Unsteady suction/injection oscillatory pulsatile blood flow through multi stenosis in an effect of Hall current With mass transfer and heat source

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Abstract

A mathematical model is developed by treating blood as a non-Newtonian fluid, unsteady MHD flow with Hall current in a multi-stenosed artery. The governing equation are solved by using perturbation technique. The fluid is subjected to the chemical reaction with mass transfer and velocity through the porous medium is consideration. The analytical solutions for velocity, temperature, concentration, shear stress and shear rate are calculated. The numerical solutions are graphically depicted for physical parameters.

Keywords: Hall current, MHD, Multi-Stenosis, Porous medium,Suction/Injection, Mass transfer, Heat source.

1 Introduction

Suction/injection velocity of the fluid is taken into account pulsatile flow as well as circular tubes of varying cross section has been investigated by

Peeyush Chandra and Krishna Prasad (1994). Misra and Pal (1999) analyzed laminar pulsatile flow of blood under the infulence of externally imposed body acceleration. Devajyoti Biswas and Uday Shankar Chakraborty (2009) studied blood flow through pulsatile a uniform artery in presence of mild stenosis, blood have been represented an Newtonian fluid. Shit and Roy (2012) investigated the unsteady pulsatile blood flow through a porous channel in an effect of magnetic field. Rajender et al., (2012) examined the steady of couple stress fluid in permeable porous channel with suction and injection of the wall. Sinha and Mondal (2015) obtained the oscillatory flow of pulsatile blood flow with porous medium and heat transfer. Nabilet et al., (2016) analyzed the MHD flow of couple stress fluid in oscillatory channel with heat and mass transfer, through a porous channel. Pulsatile blood flow, porous channel with constant permeability in an inclined with mild stenosis artery. The flow takes the body acceleration using perturbation technique examined Lukendra Kakati et al., (2017). Rafik Absi (2018) showed the oscillatory and pulsatile flows is environmental, biological (health) and industrial applications, turbulent of the oscillatory boundary layers are used different coastal engineering applications. Selvi et al., (2018) presented a incompressible conducting couple stress fluid inclined channel with pulsatile flow through injected lower permeable bed and sucked into upper permeable bed. Lukendra Kakati et al., (2018) investigated the axially symmetric, pulsatile and fully developed blood flow through inclined channel with mild stenosis artery. Effect of Hall current in oscillatory flow of a couple stress fluid in an inclined channel examined Nirmala Ratchagar et al., (2018). Venkataswamy et al., (2019) analyzed the unsteady solute transfer in a micropolar fluid impact of MHD with suction/injection. Sheeba Juliet et al., (2019) explored the unsteady oscillatory flow through a vertical channel

with porous medium, the fluid subjected to chemical reaction with heat and mass transfer. The aim of this paper is to study the unsteady MHD flow of oscillatory blood fluid in multi stenosed porous channel is considered. Solved analytically using perturbation method. The impact of magnetic field with Hall current and other parameters have been shown graphically in these results.

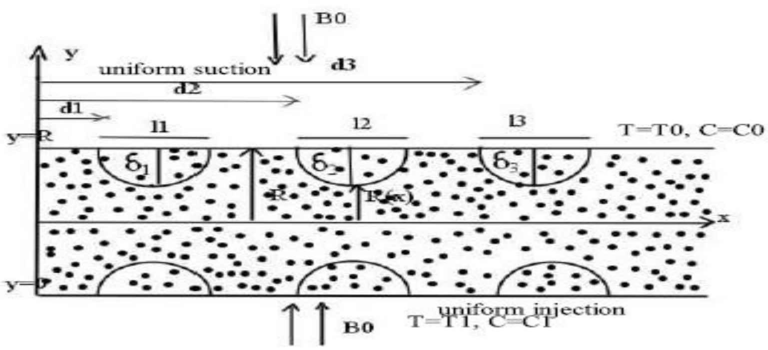

Figure: 1 Physical model.

2 Mathematical formulation

Consider the flow of unsteady, laminar, incompressible, viscous non-Newtonian blood flow through a channel in multi stenosis. The stenosis length l_1 , l_2 and l_3 is assumed to be analyzed. A uniform magnetic field B_0 is y direction of the blood flow. The following assumptions δ_1 , δ_2 and δ_3 is height of three stenosis, $R^{(*)}$ is stenosis portion. A schematic figure of the physical configuration are shown figure 1.

