KANTOWSKI-SACHS IN SAEZ –BALLESTER THEORY OF GRAVITATION WITH COLD DARK MATTER AND DARK ENERGY

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Abstract: In this study, we looked at the Kantowski- Sachs metric in the Saez-Ballester [1986] theory of gravitation filled with cold matter and dark energy. To arrive to a solution, we used a specific form of deceleration parameter presented by Singha and Debnath [2009]. There's also a discussion of the properties.

Keywords: Saez-Ballester, Cold Matter, Dark Energy

1. Introduction

Cold Dark Matter is regarded as a type of speculative dark matter in cosmology and physics. It implies that dark matter is moving at a slow rate compared with the speed of light. Dark is the opposite of this, suggesting that electromagnetic radiation and ordinary matter will interact in a fragile way. Dark matter and ordinary baryonic matter consist of stars, planets, and living organisms in the universe, respectively. Dark matter accounts for 85% of the universe's matter. In Astrophysics, the research areas most affected by these theories are those investigating the origin and evolution of galaxies. Irrefutable evidence indicates that the universe occurred around 13.6 billion years ago as a result of the Big Bang Theory. The LCDM (Lambda Cold Dark Matter) is our way of understanding the ancestry of the universe, with the idea that all the physical material in the universe is comprised of Dark matter. Dark energy is referred to here as lambda, and it's an imaginary force that is thought to accelerate the expansion of the universe. According to this theory, the universe initially existed as hot, smooth, and homogeneous. Many scientists have proven that dark energy is caused by cold, slow-moving particles that are incapable of emitting electromagnetic radiation or scattering light. Several studies have also confirmed that our universe is expanding, implying that dark energy should be investigated.
By analysing a specific version of the deceleration parameter, Bishi and Mahanta [2] studied Bianchi type-V in f(R,T) gravity. By assuming a specific form of DP, Bhoyar et al.[3] derived a Kantowski-Sachs model with EoS in f(T) gravity. In General Relativity (GR), Ghate et al [4] looked into Bianchi type-III DE models. In GR, Adhav et al. [5] discussed Bianchi type-III with dark energy. By adopting a constant decelerating parameter, Shaikh and Wankhade [6] examined dark energy cosmological models in Einstein's theory.

In the Saez-Ballester theory of gravitation, Katore and Shaikh [7] obtained a Bianchi type-VI0 DE model. In SB theory, Rao et al. [8] discussed the plane symmetric MHRDE model. Many authors [9-12] have looked into MHRDE cosmological models in this theory. In SB theory of gravitation, Rao et al[13] have found a dark energy(DE) model. In Brans-Dicke theory, Bianchi type-V model in dark energy was proposed by Rao and Jaya Sudha [14].In SB theory, Rao et al. [15] studied Bianchi II, VIII, and IX dark energy models. In Nordtvedt's general scalar tensor theory of gravitation, Rao and Neelima [16], Rao et al. [17], have examined the LRS Bianchi Type-I and Kantowski Sachs dark energy models. In the Saez-Ballester theory of gravitation, Mete et al [18] just derived Bianchi type- III, V, VI_0 containing Cold dark matter and dark energy.

Based on the foregoing, we investigate the Kantowski-Sachs in Saez-Balletser theory of gravitation, which includes cold dark matter and dark energy.

2. Metric and field equations

We consider the Kantowski -Sachs metric of the form

$$ds^2 = dt^2 - A^2\, d\psi^2 - B^2\left(d\theta^2 + \sin^2\theta\, d\chi^2\right), \tag{1}$$

where A & B are functions of cosmic time ‘t’.

Saez-Ballester [1986] proposed the field equations as

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k}\right) = -T_{ij} \tag{2}$$

and the scalar field ϕ satisfies

$$2\phi^n \phi^{i}{}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

where G_{ij} is an Einstein tensor, ω & n are constants and T_{ij} is the energy momentum tensor of matter.

The equation for energy conservation is

$$T^{ij}_{;j} = 0. \tag{4}$$

The energy momentum tensor is given by

$$T_{ij} = T_{ij}^{\ m} + T_{ij}^{\ \Lambda} \tag{5}$$

$T_{ij}^{\ m}$ & $T_{ij}^{\ \Lambda}$ are the energy momentum tensor for cold dark matter and anisotropic dark energy respectively, which are given by

$$T_{ij}^{\ m} = diag[1,0,0,0]\ \rho_m$$

and $$T_{ij}^{\ \Lambda} = diag[1, -w_\Lambda, -(w_\Lambda + \delta), -(w_\Lambda + \gamma)\ ]\rho_m \tag{6}$$

Here w_Λ is the EoS parameter of dark energy and δ *and* γ are the skewness parameters, which are the deviations from w_Λ in the direction of y and z axes respectively. These parameters may be cosmic time ‘t' functions rather than being constants. ρ_m *and* ρ_Λ are the energy densities of cold dark matter and dark energy.

