Research Article

MHD mixed convective boundary layer Casson fluid past a stagnation point of inclined stretching sheet under the influence of viscous Dissipation and mass transfer

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Abstract

This study focus as the flow behaviour of boundary layer Casson fluid embedded in an inclined stretching sheet in existence of stagnation point viscous dissipation and mass transfer. Governing equations are modified to ordinary differential equation through similarity techniques. Results of several physical parameters over velocity, temperature field, concentration, coefficient of skin friction, Nusselt and Sherwood number are demonstrated through graphical representation and discussed.

Keywords: Inclined stretching sheet, Casson fluid, Stagnation point flow,viscous dissipation, Chemical reaction.

1 Introduction

Analyzing the non-Newtonian fluid through a stretching sheet with stagnation point flow has prominent role in various industrial and engineering applications. Chaim (1996) investigated outcome of stagnation point flow field with variable thermal conductivity by analytical and numerical method of approaches. Mahapatra(2002) and Gupta evaluated stagnation point flow of boundary layer temperature field with surface heat flux embedded in a stretched sheet. Ishal et al (2009)explored numerical result of variable surface temperature on stretching sheet with consideration of stagnation points flow by utilizing Keller box method. Bhattacharyya (2013) reviewed heat transfer of time dependent surface temperature through shrinking/stretched sheet with boundary layer stagnation point flow. Mahood et.al (2015) discussed steady electrically conducting fluid flow on a permeable stretched medium with magnetohydrodynamic, stagnation point flow, first order chemical rate reaction.

Pramanik (2014) numerically examined the outcome of suction or blowing and thermal radiation on an exponentially stretched surface with Casson fluid model. Gireesha et al., (2015) scrutinized Casson fluid through a permeable stretched channel with prescribed surface temperature, prescribed heat flux, suction or blowing and non uniform heat sink/source. Jawad Raza (2019) studied Casson fluid with the outcome of stagnation point flow, velocity slip, heat radiation and chemical rate reaction embedded in a convective stretched sheet. Chamka (2000) analyzed boundary layer natural convective flow of electrical conducting fluid on an inclined permeable stretched medium with magnetic induction, heat absorption or generation and fluid wall injection or suction. Eldahab and Aziz (2004) evaluated the effect of mixed convective, internal heat absorption/generation, heat flux and suction/blowing over the power law inclined stretching surface by numerical approach. Venkateswarlu and Satya Narayana (2016) investigated the outcome of MHD Casson model over the stretched channel with the occurence of variable thermal conductivity, viscous dissipation, Soret and Dufour effects by numerical solution of shooting method.

This work analyzes the flow of stagnation point through the inclined stretched sheet with outcome of thermal radiation, viscous dissipation and mass transfer. Non-linear System of differential equations are derived by perturbation approach. Results of various parameters displayed in graphs.

2 Mathematical formulation

This investigation analyzed steady laminar two dimensional boundary layer incompressible Casson fluid past an inclined semi infinite stretched sheet coincide with plate at y=0. There is a strength of constant magnetic field B_0 is given in y-axis direction. Stagnation point $U_w(x) = ax$ and $U_\infty(x) = bx$ describes stretched velocity and free stream velocity in the direction of x-axis. which considered that the stretching sheet inclination with angle α. Sheet temperature and concentration represented by T and C. Rheological equation of isotropic Casson fluid defined by (2010),

$$\tau_{ij} = \begin{cases} 2(\frac{\mu_B + P_y}{\sqrt{2\pi}})e_{ij}, & \pi > \pi_c \\ 2(\frac{\mu_B + P_y}{\sqrt{2\pi_c}})e_{ij}, & \pi < \pi_c \end{cases}$$

here, π_{ij} defines $(i,j)^{th}$ component of stress tensor, $\pi = e_{ij}e_{ij}$ is deformation rate of (i,j) component, π indicates product deformation rate of component itself, π_c denotes product critical value, the plastic dynamic viscosity defined by μ_B, P_y represents yield stress. $\beta = \frac{\mu_B\sqrt{2\pi_c}}{P_y}$ denotes parameter of Casson fluid.

The boundary layer approximation of governing equations are written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu(1+\frac{1}{\beta})\frac{\partial^2 u}{\partial y^2} + U_\infty \frac{dU_\infty}{dx} - \sigma\frac{B_0^2}{\rho}u + g\beta_T(T-T_\infty)cos\alpha + g\beta_C(C-C_\infty)cos\alpha \quad (2)$$

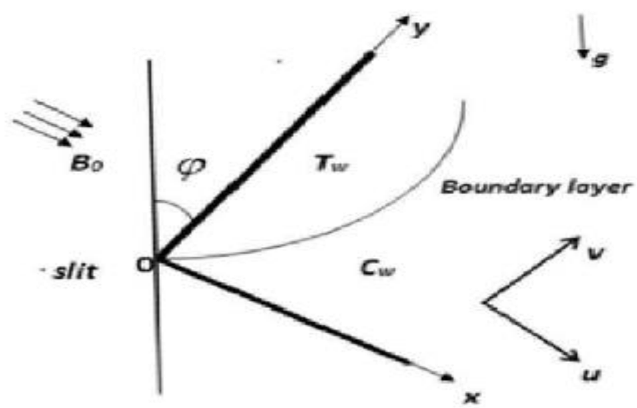

Figure 1: Physical Configuration

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - (1+\frac{1}{\beta})(\frac{\partial u}{\partial y})^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{\partial^2 C}{\partial y^2} + K_0(C-C_\infty) \quad (4)$$

here u and v are velocity over x and y direction, stretching sheet velocity is ax, where $a > 0$ is defined rate of stretching ν defines kinematic viscosity, ρ describes density, σ describes electrical conductivity, g denotes acceleration gravity, κ describes thermal conductivity, c_p denotes specific heat.