The governing equation of the fluid motion for the blood flow in cartesian form is given by,

Conservation of mass:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*} = 0 \tag{1}$$

Conservation of Momentum:

$$\frac{\partial u^*}{\partial t^*} - V_0\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \nu(\frac{\partial^2 u^*}{\partial y^{*2}}) - (\frac{\nu}{k})u^* - \frac{\sigma B_0^2 u^*}{\rho(1+m^2)}$$

$$+g\beta_t^*(T^* - T_0) + g\beta_c^*(C^* - C_0) \tag{2}$$

Conservation of Energy:

$$\frac{\partial T^*}{\partial t} - V_0\frac{\partial T^*}{\partial y^*} = \frac{K_f}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} + Q(T^* - T_0) - \frac{\partial q_r}{\partial y^*} \tag{3}$$

Conservation of Concentration:

$$\frac{\partial C^*}{\partial t^*} - V_0\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_f(C^* - C_0) + D_T\frac{\partial^2 T}{\partial y^{*2}} \tag{4}$$

with boundary condition,

$$\left.\begin{array}{ll} u^* = \frac{\sqrt{k}}{\alpha_s}\frac{\partial u^*}{\partial y^*}, & T^* = T_0, C^* = C_0 \quad at \quad y^* = 0 \\ u^* = 0, & T^* = T_1, C^* = C_1 \quad at \quad y^* = R^* \end{array}\right\} \tag{5}$$

where, u^*, v^* is velocity of the blood fluid, V_0 is suction/injection ρ is density, p^* is pressure, k is permeability, t^* is time, σ is electrically conducting fluid, m is the effect of Hall current, B_0 is the external magnetic field, μ is viscosity, σ_0 is electrical conductivity.

The blood fluid is consumed by the heart pumbing action, which give rise to a pulsatile pressure gradient approximated as Ogulu (1993)

$$-\frac{\partial p}{\partial x} = P_s + \epsilon P_0 Cos(\omega t) \tag{6}$$

The multiple stenosis is commonly construct in the femoral and lumber arteries. The configuration of the arterial portion having multiple stenosis is

$$R^*(x^*) = \begin{cases} R_0 & 0 \le x^* \le d_1^* \\ R_0 - \frac{\delta_1^*}{2}(1 + Cos\frac{2\pi}{l_1^*}(x^* - d_1^* - \frac{l_1^*}{2})) & d_1^* \le x^* \le d_1^* + l_1^* \\ R_0 & d_1^* + l_1^* \le x^* \le d_2^* \\ R_0 - \frac{\delta_2^*}{2}(1 + Cos\frac{2\pi}{l_2^*}(x^* - d_2^* - \frac{l_2^*}{2})) & d_2^* \le x^* \le d_2^* + l_2^* \\ R_0 & d_2^* + l_2^* \le x^* \le d_3^* \\ R_0 - \frac{\delta_3^*}{2}(1 + Cos\frac{2\pi}{l_3^*}(x^* - d_3^* - \frac{l_3^*}{2})) & d_3^* \le x^* \le d_3^* + l_3^* \\ R_0 & d_3^* + l_3^* \le x^* \le l^* \end{cases}$$

where, $R'(x^*)$ is the radius of the artery, R_0 is normal of the radial artery, l_i and δ_i $(i = 1, 2, 3)$ are the length and maximum thickness of three stenosis $(\delta << R_0)$, l is length of the cardiovascular artery and interval(distance) d is equispaced point.