The field equations for the metric (1) can be expressed using (5) and (6) as

$$2\frac{\ddot{B}}{B} + \frac{1}{B^2} + \frac{\dot{B}^2}{B^2} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -w_\Lambda\rho_\Lambda \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -(w_\Lambda + \delta)\rho_\Lambda \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -(w_\Lambda + \gamma)\rho_\Lambda \tag{9}$$

$$\frac{\dot{B}^2}{B^2}+2\frac{\dot{A}\dot{B}}{AB}+\frac{1}{B^2}+\frac{\omega}{2}\phi^n\dot{\phi}^2=\rho_\Lambda+\rho_m \qquad (10)$$

$$\ddot{\phi}+\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)\dot{\phi}+\frac{n}{2}\frac{\dot{\phi}^2}{\phi}=0 \qquad (11)$$

$$\dot{\rho}_\Lambda+\rho_\Lambda\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)+w_\Lambda\rho_\Lambda\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\delta\right)+\rho_\Lambda\frac{\dot{B}}{B}(\delta+\gamma)+\dot{\rho}_m+\rho_m\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=0 \qquad (12)$$

The above dot depicts differentiation with respect to 't'.

Five independent equations with eight unknowns $A, B, \phi, w_\Lambda, \delta, \gamma, \rho_m$ & ρ_Λ.make up the field equations (7) to (12).

From (8) & (9), we have

$$\delta=\gamma \qquad (13)$$

To obtain a solution, First, we looked at Singh and Debnath's [19] particular kind of deceleration parameter.

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-1+\frac{r}{1+a^r} \qquad (14)$$

where $r>0$ is a constant and a is scale factor of the universe.

The equation above can then be used to find out the value of Hubble's parameter. $H=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=\frac{\dot{a}}{a}=k\left(1+a^{-r}\right)$ (15)

On integrating (15), we can get the value of scale factor as

$$a=\left(e^{krt}-1\right)^{\frac{1}{r}} \qquad (16)$$

where k is constant of integration

Second, we make the assumption that the shear scalar and the expansion scalar are proportional, resulting in

$$A=B^m,\ m\neq 0. \qquad (17)$$

From (15), we have

$$B=\left(c_1\left(e^{krt}-1\right)^{\frac{3}{r}}\right)^{\frac{1}{m+2}} \qquad (18)$$

$$A = \left(c_1 \left(e^{krt} - 1 \right)^{\frac{3}{r}} \right)^{\frac{m}{m+2}} \tag{19}$$

where c_1 is constant of integration

As a result, Now it is possible to write the metric (1) as

$$ds^2 = dt^2 - \left(c_1 \left(e^{krt} - 1 \right)^{\frac{3}{r}} \right)^{\frac{2m}{m+2}} d\psi^2 - \left(c_1 \left(e^{krt} - 1 \right)^{\frac{3}{r}} \right)^{\frac{2}{m+2}} \left(d\theta^2 + \sin^2\theta \, d\chi^2 \right) \tag{20}$$

Hence from equation (11) using equations (18) and (19), we have

$$\phi = c_3 \left(e^{krt} - 1 \right)^{\frac{2(r-3)}{r(n+2)}} \tag{21}$$

where $c_3 = \left(\frac{(n+2)c_2}{2k(r-3)c_1} \right)^{\frac{2}{n+2}}$, c_2 is constant of integration

Now the energy conservation equation (12) can be written as

$$\dot{\rho}_\Lambda + \rho_\Lambda \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) + w_\Lambda \rho_\Lambda \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\delta \right) + \rho_\Lambda \frac{\dot{B}}{B}(\delta + \gamma) = 0$$

$$\dot{\rho}_m + \rho_m \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) = 0 \tag{22}$$

From (22), we can get the energy density of CDM as

$$\rho_m = c_5 \left(e^{krt} - 1 \right)^{\frac{-3}{r}} \tag{23}$$

where $c_5 = \frac{c_4}{c_1}$, c_4 is constant of integration

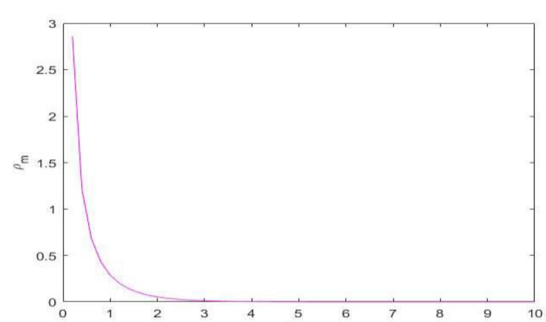

Figure.1. Plot of ρ_m versus time

Also from the field equations (7) to (10), we can obtain

Energy density of dark energy

$$\rho_\Lambda = \frac{9(1+2m)k^2}{(m+2)^2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^2} + \frac{1}{c_1^{\frac{2}{m+2}}\left(e^{krt}-1\right)^{\frac{6}{r(m+2)}}} + \frac{\omega c_6}{2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^{\frac{6}{r}}} - \frac{c_5}{\left(e^{krt}-1\right)^{\frac{3}{r}}}, \tag{24}$$

