E)$.

Definition 1.6 *[2] Let* (V, τ_ζ) *be a* ζ*-nano topological space.*

1. The two membersu andw inVareζ-topologically distinguishableif they do not have exactly the sameζ-neighbourhoods.
2. Two subsetsUandWofVareζ-separatedif each is disjoint from the other's ζ_c.
3. UandWare two subsets of V. Then it said to be ζ- separated by ζ-neighbourhoodsif they have disjoint ζ-neighbourhoods.

Remark 1.7 *[2] In* (V, τ_ζ), *the two membersu andw inVare have exactly the same*ζ*-neighbourhoods then is called* ζ*-topologically indistinguishable.*

Proposition 1.8 *[2] Any two* ζ*- topologically distinguishable points in* (V, τ_ζ) *are* ζ*-separated (another name is* ζ*- R0 or* ζ*- symmentry).*

Remark 1.9 *[2] In* (V, τ_ζ), *any of the two* ζ*- topologically distinguishable points are not* ζ*-separated. (another name is* ζ*- Sierpinski space).*

The structure of this paper is represent: In Section 2, represents some new separation axioms ζ-T_0, ζ-T_1 and ζ-T_2 in (V, τ_ζ). ζ-T_0 gives the result that the distinct points is does not contain in same ζ - open set. ζ-T_1 says that the distinct points contain in different ζ-open set. ζ-T_2 represents that the distinct points contain in disjoint ζ-open set. We compare relationship between $\mathcal{N}$-T_0 (resp. $\mathcal{N}$-T_1, $\mathcal{N}$-T_2) and ζ-T_0 (resp. ζ-T_1, ζ-T_2). Few characterization of ζ-topologically distinguishable, ζ-symmentry space are relate with these separation axioms in ζ-Nano Topology. The conclusion of this paper is set forth in section 3.

Note:$\tau_{\mathcal{N}}$ or $\mathcal{N}$ denotes Nano Topology and τ_ζ or ζ denotes ζ-Nano Topology.

2 Separation axioms

In this section, We study about ζ-T_0, ζ-T_1 and ζ-T_2 in τ_ζ. Some of it's properties are handle.

Definition 2.1 *A space V is said to be* ζ-T_0 *if for each pair of points u, w of V are distinct, there exists a* ζ*-open set X such* *that $u \in X$ and $w \notin X$ in (V, τ_ζ).*

Example 2.2 *In $V = \{s_1, s_2, s_3\}, V / \mathcal{R} = \{\{s_1\}, \{s_2\}, \{s_3\}\}, Y = \{s_1, s_2\}$ and $\tau_\mathcal{N}(Y) = \{\phi, V, \{s_1, s_2\}\}$. Then, $\zeta = \{s_1, s_2\} \Rightarrow \tau_\zeta(Y) = \{\phi, V, \{s_1\}, \{s_2\}, \{s_1, s_2\}\}$. Therefore, (V, τ_ζ) is ζ-T_0 space because:*

1. for s_1 and s_2, $\exists$ an open set $\{s_1\}$ such that $s_1 \in \{s_1\}$ and $s_2 \notin \{s_1\}$.
2. for s_1 and s_3, $\exists$ an open set $\{s_1\}$ such that $s_1 \in \{s_1\}$ and $s_3 \notin \{s_1\}$.
3. for s_2 and s_3, $\exists$ an open set $\{s_2\}$ such that $s_2 \in \{s_2\}$ and $s_3 \notin \{s_2\}$.

Theorem 2.3 *In (V, τ_ζ), every ζ-T_0 has a pair of distinct points.*

Proof. Let assume that u and w are ζ-T_0 in V. Then, $\exists$ u and w are in V and the open subset U such that $u \in U$ and $w \notin U$ by the definition of ζ-T_0. From our assumption, clearly it gives u and w are distinct points.