Introducing the following non dimensional quantities,

$$x = \frac{x^*}{R_0}, y = \frac{y^*}{R_0}, u = \frac{u^*}{u_0}, R_e = \frac{u_0 R_0}{\nu}, k = \frac{k^*}{R_0^2 \rho}, t^* = \frac{t R_0^2}{\nu}, p^* = \frac{p}{\rho u_0^2}, s = \frac{V_0 R_0}{\nu},$$

$$Da = \frac{K}{R_0^2}, \gamma = \frac{\sqrt{k}}{\alpha_s R_0}, S_r = \frac{D(T_1 - T_0)}{\gamma(C_1 - C_0)}, P_r = \frac{\rho c_p \nu}{K}, G_r = \frac{g\beta_t (T_1 - T_0) R_0^2}{u_0 \nu},$$

$$G_c = \frac{g\beta_c (C_1 - C_0) R_0^2}{u_0 \nu}, M = \frac{\sigma B_0^2 R_0^2}{\rho \nu}$$

The governing equations in dimensionless form are reduced to:

$$\frac{\partial u}{\partial t} - V_0\frac{\partial u}{\partial y} = -\frac{1}{R_e}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (\frac{M^2}{1+m^2} + \frac{1}{Da})u + G_r\theta + G_c\phi \tag{7}$$

$$\frac{\partial \theta}{\partial t} - V_0\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + \delta_0\theta + V_0\theta \tag{8}$$

$$\frac{\partial \phi}{\partial t} - V_0\frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} + S_r\frac{\partial^2 \theta}{\partial y^2} - K_c\phi \tag{9}$$

The boundary conditions in equation (5) non dimensionless form we get,

$$\left.\begin{array}{ll} u = \gamma\frac{\partial u}{\partial y}, & \theta = 0, \phi = 0 \quad at \quad y = 0 \\ u = 0, & \theta = 1, \phi = 1 \quad at \quad y = R \end{array}\right\} \tag{10}$$

The multistenosis in non dimensionless form is,

$$R(x) = \begin{cases} 1 & 0 \le x \le d_1 \\ 1 - \frac{\delta_1}{2}(1 + Cos\frac{2\pi}{l_1}(x - d_1 - \frac{l_1}{2})) & d_1 \le x \le d_1 + l_1 \\ 1 & d_1 + l_1 \le x \le d_2 \\ 1 - \frac{\delta_2}{2}(1 + Cos\frac{2\pi}{l_2}(x - d_2 - \frac{l_2}{2})) & d_2 \le x \le d_2 + l_2 \\ 1 & d_2 + l_2 \le x \le d_3 \\ 1 - \frac{\delta_3}{2}(1 + Cos\frac{2\pi}{l_3}(x - d_3 - \frac{l_3}{2})) & d_3 \le x \le d_3 + l_3 \\ 1 & d_3 + l_3 \le x \le l \end{cases}$$

3 Method of solution

Velocity, temperature and concentration distribution:

The ordinary differential equation of the form is

$$u(y,t) = u_0(y)e^{i2\pi t} \tag{11}$$

$$\theta(y,t) = \theta_0(y)e^{i2\pi t} \tag{12}$$

$$\phi(y,t) = \phi_0(y)e^{i2\pi t} \tag{13}$$

Substituting equations (11) to (13) and the equation (7), (8) and (9) equating the coefficient we have,

$$\frac{\partial^2\theta_0}{\partial y^2} + sP_r\frac{\partial\theta_0}{\partial y} + (\delta_0 + s_0 - i\omega)P_r\theta_0 = 0 \tag{14}$$

$$\frac{1}{S_c}\frac{\partial^2\phi_0}{\partial y^2} + s\frac{\partial\phi_0}{\partial y} - (i\omega + K_c)\phi_0 = -S_r\frac{\partial^2\theta_0}{\partial y^2} \tag{15}$$

$$\frac{\partial^2 u_0}{\partial y^2} + s\frac{\partial u_0}{\partial y} - (i\omega + \frac{M^2}{1+m^2} + \frac{1}{Da})u_0 = -(P_s + Cos[2\pi t]) - G_r\theta - G_c\phi \tag{16}$$

with boundary condition,

$$\left.\begin{array}{l} u_0 = \gamma\frac{\partial u_0}{\partial y},\ \theta_0 = 0, \phi_0 = 0 \quad at \quad y = 0 \\ u_0 = 0, \qquad \theta_0 = 1, \phi_0 = 1 \quad at \quad y = R \end{array}\right\} \tag{17}$$