$$where\ c_6 = c_3^{\,n+2}\left(\frac{2k(r-3)}{(n+2)}\right)^2$$

EoS parameter of dark energy

$$w_\Lambda = \frac{\frac{-27k^2}{(m+2)^2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^2} + \frac{6k^2 r}{(m+2)}\frac{e^{krt}}{\left(e^{krt}-1\right)^2} - \frac{1}{c_1^{\frac{2}{m+2}}\left(e^{krt}-1\right)^{\frac{6}{r(m+2)}}} + \frac{\omega c_6}{2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^{\frac{6}{r}}}}{\frac{9(1+2m)k^2}{(m+2)^2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^2} + \frac{1}{c_1^{\frac{2}{m+2}}\left(e^{krt}-1\right)^{\frac{6}{r(m+2)}}} + \frac{\omega c_6}{2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^{\frac{6}{r}}} - \frac{c_5}{\left(e^{krt}-1\right)^{\frac{3}{r}}}} \tag{25}$$

Skew ness parameter

$$\delta = \gamma = \frac{\frac{-9\left(m^2+m-2\right)k^2}{(m+2)^2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^2} + \frac{3(m-1)rk^2}{(m+2)}\frac{e^{krt}}{\left(e^{krt}-1\right)^2} + \frac{1}{c_1^{\frac{2}{m+2}}\left(e^{krt}-1\right)^{\frac{6}{r(m+2)}}}}{\frac{9(1+2m)k^2}{(m+2)^2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^2} + \frac{1}{c_1^{\frac{2}{m+2}}\left(e^{krt}-1\right)^{\frac{6}{r(m+2)}}} + \frac{\omega c_6}{2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^{\frac{6}{r}}} - \frac{c_5}{\left(e^{krt}-1\right)^{\frac{3}{r}}}} \tag{26}$$

As a result, the metric (1), together with (21) and (23) to (26) in the Saez-Ballester theory of gravity, represents the Kantowski- Sachs metric filled with cold matter and dark energy.
The cold dark matter density parameter is equal to

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{c_5}{3k^2}\frac{1}{e^{2krt}\left(e^{krt}-1\right)^{\frac{3-2r}{r}}} \tag{27}$$

The anisotropic dark energy density parameter is provided by

$$\Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} = \frac{3(1+2m)}{(m+2)^2} + \frac{1}{3k^2 c_1^{\frac{2}{m+2}} e^{2krt}\left(e^{krt}-1\right)^{\frac{6}{r(m+2)}-2}} + \frac{\omega c_6\left(e^{krt}-1\right)^{\frac{2r-6}{r}}}{6k^2} - \frac{c_5}{3k^2}\frac{1}{e^{2krt}\left(e^{krt}-1\right)^{\frac{3-2r}{r}}} \tag{28}$$

Total energy density parameter

$$\Omega = \frac{\rho}{3H^2} = \frac{3(1+2m)}{(m+2)^2} + \frac{1}{3k^2 c_1^{\frac{2}{m+2}} e^{2krt}\left(e^{krt}-1\right)^{\frac{6}{r(m+2)}-2}} + \frac{\omega c_6\left(e^{krt}-1\right)^{\frac{2r-6}{r}}}{6k^2} \tag{29}$$

Coincident parameter

$$\frac{\rho_\Lambda}{\rho_m} = \frac{1}{c_5}\left[\frac{9(1+2m)k^2}{(m+2)^2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^{\frac{2r+3}{r}}} + \frac{1}{c_1^{\frac{2}{m+2}}\left(e^{krt}-1\right)^{\frac{3(m+4)}{r(m+2)}}} + \frac{\omega c_6}{2}\frac{e^{2krt}}{\left(e^{krt}-1\right)^{\frac{9}{r}}}\right] \tag{30}$$

3. Important characteristics of the model

Spatial volume for the model (1) is

$$V = AB^2\, Sin\theta = c_1\left(e^{krt}-1\right)^{\frac{3}{r}} Sin\theta \tag{31}$$

Expansion scalar

$$\theta = 3\frac{ke^{krt}}{e^{krt}-1} \tag{32}$$

Hubble's parameter

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) = \frac{ke^{krt}}{e^{krt}-1} \tag{33}$$

Deceleration parameter $q = -\left(\frac{3}{\theta^2}\dot{\theta}+1\right) = \frac{r}{e^{krt}}-1$ (34)