Remark 2.4 *The converse of the theorem 2.3 cannot be true.*

Example 2.5 *$V = \{s_1, s_2, s_3, s_4, s_5\}$ with $V / \mathcal{R} = \{\{s_1\}, \{s_2\}, \{s_3, s_4, s_5\}\}, Y = \{s_1, s_2, s_3\} \subset V$. $\tau_\mathcal{N}(Y) = \{\phi, V, \{s_1, s_2\}, \{s_3, s_4, s_5\}\}$. And $\zeta = \{s_1, s_2\} \Rightarrow \tau_\zeta(Y) = \{\phi, V, \{s_1\}, \{s_2\}, \{s_1, s_2\}, \{s_3, s_4, s_5\}, \{s_1, s_3, s_4, s_5\}, \{s_2, s_3, s_4, s_5\}\}$. If a pair of distinct points are s_3 and s_4, then $\exists \{s_3, s_4, s_5\}$ such that $\{s_3\} \subseteq \{s_3, s_4, s_5\}$. Similarly, $\{s_4\} \subseteq \{s_3, s_4, s_5\}$. Hence, it is not ζ-T_0 space.*

Theorem 2.6 *Every $\mathcal{N}$-T_0 is ζ-T_0 in (V, τ_ζ).*

Proof. Let u and w are ζ-topologically distinguishable in $\mathcal{N}$-T_0. Then, $\exists$ an ζ-open subset U such that $u \in U$ and $w \notin U$. This implies, $u \in \mathcal{N}_i(U) \subseteq (\mathcal{N}_i(U) \cup \zeta)$ and $w \in \mathcal{N}_i(W) \subseteq (\mathcal{N}_i(W) \cup \zeta)$ by our assumption is ζ-topologically distinguishable. Now, we can hold that every $\mathcal{N}$-T_0 is ζ-T_0.

Remark 2.7 *The converse of the before theorem 2.6 cannot be true.*

Example 2.8 *In example 2.5, $\{s_1\}$ and $\{s_2\}$ are ζ-T_0 but not $\mathcal{N}$-T_0.*

Theorem 2.9 *In (V, τ_ζ), Z is ζ- closed set iff Z = ζ- closure set.*

Proof. Let Z is ζ-closed and $y \in Z$. We know that, $Z \subseteq \zeta_c(Z)$. Suppose that $y \notin Z$. Then Z is ζ-closed, $Y = V - Z$ is ζ-open and contains y, but Z not contains $\Rightarrow y \notin \zeta_c(Z)$. Contradiction $y \in Z \Rightarrow y \in \zeta_c(Z)$ and $y \notin Z \Rightarrow y \notin \zeta_c(Z)$. So, $Z = \zeta_c(Z)$. $\Leftarrow$(Conversely) If $Z = \zeta_c(Z)$, then Z is ζ-closed. Let $y \in V - Z$. Then $y \notin \zeta_c(Z)$, so there is few ζ-open sets Y such that $y \in Y$ and $Y \cap Z = \phi \Rightarrow Y \subseteq V - Z$, This gives that all points of $V - Z$ are contained in Y, $V - Z = \cup \{Y: y \in V - Z\}$, whose union of ζ-open sets. So, $V - Z$ is ζ-open sets $\Rightarrow \zeta$-closed.

Corollary 2.10 *In (V, τ_ζ), Z is ζ-open set iff Z = ζ-interior set.*

Proof. The proof is follows from the contradiction result of the theorem 2.9.

Lemma 2.11 *In (V, τ_ζ), $\zeta_c(S)$ is the smallest ζ-closed set containing S.*

Proof. Let S is the ζ-closed set and $\zeta_c(S) = S \cup \zeta_f(S) \Rightarrow \zeta_c(S) \subseteq S$ or $\zeta_c(S) \subseteq \zeta_f(S)$.

Case (i): If $\zeta_c(S) \subseteq S$, then $\zeta_c(S)$ is the smallest ζ-closed set containing S because S is the ζ-closed set, by the theorem 2.9.

Case (ii): If $\zeta_c(S) \subseteq \zeta_f(S)$, then $\zeta_c(S)$ is the smallest ζ-closed set because $\zeta_c(S) \subseteq \zeta_f(S) = \zeta_c(S) \cap \zeta_c(V - S)$. Hence, the proof.