Solving equation (14) to (16) and (18) to (20) subject to boundary condition (17) and (21) we have,

$$\theta(y,t) = (c_1e^{a_1y} + c_2e^{a_2y})e^{i2\pi t} \tag{18}$$

$$\phi(y,t) = (c_3e^{a_3y} + c_4e^{a_4y} + c_5e^{a_1y} + c_6e^{a_2y})e^{i2\pi t} \tag{19}$$

$$u(y,t) = c_7e^{a_5y} + c_8e^{a_6y} + c_9 + c_{10}e^{a_1y} + c_{11}e^{a_2y} + c_{12}e^{a_3y} + c_{13}e^{a_4y} + c_{14}e^{a_1y} + c_{15}e^{a_2y} \tag{20}$$

Shear stress at the wall

$$\tau_w = [\frac{\partial u}{\partial y}]_{y=R(x)} \quad (21)$$

Substituting from equation (20) in equation (21) and calculating the shear stress of the wall can be written as,

$$\tau_w = c_7 a_5 e^{a_5 y} + c_8 a_6 e^{a_6 y} + c_{10} a_1 e^{a_1 y} + c_{11} a_2 e^{a_2 y} + c_{12} a_3 e^{a_3 y} + c_{13} a_4 e^{a_4 y} + c_{14} a_1 e^{a_1 y} + c_{15} a_2 e^{a_2 y}$$

where,

$$a_1 = \frac{-SP_r + \sqrt{(SP_r)^2 - 4(1)(\delta_0 + S_0)P_r}}{2}$$

$$a_2 = \frac{-SP_r - \sqrt{(SP_r)^2 - 4(1)(\delta_0 + S_0)P_r}}{2}$$

$$c_1 = \frac{1}{e^{a_1 R} - e^{a_2 R}}$$

$$c_2 = \frac{-1}{e^{a_1 R} - e^{a_2 R}}$$

$$a_3 = \frac{-SS_c + \sqrt{(SS_c)^2 + 4S_c K_c}}{2}$$

$$a_4 = \frac{-SS_c - \sqrt{(SS_c)^2 + 4S_c K_c}}{2}$$

$$c_3 = -(c_4 + c_5 + c_6)$$

$$c_4 = \frac{-1 - c_5(e^{a_3 R} - e^{a_1 R}) - c_6(e^{a_3 R} - e^{a_2 R})}{e^{a_3 R} - e^{a_4 R}}$$

$$c_5 = \frac{S_r c_1 a_1^2}{a_1^2 + S_c S a_1 - S_c K_c}$$

$$c_6 = \frac{S_r c_2 a_2^2}{a_2^2 + S_c S a_2 - S_c K_c}$$

$$a_5 = \frac{-S + \sqrt{S^2 + 4(\frac{M^2}{1+m^2} + \frac{1}{Da})}}{2}$$

$$a_6 = \frac{-S - \sqrt{S^2 + 4(\frac{M^2}{1+m^2} + \frac{1}{Da})}}{2}$$

$$c_9 = \frac{P_s + P_0 Cos[i2\pi t]}{(\frac{M^2}{1+m^2} + \frac{1}{Da})}$$

$$c_{10} = \frac{-G_r c_1}{a_1^2 + S a_1 - (\frac{M^2}{1+m^2} + \frac{1}{Da})}$$

$$c_{11} = \frac{-G_r c_2}{a_2^2 + S a_2 - (\frac{M^2}{1+m^2} + \frac{1}{Da})}$$

$$c_{12} = \frac{-G_c c_3}{a_3^2 + S a_3 - (\frac{M^2}{1+m^2} + \frac{1}{Da})}$$

$$c_{13} = \frac{-G_c c_4}{a_4^2 + S a_4 - (\frac{M^2}{1+m^2} + \frac{1}{Da})}$$

$$c_{14} = \frac{-G_c c_5}{a_1^2 + S a_1 - (\frac{M^2}{1+m^2} + \frac{1}{Da})}$$

$$c_{15} = \frac{-G_c c_6}{a_2^2 + S a_2 - (\frac{M^2}{1+m^2} + \frac{1}{Da})}$$

$$f1 = c_9[e^{a_5 R} + (\gamma a_5 - 1)] - c_{10}[(\gamma a_1 - 1)e^{a_5 R} - (\gamma a_5 - 1)e^{a_1 R}]$$