The flow is considered in the following boundary condition,

$$\left.\begin{aligned} &u = u_w(x) = ax, v = 0, T = T_w, C = C_w, \quad at \quad y \to 0, \\ &\frac{\partial u}{\partial y} = 0, v = 0, T = T_\infty, C = C_\infty \quad at \quad y \to \infty \end{aligned}\right\} \qquad (5)$$

Rosseland approximation for radiation is defined as(Pantokratoras (2014))

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \qquad (6)$$

here σ^* denotes Stefan's constant and k^* defines coefficient of mean absorption. Expanding the T^4 over T_∞ by applying Taylor series and omitting the higher terms we get,

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \qquad (7)$$

substituting the eqn.(7) in eqn.(6), it becomes,

$$q_r = -\frac{4\sigma^* 3k^* \partial(4T_\infty^3 T - 3T_\infty^4)}{\partial y^*}$$

$$q_r = -\frac{16T_\infty^3}{3k^*}\frac{\partial T}{\partial y} \qquad (8)$$

applying equation(8) in (3) we get,

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - (1 + \frac{1}{\beta})(\frac{\partial u}{\partial y})^2 - \frac{16T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \qquad (9)$$

make use of the similarity transformations below,

$$\left.\begin{aligned}&\psi = \sqrt{a\nu}xf(\eta), \eta = \sqrt{\tfrac{a}{\nu}}y,\\ &\theta(\eta) = \tfrac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \tfrac{C-C_\infty}{C_w-C_\infty}.\end{aligned}\right\} \quad (10)$$

Stream function ψ is described by,

$$u = \frac{\partial\psi}{\partial y} \quad and \quad v = -\frac{\partial\psi}{\partial x} \quad (11)$$

Applying the equations (10) and (11) in the equations (2), (4) and (9) the governing equations and boundary condition becomes,

$$(1+\frac{1}{\beta})f'''(\eta) + f(\eta)f''(\eta) - f'(\eta)^2 + \varepsilon^2 - Mf'(\eta) + Gr\theta(\eta)cos\alpha + Gc\phi(\eta)cos\alpha = 0 (12)$$

$$(1+R)\theta''(\eta) + Pr(f(\eta)\theta(\eta) - (1+\frac{1}{\beta})Ec[f''(\eta)]^2 = 0 \quad (13)$$

$$\phi(\eta) + Sc(f(\eta)\phi(\eta) + \gamma\phi(\eta)) = 0 \quad (14)$$

transformed boundary conditions are modified by

$$\left.\begin{aligned}&f'(\eta) = 1, f(\eta) = 0, \theta(\eta) = 1, \phi(\eta) = 1 \quad at \quad \eta \to 0,\\ &f''(\eta) = 0, f(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0 \quad at \quad \eta \to \infty\end{aligned}\right\} \quad (15)$$

here $\varepsilon = \frac{b}{a}$ defined as velocity ratio parameter, $M = \frac{\sigma B_0^2}{\rho a}$ defines magnetic parameter, $Gr = \frac{g\beta_T(T_w-T_\infty}{a^2x}$ is Grashof number, $Gc = \frac{g\beta_C(C_w-C_\infty}{a^2x}$ is modified Grashof number, $Pr = \frac{\mu c_p}{\kappa}$ is Prandtl number, $Ec = \frac{\nu}{c_p}\frac{ax}{T_w-T_\infty}$ is Eckert number, $R = \frac{16\sigma^* T_\infty^3}{3\kappa k^*}$ is parameter of radiation , $Sc = \frac{D}{\nu}$ is Schmidth number, $\gamma = \frac{K}{a}$ is parameter of chemical reaction.

The coefficient of skin friction, Nusselt and Sherwood number are described by

$$C_f = \frac{\tau_w}{\rho_f u_w^2(x)}, Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, Sh_x = \frac{x q_m}{D_B(C_w - C_\infty)}. \tag{16}$$

Here τ_w denotes shear stress over stretched sheet, q_w represents surface heat flux, q_m describes mass flux, which are denoted as

$$\tau_w = -\mu(\frac{\partial u}{\partial y})_{y=0}, q_w = -\alpha(\frac{\partial T}{\partial y})_{y=0} \quad and$$

$$q_m = -D_B(\frac{\partial C}{\partial y})_{y=0} \quad respectively. \tag{17}$$

By using the eqn.(10) the above equation become

$$\sqrt{Re_x} = -f''(0), Nu_x = -\sqrt{Re_x}(1 + \frac{4R}{3})\theta'(0) = 0 \quad and \quad Sh_x = -\sqrt{Re_x}\phi'(0)(18)$$

where $Re_x = \frac{xU_w(x)}{\nu}$ denotes Reynolds number

3 Method of solution

Velocity, temperature and concentration are calculated using perturbation technique with Mathematica software. Since, the system of differential equations are decompose into base part and perturbed part.

$$\left.\begin{aligned} f(\eta) &= f_0(\eta) + \epsilon f_1(\eta) + \epsilon^2 f_2(\eta) + ... \\ g(\eta) &= g_0(\eta) + \epsilon g_1(\eta) + \epsilon^2 g_2(\eta) + ... \\ \theta(\eta) &= \theta_0(\eta) + \epsilon\theta_1(\eta) + \epsilon^2\theta_2(\eta) + ... \\ \phi(\eta) &= \phi_0(\eta) + \epsilon\phi_1(\eta) + \epsilon^2\phi_2(\eta) + ... \end{aligned}\right\} \tag{19}$$

The above equation are substituting into equation (12) to(15), the higher orders of perturbation parameter (ϵ^2) neglecting and equating the zeroth and first order terms.