Lemma 2.12 *In (V, τ_ζ), if $\zeta_c(S) = (\zeta_i(S))^c$, then $\zeta_i(S) \neq \zeta_c(S)$.*

Proof. Let $S = \zeta_c(S)$ in V and $S \subseteq \zeta_c(S) = (\zeta_i(S))^c$, by the definition of τ_ζ. Then, $(\zeta_i(S))^c = V - (\zeta_i(S))$ is a

ζ-closed set. From our assumption and lemma 2.11, $V-(\zeta_i(S))=\zeta_c(S)\neq\zeta_i(S)$. Hence, we hold $\zeta_i(S)\neq\zeta_c(S)$.

Theorem 2.13 *In (V, τ_ζ), Z is a subset of V, then every*

1. ζ-T_0 if and only if $(\Leftrightarrow)\zeta$-topologically distinguishable points.
2. ζ-$T_0 \Leftrightarrow \zeta$-separated points.

*Proof.*1. To prove that ζ-$T_0 \Rightarrow \zeta$-topologically distinguishable points. Assume that u and w are ζ-T_0 and its open. By the definition of ζ-T_0, there exists a open set U such that $u\in U$ and $w\notin U$ then $u\subseteq(\zeta\text{-}int(U)\cup\zeta)$. That is, $u\subseteq(U\cup\zeta)\Rightarrow u\subseteq U$ by our assumption. From the definition of ζ-neighbourhood, $\exists$ an open sets U and W such that $u\subseteq U\subseteq N_{\zeta U}$. Similarly, for $w\subseteq W\subseteq N_{\zeta W}$ by our assumption. Now, we get two disjoint neighbourhood for two point u and w. Therefore, ζ-$T_0 \Rightarrow \zeta$-topologically distinguishable points.

The converse part of theorem, prove that ζ-topologically distinguishable points $\Rightarrow \zeta$-T_0. Let there are two points u and w are distinct in ζ-topologically distinguishable points. From the definition, its gives that it has disjoint ζ-neighbourhood. That is, $\exists$ an open sets U and W such that $u\subseteq U\subseteq N_{\zeta U}$ and $w\subseteq W\subseteq N_{\zeta W}\Rightarrow$ U and W are distinct $\Rightarrow N_{\zeta U}$ and $N_{\zeta W}$ are distinct. Finally, its satisfy the ζ-T_0 condition. Hence ζ-$T_0\Rightarrow\zeta$-topologically distinguishable points.

2. Prove that ζ-$T_0\Rightarrow\zeta$-separated points. Let u and w are ζ-T_0 with w is closed in V. If u and w are ζ-T_0, then $u\in U$ and $w\notin U$ where U is open. This implies $w\in(\zeta_i(U))^c\Rightarrow w\in\zeta_c(U)\Rightarrow U\cap\zeta_c(U)=\phi$ by lemma 2.12.Thus, the result is proved.

The another part, prove ζ-separated points $\Rightarrow\zeta$-T_0. Suppose x and y are ζ-separated points with u is open in V $(i.e\{u\}\in U)$. Then, $U\cap\zeta_c(U)=\phi\Rightarrow U\neq\zeta_c(U)\Rightarrow\{u\}\in U$ and $\{u\}\notin V-U=\zeta_c(U)\ni w$, by our assumption and lemma 2.12. Thence, the another part is proved.

Corollary 2.14 *In (V,τ_ζ), every u and w are pair of disjoint are ζ-T_0 $\Longleftrightarrow$ every u and w are pair of disjoint are ζ-symmentry.*

Proof. From the before theorem 2.13 (1) and (2), we can say ζ-$T_0\Longleftrightarrow\zeta$-topologically distinguishable and ζ- separated $\Rightarrow$ ζ- symmetry.