$$f2 = -c_{11}[(\gamma a_2 - 1)e^{a_5 R} - (\gamma a_5 - 1)e^{a_2 R}] - c_{12}[(\gamma a_3 - 1)e^{a_5 R} - (\gamma a_5 - 1)e^{a_3 R}]$$

$$f3 = -c_{13}[(\gamma a_4 - 1)e^{a_5 R} - (\gamma a_5 - 1)e^{a_4 R}] - c_{14}[(\gamma a_1 - 1)e^{a_5 R} - (\gamma a_5 - 1)e^{a_1 R}]$$

$$c_8 = \frac{f1 + f2 + f3 - c_{15}[(\gamma a_2 - 1)e^{a_5 R} - (\gamma a_5 - 1)e^{a_2 R}]}{[(\gamma a_6 - 1)e^{a_5 R} - (\gamma a_5 - 1)e^{a_6 R}]}$$

$$f4 = c_9[e^{a_6 R} + (\gamma a_6 - 1)] - c_{10}[(\gamma a_1 - 1)e^{a_6 R} - (\gamma a_6 - 1)e^{a_1 R}]$$

$$f5 = -c_{11}[(\gamma a_2 - 1)e^{a_6 R} - (\gamma a_6 - 1)e^{a_2 R}] - c_{12}[(\gamma a_3 - 1)e^{a_6 R} - (\gamma a_6 - 1)e^{a_3 R}]$$

$$f6 = -c_{13}[(\gamma a_4 - 1)e^{a_6 R} - (\gamma a_6 - 1)e^{a_4 R}] - c_{14}[(\gamma a_1 - 1)e^{a_6 R} - (\gamma a_6 - 1)e^{a_1 R}]$$

$$c_7 = \frac{f4 + f5 + f6 - c_{15}[(\gamma a_2 - 1)e^{a_6 R} - (\gamma a_6 - 1)e^{a_2 R}]}{[(\gamma a_5 - 1)e^{a_6 R} - (\gamma a_6 - 1)e^{a_5 R}]}$$

4 Results and Discussion

In order to investigated numerical solutions velocity, temperature, concentration and shear stress are obtained for different values of Magnetic field (M), Grashof number (Gr), Prandtl number (Pr), warm radiation parameter (δ_0), Reynolds number (Re), Hall parameter (m), and soret number (Sr), are presented graphically. Further, it is assumed to the multiple stenosis $l_1 = 0.2\,,\ l_2 = 0.3\,,\ l_3 = 0.5\,,\ \leq epsilon \leq 1\,,\ d_1 = 0.2\,,\ d_2 = 0.4\,,\ d_3 = 0.7\,,$

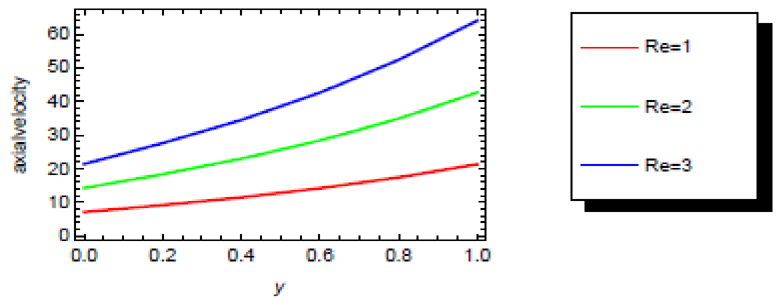

Figure 2

Plots of velocity while distance axis for various values "R_e" $(\delta_1 = 0.1, \delta_2 = 0.4 and \delta_3 = 0.6)$.

figures (2)-(6) describes the variation of velocity profile for the various values R_e, M, S, m, Da. Figure 2 it is clear that the velocity increases with Reynolds number increases. Figure 3 velocity profile b decreases with magnetic parameter increases. Figure 4 suction/injection parameter and velocity profiles are stenotic region is increasing blood flow to the injury. Figure 5 impact of Hall parameter increases with velocity profile increases. Figure 6 velocity decreases with darcian parameter increases.