Base part:

$$(1+\frac{1}{\beta})f_0''' + f_0 f_0'' - f_0'^2 + \varepsilon^2 - Mf_0' + Gr\theta_0 cos\alpha + Gr\phi_0 cos\alpha = 0 \quad (20)$$

$$(1+R)\theta_0'' + Pr(f_0\theta_0' + \lambda\theta_0) = 0 \quad (21)$$

$$\phi_0'' + Sc(f_0\phi_0' - \gamma\phi_0) = 0 \quad (22)$$

Perturbed part:

$$(1+\frac{1}{\beta})f_1''' - f_1' + f_0 f_1'' + f_1 f_0'' - 2f_0 f_1 - Mf_1' + Gr\theta_1 cos\alpha + Gr\phi_1 cos\alpha = 0 (23)$$

$$(1+R)\theta_1'' + Pr(f_0\theta_1' + f_1\theta_0' - \theta_1 + \lambda\theta_1) = 0 \quad (24)$$

$$\phi_1'' + Sc(\phi_1' f_0 + f_1\phi_0' - \phi_1 - \gamma\phi_1) = 0 \quad (25)$$

Base part boundary condition:

$$\left.\begin{array}{ll} f_0' = 1, f_0 = 0, \theta_0 = 1, \phi_0 = 1 & at \quad \eta \to 0 \\ f_0'' = 0, f_0 = 0, \theta_0 = 0, \phi_0 = 0 & at \quad \eta \to \infty \end{array}\right\} \quad (26)$$

Perturbed part boundary condition:

$$\left.\begin{array}{ll} f_1' = 0, f_1 = 0, \theta_1 = 0, \phi_1 = 0 & at \quad \eta \to 0 \\ f_1'' = 0, f_1 = 0, \theta_1 = 0, \phi_1 = 0 & at \quad \eta \to \infty \end{array}\right\} \quad (27)$$

These base and perturbed non linear partial differential equation are derived numerically subject to the boundary condition equations (26) and (27). The graphs are presented for the axial velocity, temperature and concentration.

4 Results and Discussion

Major intention of this study evaluates heat and mass transfer in the presence of stagnation point flow of Casson fluid on an inclined stretched sheet. Results of various values over velocity profile, temperature and concentration field displayed in graphs.

Figure 2 illustrates the velocity profile $f(\eta)$ increases with raising cases of Casson fluid parameter (β). Figure 3 represents velocity field $f(\eta)$ increase with raising values of M. Figure 4 displays the Grashof number velocity field $f(\eta)$ which falling with various values of Gr. figure 5 demonstrates velocity distribution $f(\eta)$ which decreases with several cases of Gc.

Figure 6 demonstrates the impact of radiation parameter over temperature profile which decrease with increasing cases of R. Figure 7 displays the outcome of Prandtl number over temperature profile which decrease with increases values of Pr. Figure 8 demonstrates that result of Eckert number on temperature distribution which is parabolic in nature with increasing cases of Ec.

Figures 9 and 10 displays impact of Schmidt number (Sc) and chemical

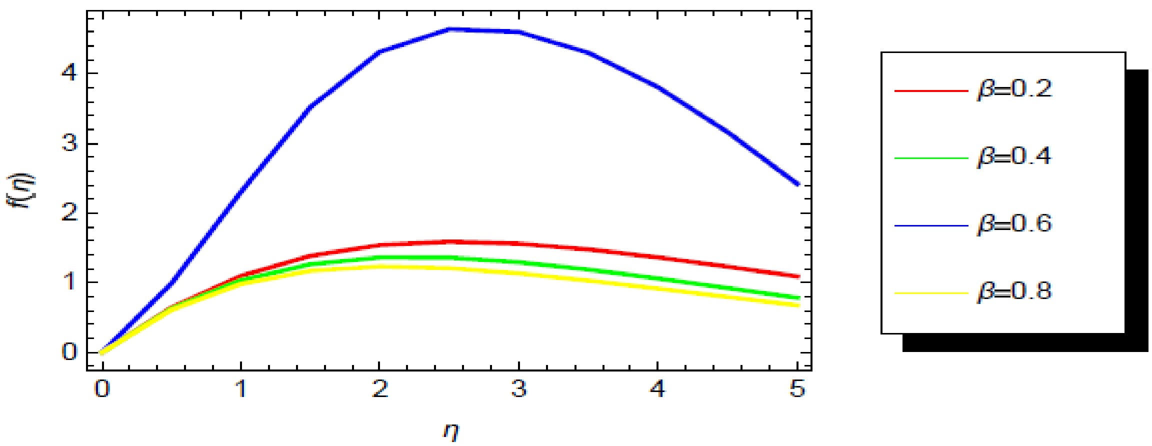

Figure 2: Variation of β on velocity profile

reaction term γ over concentration $\phi(\eta)$ which describes both are falling with increasing cases of Sc and γ.

Figure 11 describes the influence of Casson fluid parameter over coefficient of skin friction. It predicts that skin friction falling with various value of β. Figure 12 displays the variation of Casson fluid parameter over Sherwood number which is increasing with various values of β. Figure 13 depicts that effect of Schmidth number on Nusselt number which is decreases with different values of Sc.