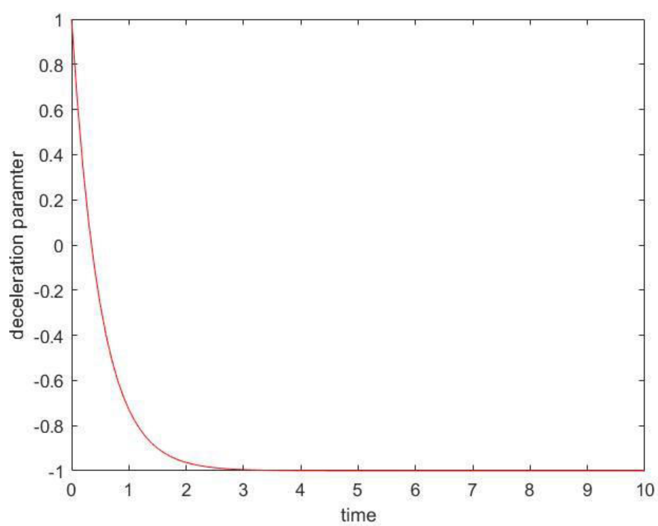

Figure.2. Plot of deceleration parameter versus time

Jerk parameter

$$J = \frac{1}{H^3}\frac{\dddot{a}}{a} = 1+\frac{r(r-2)}{e^{krt}}+\frac{r^2}{e^{2krt}} \tag{35}$$

Mean anisotropy parameter

$$A_m = \frac{2(m-1)^2}{(m+2)^2} \tag{36}$$

Red shift is defined as

$$z = \frac{a_0(t)}{a(t)} - 1$$

where a_0 denotes the current scale factor, which is set to 1.

Therefore $z = \frac{1}{a(t)} - 1 = \frac{1}{(e^{krt}-1)^{\frac{1}{r}}} - 1$ (37)

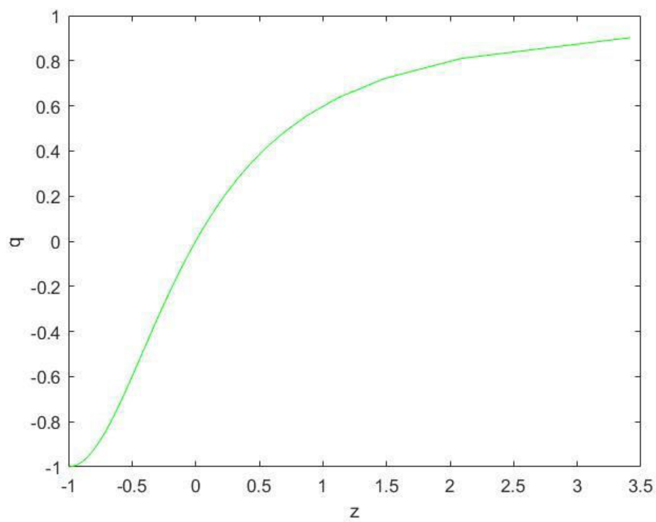

Figure.3. Plot of deceleration parameter versus red shift

Luminosity distance is defined as

$d_L = r_1(1+z)$, where

$$r_1 = \int_t^{t_0} \frac{1}{a(t)} dt$$
$$= \frac{1}{k(r-1)}\left[\left(e^{krt_0}-1\right)^{\frac{r-1}{r}} - \left(e^{krt}-1\right)^{\frac{r-1}{r}}\right]$$

$$d_L = r_1(1+z)$$
$$= \frac{1}{k(r-1)}\left[\left(e^{krt_0}-1\right)^{\frac{r-1}{r}}\left(e^{krt}-1\right)^{\frac{-1}{r}} - \left(e^{krt}-1\right)^{\frac{r-2}{r}}\right] \quad (38)$$

4. Conclusions

We studied the Kantowski-Sachs metric with CDM and anisotropic DE in the Saez-Ballester theory of gravity using a specific kind of deceleration parameter proposed by Singha and Debnath[2009] in this study. At the first epoch, t=0, volume will disappear, and the expansion scalar will go to infinity whereas as $t \to \infty$ volume tends to infinity and θ become constant. The EoS parameter, skewness parameters and ρ_Λ leads to constant value as $t \to \infty$. From Figure 1, we can clearly observe that ρ_m will vanish with the increase of time. From (34), as t goes to infinity, the deceleration parameter $q = -1$ may be seen i.e. exponential or desitter expansion. From Figure 2, we can observe that at t=0, q is positive (decelerating phase) and as t increases q will become negative (accelerating phase). The jerk parameter is employed in cosmology to explain the universe's shift from a decelerating to an accelerating state. Also the behavior of deceleration parameter with red shift is plotted in Figure 3. As we know that for $j = 1$, we will get ΛCDM models, here it is possible for r=0.Also the model presented here represent isotropic for $m = 1$.

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