Theorem 2.15 *A space (V,τ_ζ) is ζ-T_0 $\Leftrightarrow$ for each pair of points u, w of V are distinct, $\zeta_c(\{u\})\neq\zeta_c(\{w\})$.*

Proof. To prove: ζ-$T_0\Rightarrow\zeta_c(\{u\})\neq\zeta_c(\{w\})$. Let V be a ζ-T_0 with u and w are ζ-topologically distinguishable and clopen set. Then, $\exists$ an ζ-open set U and W such that $u\in U\subseteq N_{\zeta U}$ and $v\in W\subseteq N_{\zeta W}$. Since $(U\cap W)\subseteq N_{\zeta U}\cap N_{\zeta V}=\phi$. Now, $U\cap W=\phi\Rightarrow U^c\cap W^c=\phi=\zeta_c(\{u\})\cap\zeta_c(\{w\})$, by lemma 2.12 and our assumption. Therefore, $\zeta_c(\{u\})\neq\zeta_c(\{w\})$ because it is a disjoint. Converse part of the theorem is the reverse part of the theorem.

Corollary 2.16 *In a space, (V,τ_ζ) is ζ-T_0 $\Leftrightarrow$ for each pair of points u, w of V are distinct, $u\notin\zeta_c(w)$ and $w\notin\zeta_c(u)$.*

Proof. Assume that u and w are ζ-T_0 in V. From the theorem 2.15, $u\in U=\zeta_c(\{u\})$ and $w\in W=\zeta_c(\{w\})$, where U and W are open. This implies, $\zeta_c(\{u\})$ and $\zeta_c(\{w\})$ are disjoint. From this, we can conclude that $u\notin\zeta_c(w)$ and $w\notin\zeta_c(u)$.$\Leftarrow$, Let each pair of points u, w of V are distinct, $u\notin\zeta_c(w)$ and $w\notin\zeta_c(u)$ are ζ-topologically distinguishable. This gives $u\in(V-\zeta_c(w))\Rightarrow u\in U$, where U is open. Similarly,$w\in W$. From our assumption, $U\cap W=\phi$. Clearly, each pair of u and w are ζ-T_0.Thence, it is proved.

Definition 2.17 *A space V in τ_ζ is said to be a ζ-T_1. If for any points of u and w are disjoint, then $\exists$ two ζ- open sets U and W such that $u\in U, w\notin U$ and $u\notin W, w\in W$.*

Theorem 2.18 *Every $\mathcal{N}$-T_1 is ζ-T_1.*

Proof. Assume that u and w are $\mathcal{N}$-T_1 in $(V,\tau_\mathcal{N})$.Then, $\exists$ the ζ-open subset U such that $u\in U\subseteq(U\cup\zeta)\Rightarrow u\in\zeta_i(\{u\})$ and $w\in\zeta_c(\{w\})\subseteq U^c$ by the lemma 2.11 and the definition of ζ-T_1 are separated. It gives, every $\mathcal{N}$-T_1 is ζ-T_1.

Remark 2.19 *The converse of the theorem* 2.18 *cannot be true.*

Example 2.20 $V = \{s_1, s_2, s_3, s_4\}$ *with* $V / \mathcal{R} = \{\{s_1\}, \{s_3\}, \{s_2, s_4\}\}, Y = \{s_1, s_2\} \subset V$. $\tau_{\mathcal{N}}(Y) = \{\phi, V, \{s_1\}, \{s_2, s_4\}, \{s_1, s_2, s_4\}\}$. *And* $\zeta = \{s_1, s_2, s_4\} \Rightarrow \tau_\zeta(Y) = \{\phi, V, \{s_1\}, \{s_2\}, \{s_4\}, \{s_1, s_2\}, \{s_1, s_4\}, \{s_2, s_4\}, \{s_1, s_2, s_4\}\}$.Now $\{s_2\}$ and $\{s_4\}$ are ζ-T_1 but not $\mathcal{N}$-T_1.

Theorem 2.21 *For a space* (V, τ_ζ)*, then the following are equivalent:*

1. V is ζ-T_1.
2. For every $u \in V, \{u\} = \zeta_c(\{u\})$.
3. For each $u \in V$, the intersection of all ζ open sets containing u is {u}.

Proof. 1. Case(1): Prove (1) $\Rightarrow$ (2). Suppose V is ζ-T_1 and it is a clopen set. From the definition of $\zeta - T_1, \Rightarrow \exists$ an open set U such that $\{u\} \in U$ but $\{w\} \notin U \Rightarrow \{u\} = U$. By our assumption, $\{u\} = \zeta_c(u)$. Hence, the result is proved.