5 Conclusion

The stagnation point flow on stretching surface used in cooling transpiration, wire drawing, designing of thrust bearings, polymer of extrusion drag reduction, radial diffusers, drawing of plastic sheets, recovery of thermal oil and involved in many hydrodynamic process.

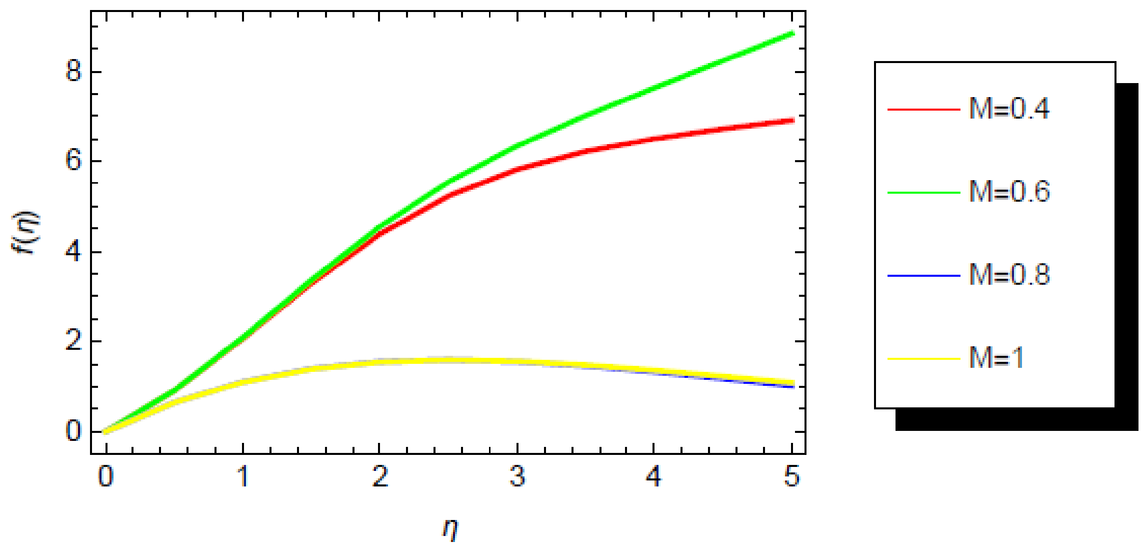

Figure 3: Variation of velocity profile for different value of M

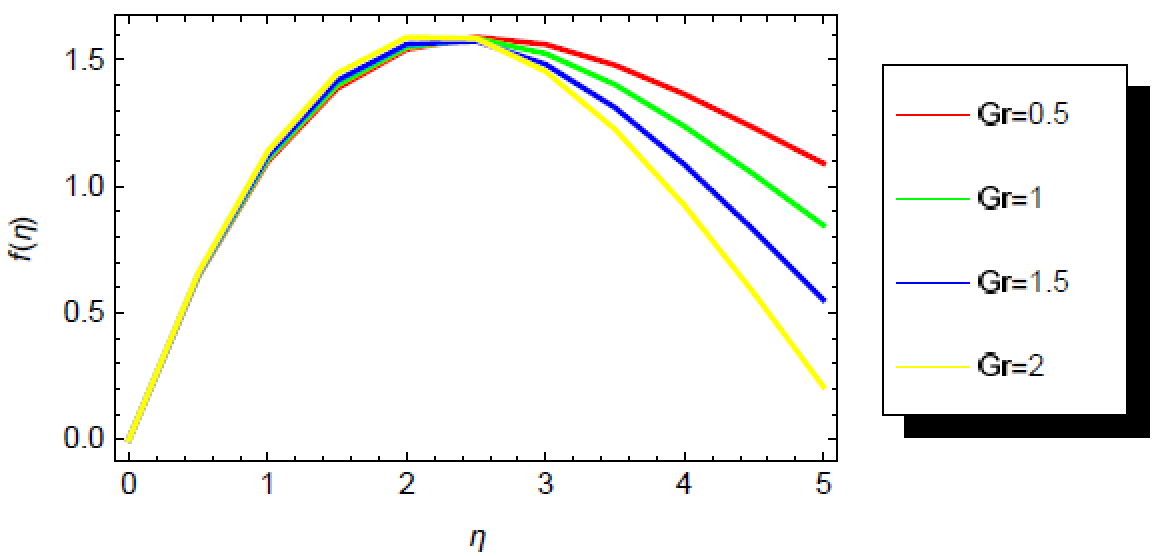

Figure 4: Variation of Gr of velocity profile

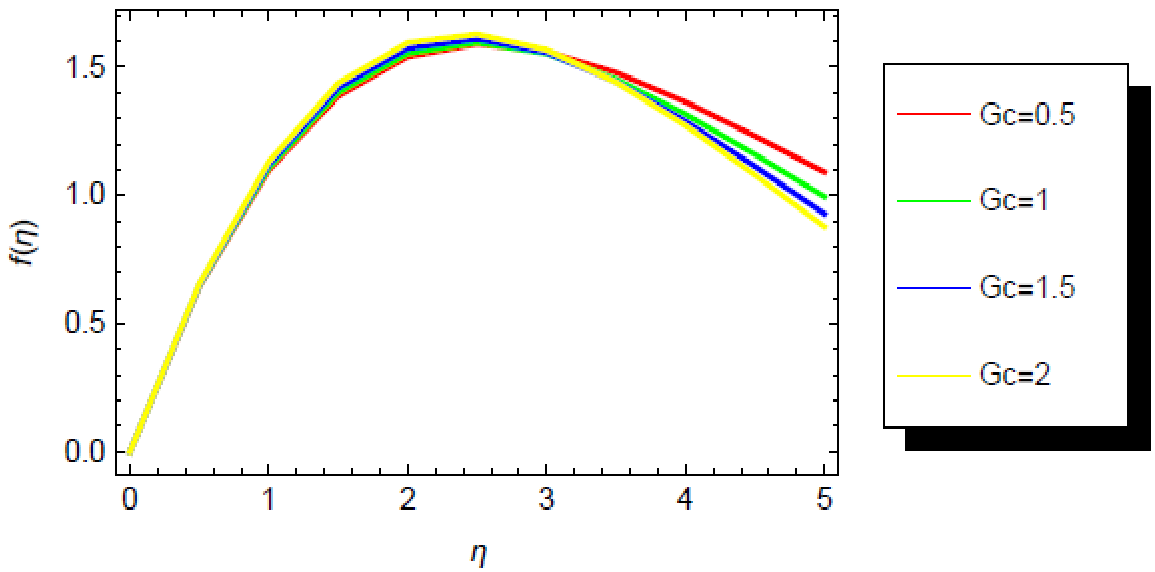

Figure 5: Variation of Gc on velocity profile

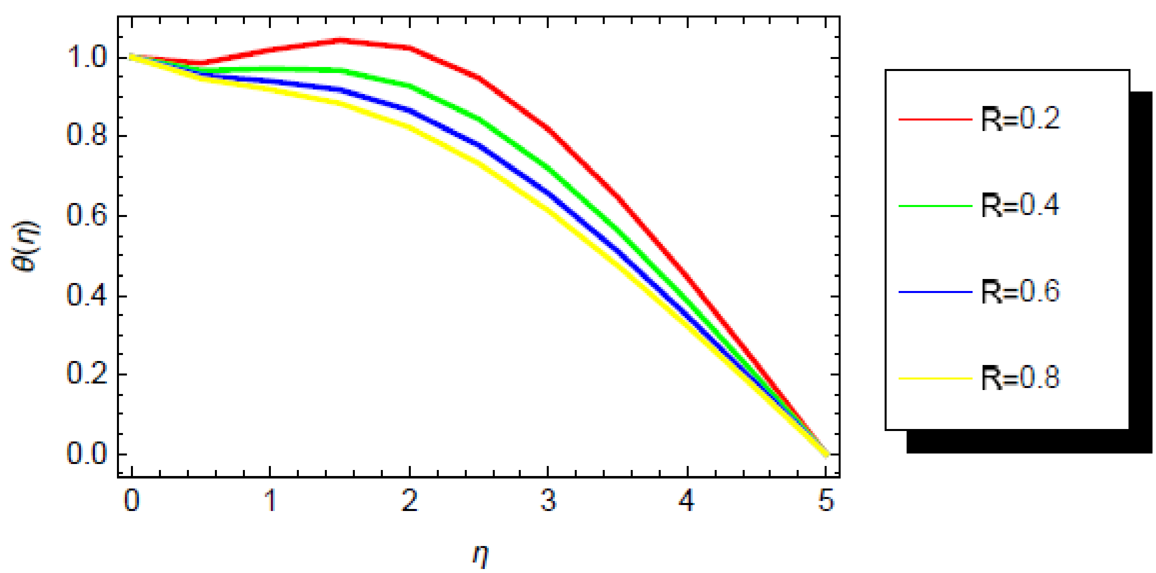

Figure 6: Variation of R on rate of heat transfer

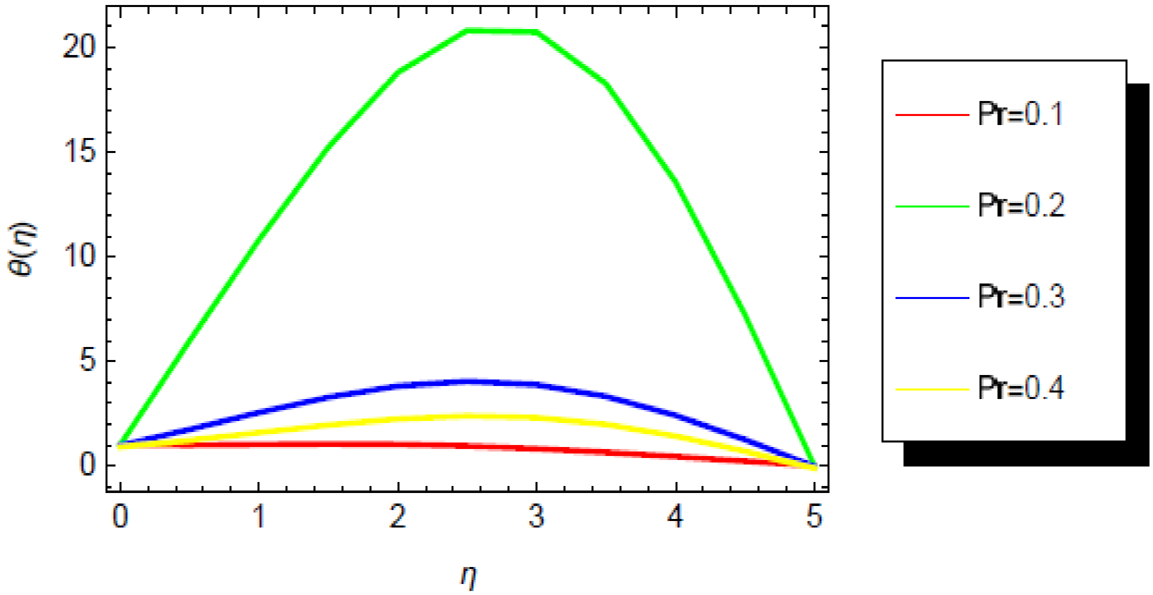

Figure 7: Variation of Pr on rate of heat transfer