Figure (7)-(9) depict the variation of temperature profile are variation of the parameter $S, P_r and \delta_0$. Figure 7 shows that the temperature field is parabolic. The temperature decreases with increasing warm radiation parameter. Figure 8 illustrates the impact of prandtl number on temperature profiles. It is clearly temperature decreases while prandtl number increases. Figure 9 displays that decreases temperature and increasing δ_0.

Figure (10)-(11) concentration field is blood flow is increases with schmidt number increases in Figure 10. Figure 11 concentration decreases with increasing the chemical reaction.

5 Conclusion

The effect of Hall parameter and suction/injection on the multi stenosis geometry has been considered. The rate of the blood flow through decreasing with increasing thickness of the stenosis (δ). This study is beneficial for

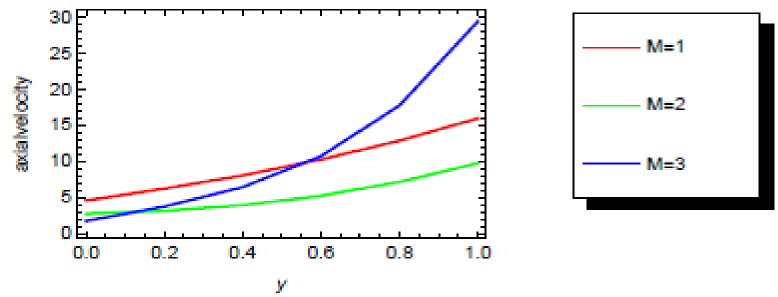

Figure 3

Plots of velocity while distance axis for various values "M"

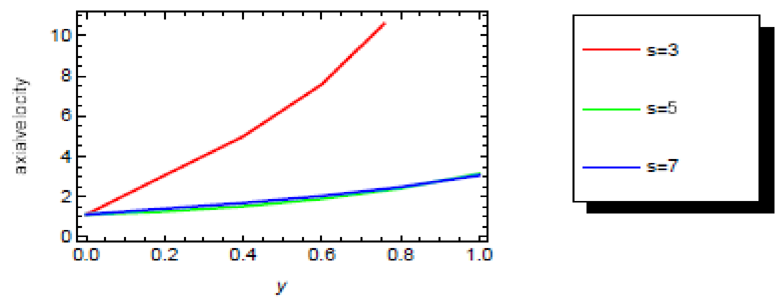

Figure 4

Plots of velocity while distance axis for various values "s"

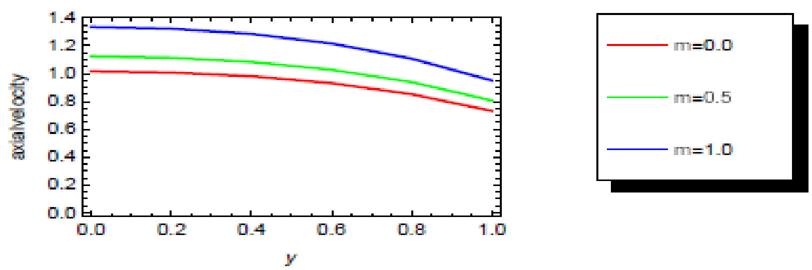

Figure 5

Plots of temperature while distance axis for various values "m"

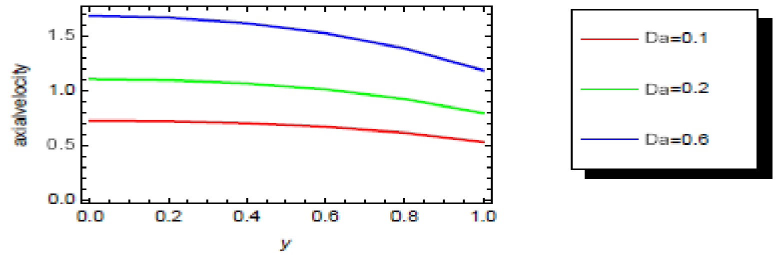

Figure 6

Plots of temperature while distance axis for various values "Da"