2. Case(2): To prove(2) $\Rightarrow$ (3). We take, every $u \in V$, $\{u\} = \zeta_c(u)$ are clopen set. Then,$\exists$ is ζ - open set such that $\{u\} \subseteq U. \{u\} = \cap_{x \in V} U$ where x is index value. Thus, it is proved.

3. Case(3): Prove (3) $\Rightarrow$ (1). Let for each $u \in V, \{u\} = \cap_{x \in V} U$ where x is a index value and ζ-separated by ζ-neighbourhood. Then, there is a distinct pair u and w in V, $\exists$ an open set U and W such that $u \in U \subseteq N_u$ and $w \in W \subseteq N_w \Rightarrow N_u \cap N_w = \phi$, by our assumption. Its clearly, says that each distinct ζ-points has distinct ζ-open set which implies distinct ζ-neighbourhood. Therefore, we found the result.

Theorem 2.22 *In* (V, τ_ζ)*, every u and w are pair of disjoint are* ζ-T_1 $\Longleftrightarrow$ *V is* ζ*-topologically distinguishable.*

Proof. Let $u \neq w$ are ζ-T_1.So u and w are ζ-T_1, then $\exists\zeta$-open U and W which contains a member does not contain another member. From the result, we get $u \in U$ and $w \notin U$ (i.e) each member has an individual ζ-open and also ζ-neighbourhood. Thus, V is ζ-topologically distinguishable. Reverse part, assume that the pair u and w are pair of disjoint with ζ-topologically distinguishable. That is, $\{u\} \in U \subseteq N_U$ and $\{w\} \in W \subseteq N_W$. Therefore, it is ζ-T_1 because it does not have same ζ-neighbourhood and also ζ-open. Hence, ζ-topologically distinguishable is ζ-T_1.

Theorem 2.23 *A space* (V, τ_ζ) *is* ζ-T_1 $\Leftrightarrow$ *the singletons are* ζ*-closed sets.*

Proof. Suppose $u \in U$. $\forall w \in \{u\}^c$, there is a ζ-open set W with $w \in W$ and $u \notin W$. Then, $W_o = \cup_{w \in \{u\}^c} W$ is ζ-open and it's complement of $\{u\}$ is exactly ζ-closed. $\Leftarrow$ (Conversely), Assume $u, w \in V$ with $u \neq w$. So, $\{u\}^c$ is a ζ - open set with $w \in \{u\}^c$ and $u \notin \{u\}^c$. Hence, it is ζ-T_1.

Definition 2.24 *A space V is said to be* ζ-T_2 *if for each pair of points u and w in V are distinct,* $\exists$ *a* ζ*- open sets U and a* ζ*- open sets W are disjoint in V such that* $u \in U$ *and* $w \in W$.

Theorem 2.25 *Every* $\mathcal{N}$-T_2 *is* ζ-T_2.

Proof. Suppose u and w are ζ-open in $\mathcal{N} - T_2$. So, $\exists$ an open subset U and W such that $u \in U \subseteq (U \cup \zeta) \Rightarrow u \in \zeta_i(\{u\})$ and similarly, $w \in \zeta_i(\{w\})$ by the definition of ζ-T_2. Then, $\zeta_i(\{u\}) \cap \zeta_i(\{w\}) = \phi$ by our assumption. It says that every $\mathcal{N}$-T_1 is ζ-T_1.

Remark 2.26 *The converse of the theorem* 2.25 *cannot be true.*

Example 2.27 *In example 2.20,* $\{s_4\}$ *and* $\{s_2\}$ *are* ζ-T_1 *but not* $\mathcal{N}$-T_1.

Theorem 2.28 *For a space* (V, τ_ζ)*, then the following are equivalent:*

1. V is ζ-T_2.
2. If $u, w \in V$, for each $u \neq w$, then there is a ζ-open set U containing u such that $w \notin \zeta_c(U)$.