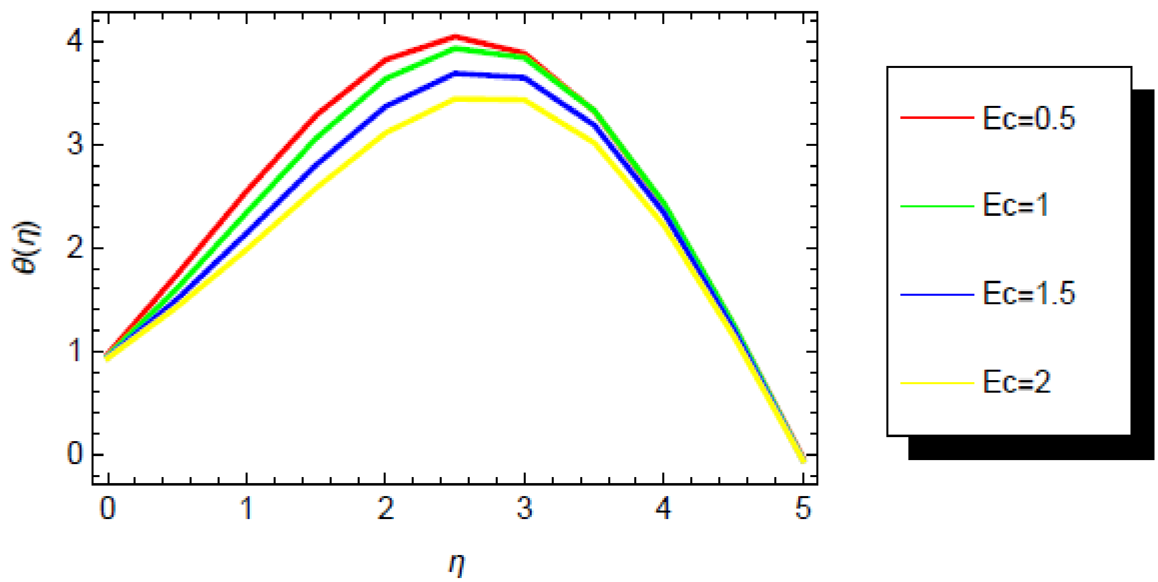

Figure 8: Variation of Ec on rate of heat transfer

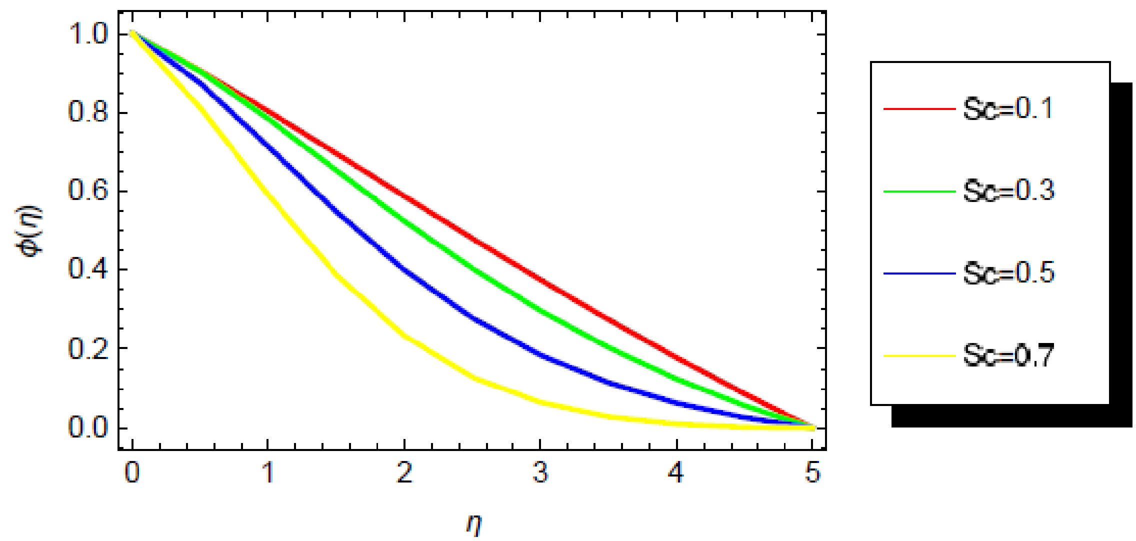

Figure 9: Effect of Sc on concentration profile

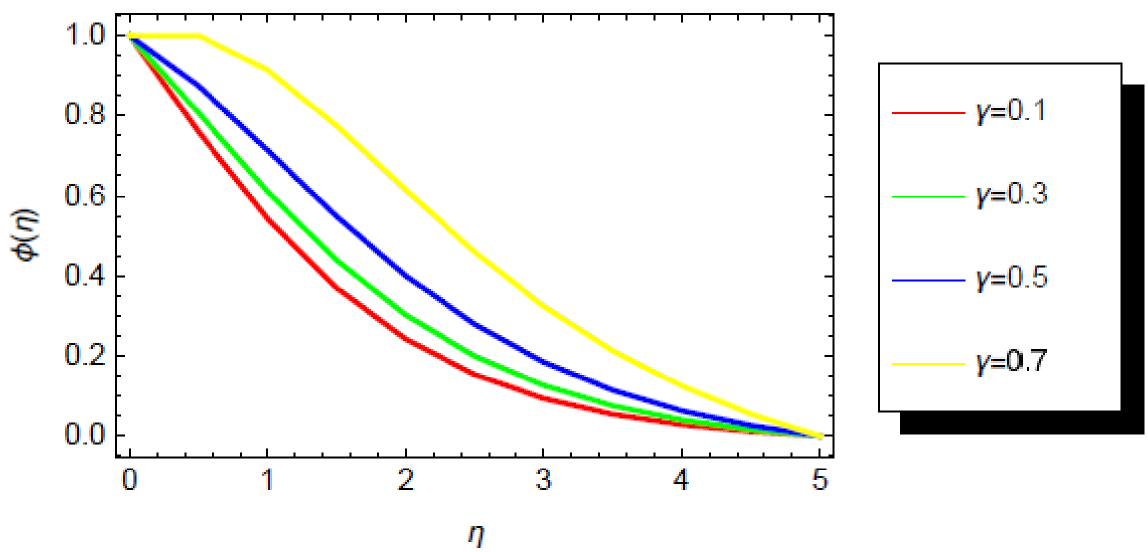

Figure 10: Consequence of γ on concentration profile

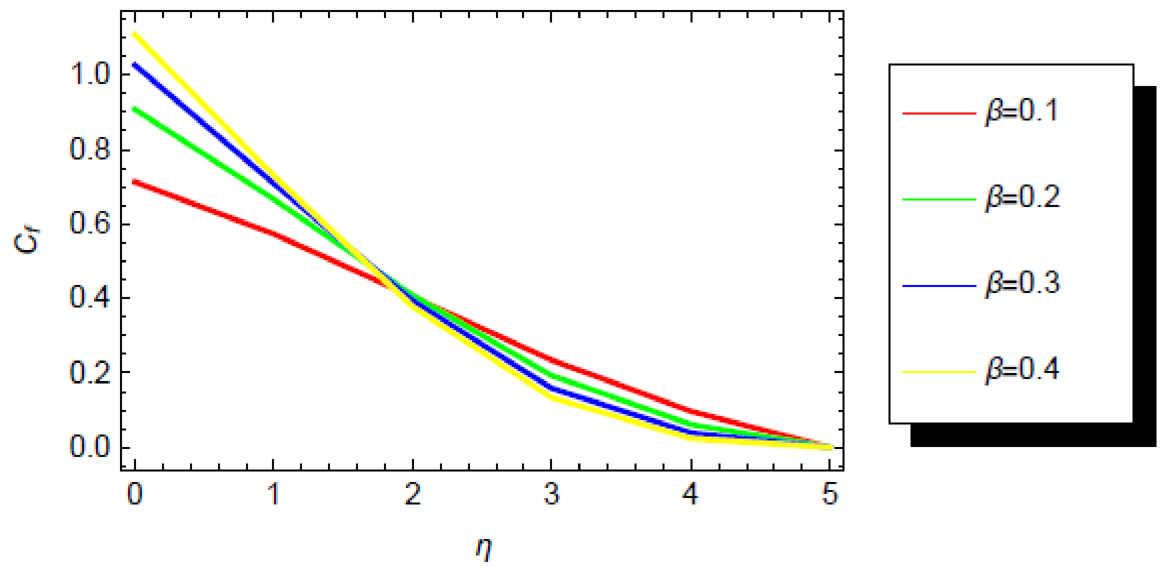

Figure 11: Variation of β on Skin friction

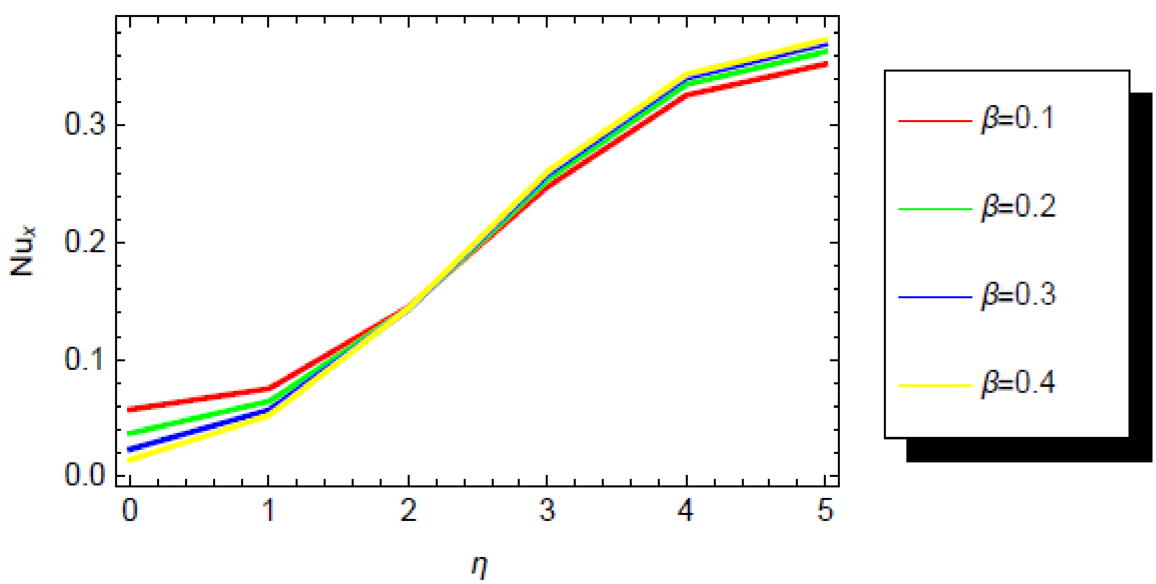

Figure 12: Effect of β over Nusselt number

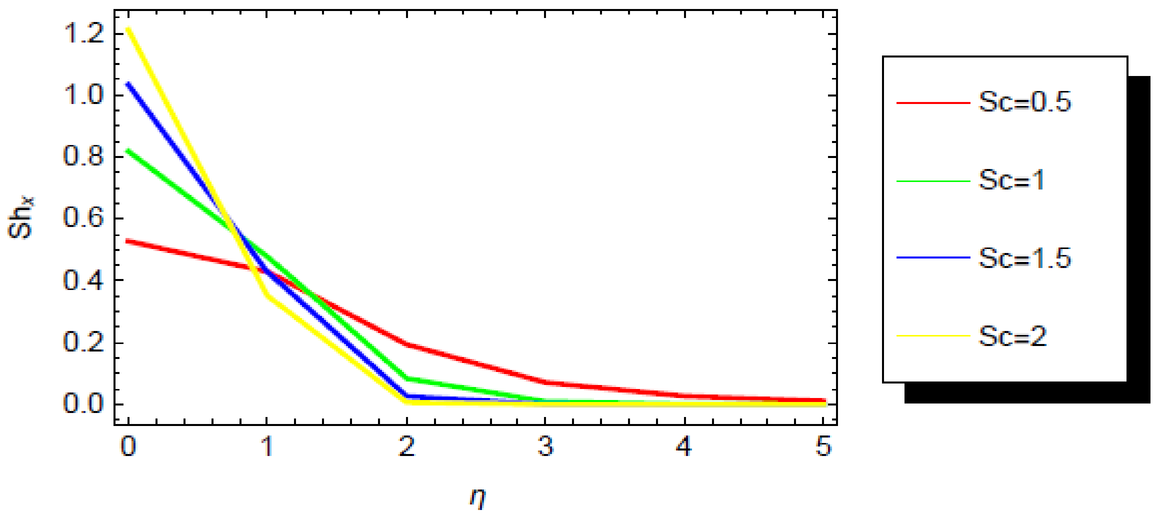

Figure 13: Influence of Sc on Shearwood number

References

[1] Chaim, T.C., (1996), Heat transfer with variable conductivity in a stagnation-point flow towards a stretching sheet, *International Communications in Heat and Mass Transfer*, **23(2)**, 239-248.