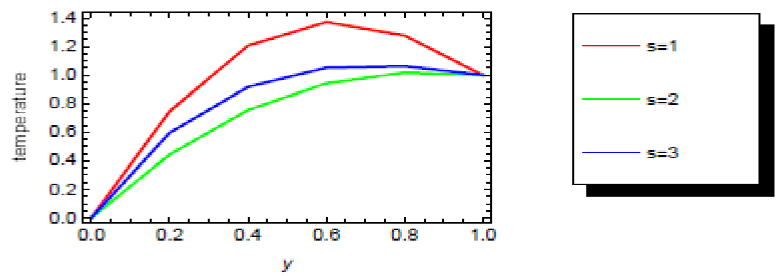

Figure 7

Plots of temperature while distance axis for various values "s"

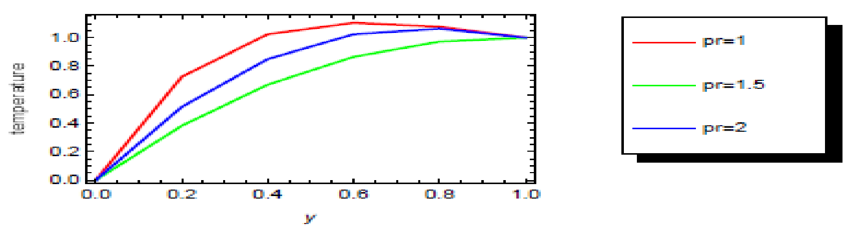

Figure 8

Plots of temperature while distance axis for various values "pr"

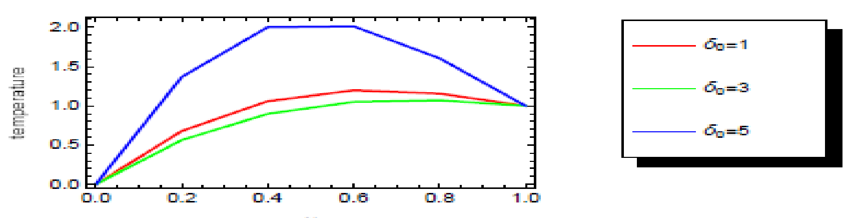

Figure 9

Plots of temperature while distance axis for various values "δ_0"

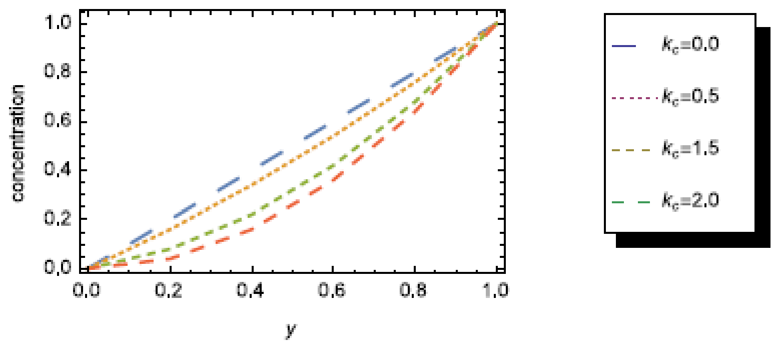

Figure 10

Plots of concentration while distance axis for various values K_c

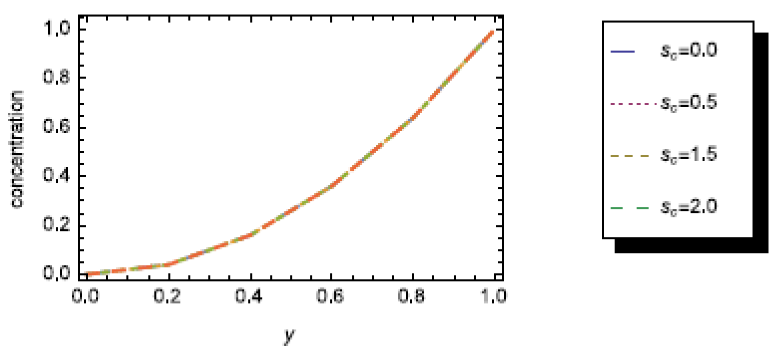

Figure 11

Plots of concentration while distance axis for various values S_c

analyzing the behaviour of temperature distribution inside stenosis artery during hyperthermia and cryosurgery.

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