Proof.(1) $\Rightarrow$ (2). Suppose V is ζ-T_2 and clopen sets. Then, $u \neq W \exists \zeta$-open U and W such that $u \in U, w \in W$ and $U \cap$

$W=\phi$. Since $U=\zeta_c(U)$ and $W=\zeta_c(W)\Rightarrow\zeta_c(U)\cap\zeta_c(W)=\phi$, by our assumption. Hence, $w\notin\zeta_c(U)$.

(2) $\Rightarrow$ (1). Let $u,w\in V$, for each $u\neq w$, then $\exists$ a ζ-open U where $u\in U$ such that $w\notin\zeta_c(U)$. Then, $w\in V-\zeta_c(U)$ by lemma 2.11. So, $u\in U\subseteq\zeta_c(U)$ and $V-\zeta_c(U)$ are open $\Rightarrow U\cap\zeta_c(U)=\phi$. Thus, it is ζ-T_2.

Theorem 2.29 *In (V,τ_ζ), every u and w are pair of disjoint are ζ-T_2 $\Leftrightarrow$ every u and w are pair of disjoint are ζ-symmentry space.*

Proof. Suppose $u\neq w$ are ζ-T_2 with ζ-topologically distinguishable and ζ-clopen set in V. Then, $\exists\zeta$-open set U and W such that $u\in U$ and $w\in W\Rightarrow U\cap W=\phi$. From our assumption of clopen, $\Rightarrow U\cap W=\phi\Rightarrow U\cap\zeta_c(W)=\phi$. Therefore, it is ζ-seperated $\Rightarrow$ ζ-symmentry. Suppose V is ζ-symmentry. Then, $\exists$two ζ-open sets U and W such that $U\cap\zeta_c(W)=\phi\subseteq U\cap W=\phi$. We have, the two sets are distinct $U\cap W=\phi$, by the definition ζ-topologically distinguishable.Thence, it is ζ-T_2.

Theorem 2.30 *For a space (V,τ_ζ), ζ-$T_2\Rightarrow\zeta$-$T_1\Rightarrow\zeta$-T_0*

Proof. Case(1): Let (V,τ_ζ) be a ζ-T_2. Prove ζ-$T_2\Rightarrow\zeta$-T_1. Assume $u,w\in V$, $u\neq w$. Since V is ζ-T_2, $\exists two\zeta$ -open set U and W in V such that $u\in U$, $w\in W$ and $U\cap W=\phi$. It represents $u\in U$ and $U\cap W=\phi\Rightarrow w\notin U$ and $w\in W$ and $U\cap W=\phi\Rightarrow u\notin W$. Therefore, ζ-$T_2\Rightarrow\zeta$-T_1. Case(2): Let (V,τ_ζ) be a ζ-T_1. Prove ζ-$T_1\Rightarrow\zeta$-T_0.From the definition of ζ-T_1, $\exists\zeta$-open set U such that $u\in U$ and $w\notin U$. It gives $u\in U$ and not containing other member $\Rightarrow$ ζ-T_0. Hence, ζ-$T_1\Rightarrow\zeta$-T_0.

Remark 2.31 *The converse of the theorem* 2.30 *cannot be true.*

Example 2.32 1. $V=\{s_1,s_2\}$ with $V/\mathcal{R}=\{\{s_1\},\{s_2\}\}, Y=\{s_1\}\subset V$. $\tau_{\mathcal{N}}(Y)=\{\phi,V,\{s_1\}\}$. And $\zeta=\{s_1\}\Rightarrow$ $\tau_\zeta(Y)=\{\phi,V,\{s_1\}\}$. Here, s_2 and s_1 are ζ-T_0 but not ζ-T_1. This is also called ζ - Sierpinski space.

2. In example 2.2, $\{s_1\}$ and $\{s_3\}$ are ζ-T_1 but not ζ-T_2.

3 Conclusion

We can established the ζ nanotopology using ζ-T_0, ζ-T_1 and ζ-T_2. In future, We can extend into the idea of ζ-T_3, ζ-T_4, ζ-T_5, ζ-$T_{1/2}$ *and* ζ-$T_{3/2}$. Also, we can approach in another field like an ideal nano topology, fuzzy nano topology, grill nano topology, bi-nano topology, neutrosophic nano topology, graph structures in nano topology, micro topology etc with few results related on some applications.

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