[2] Ray Mahapatra, T., and Gupta, A.S., (2002), Heat transfer in stagnation-point flow towards a stretching sheet,*Heat and Mass Transfer*, **38(6)**, 172-176.

[3] Ishak, A., Jafar, K., Nazar, N., and Pop, I., (2009), MHD stagnation point flow towards a stretching sheet, textitPhysica A: Statistical mechanics and its applications, **388(17)**, 3377-3383.

[4] Krishnendu Bhattacharyya, (2013), Heat transfer analysis in unsteady boundary layer stagnation point flow towards a shrinking/stretching sheet, *Ains Shams Enginnering Journal*, **4(2)**, 259-264.

[5] Mabood, F., Khan, W.A., Ismail, and A.I.Md., (2015) MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration, *Chemical Engineering Journal*,**273**, 430-437.

[6] Pramanik, S.,, (2014), Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation, *Ain Shams Engineering Journal*,**5(1)**,205-212.

[7] Gireesha, B.J., Mahanthesh, B., and Rashidi, M.M., (2015), MHD boundary layer heat and mass transfer of a chemically reacting Casson fluid over a permeable stretching surface with non-uniform heat source/sink, *International Journal of Industrial Mathematics*, **7(3)**,247-260.

[8] Jawad Raza, (2019), Thermal radiation and slip effects on magnetohydrodynaamics(MHD) stagnation point flow of Casson fluid over a convective stretching sheet, *Propulsion and Power Research*, **8(2)**, 138-146.

[9] Chamkha, A.I., (2000), Transient hydromagnetic three-dimensional natural convection from an inclined stretching permeable surface, *Chemical Engineering Journal* **76(2)**,159-168.

[10] .Abo-Eldahab, E.M., and Mohamed Abd El-Aziz, (2004), Blowing/suction effect on hydromagnetic heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption, *International Journal of Thermal Science*,**43(7)**,709-719.

[11] Venkateswarlu, B., and Satya Narayana, P.V., (2016),Influence of variable thermal conductivity on MHD Casson fluid flow over a stretching sheet with viscous dissipation, Soret and Dufour effects, *Frontiers in Heat and Mass Transfer*,**7(16)**,1-9.

[12] Hayat, T., Shehzad, S.A., and Alsaedi, A., (2010), Soret and Dufour effects on magnetohydrodynamic(MHD) flow of Casson fluid, *Applied Mathematics and Mechanics*, **33(10)**,1301-1312.

[13] Pantokratoras, A., and Fang, T., (2014), Blasius flow non-linear Rosseland thermal radiation,*Meccanica*,**49(6)**,1539-1545