



Students' Mental Model in Understanding Quadrilateral Concept

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Abstract: The purpose of this study was to describe the 5 levels of students' mental models, namely initial, transition 1, synthesis, transition 2 and formal understanding of the quadrilateral concept. The subjects of this study were students of 7th grades, which amounted to 34 students, but there were only 5 students who were representatives in each level of the mental model. The results of this study are students with this initial level do not have the concept of a quadrilateral in themselves or the concept that is in them cannot be recalled when they answer the questions given. At transition level 1, student is able to answer correctly using the concepts that are in him. So when describing the answer is also close to the truth. For student who are at the synthesis level, they have started to be able to answer correctly with a concept model that is starting to form perfectly. Student who is at transition level 2 is already close to the truth both in answering and describing the answers in his self-concept about quadrilaterals. Student with a formal mental model level has a perfect quadrilateral concept and can use the concept well in answering the questions given. In describing the answers that have been written are also correct and the answers given can be accepted scientifically.

Keywords: mental model, concept understanding, quadrilateral

1. Introduction

Understanding the concept is the basis and an important stage in a series of mathematics learning. Understanding of mathematical concepts is an understanding of concepts that is the center of attention in the development of mathematics curricula in each country (Pirie&Kieren, 1994; Mousley, 2005). Students' ability to learn mathematics is directly related to their understanding of mathematical concepts and principles. Concepts are the basis for higher-order thinking processes, it can be interpreted that students who understand concepts well will be able to generalize and transfer their knowledge more than students who only memorize definitions. The lack of understanding of basic mathematical concepts in addition to the use of inadequate learning media is also due to students' thinking processes that are still not at a high level (Fatmawati, 2021). There are many studies on understanding the concept, one of which is understanding the concept of a quadrilateral. Quadrilateral is part of learning geometry.

Geometry is a field of mathematics faced by almost every individual during their educational process. Geometry is an integral part of the curriculum, which provides information about how students should learn about concepts. Geometry comes from the Greek, which means the scientific field that analyzes the size and shape of objects. One of the principles and standards of the National Board of Teachers of Mathematics (NCTM, 2000) regarding school mathematics is geometry. Geometry is a science that helps to analyze and interpret events happening around us. In this context, correcting errors in geometry learning is very important, especially in terms of developing students' thinking systems (Ozkan&Bal, 2017).

Learning geometry should start with the most basic geometric tasks such as drawing, playing with geometric shapes, and naming geometric shapes (Browning, 2008). Learning geometric concepts is a complex process and as a component of the geometry curriculum, defining and classifying a quadrilateral is considered a difficult subject by many students (Clement&Battista, 1992; De Villiers, 1994; Erez&Yerushalmy, 2006; Fujita&Jones, 2007; Fujita, 2012; Okazaki&Fujita, 2007).

Several researchers have conducted research on understanding concepts in quadrilaterals, including (Ozkan&Bal, 2017), which examined the analysis of students' errors in their understanding of quadrilaterals. Budiarto (2017) researched on testing the abstraction thinking of mathematical concepts by considering the abstractions built previously on the quadrilateral concept. Ulger&Broutin (2017) examined the understanding of prospective teachers to express the conception, definition of a quadrilateral and its internal relationships in figural and formal concepts in the parallelogram case.

From some of these studies, it can be concluded that understanding the concept of a quadrilateral is very important to study. Mathematical thinking processes focus on understanding concepts. Mathematical thinking can be summarized as a method for accessing from the unknown to the known which consists of making assumptions, gathering evidence and the process of generalizing about the case. In order to actively participate in the mathematical thinking process, it is necessary to have mathematical abilities and develop them, one of which is the formation of mental models (Baltaci, 2016). The formation of a complete mental model of students, one of which is the role of the teacher as a student educator during learning. The teacher's teaching strategy and the processing of the teacher's teaching materials greatly affect the development of the students' mental models.

According to (Duffy, 2012), the concept of a mental model was first proposed by Craik (1943), according to Craik the mind builds a small-scale model of reality that is used to anticipate events, reasons and to base explanations. Mental model according to some other experts (Zimet, 2017; Turk, 2016; Ozkan&Bal, 2017), according to (Zimet, 2017) mental model is a cognitive structure that has process or product properties. Mental models also have several functions that allow them to store data and enforce strategies to get results. According to (Turk, 2016), mental models are internal and cognitive representations, psychological representations of real and imaginary situations. Mental models are formed with people who understand and conceptualize events in the world. In their thinking, mental models are internal representations of the actual situation in people's minds to understand. According to (Ozkan&Bal, 2017), mental models are internal representations that act as structural analogies of situations or processes. Its role is to account for the reasoning of both when they try to understand discourse and when they try to explain and predict the behavior of the world.

Thus, an analysis of mental models can provide us with very valuable information, understanding students' feelings and learning processes. One study on mental models that discusses mathematical problems, namely research by (Bofferding, 2014) which examines initial mental models can provide an overview of how students process the information that has been given in solving integer problems so that teachers can find out students' difficulties and even misconceptions that students usually have. Bofferding (2014) identified five categories (levels) of mental models, namely initial (initial), transition I, synthetic, transition II and formal.

Mental models are interesting to study for two reasons. First, that mental models affect cognitive function and second, mental models can provide valuable information for science education researchers about the structure of students' concepts (Laliyo, 2011). Based on the description above, it is very important to know how the mental model of students in understanding the concept of a quadrilateral is. So that we can describe each level in the mental model of how to understand the concept of quadrilateral material.

2. Research Method

2.1 Research Approach

This study uses a qualitative approach, this is in accordance with the characteristics of qualitative research described (Creswell, 2012) as follows, namely the natural environment, the researcher as a key instrument, various data sources, inductive data analysis, the meaning of the participants, the evolving design, the theoretical perspective, the nature of interpretation, and overall view. The type of research used is descriptive qualitative research which aims to describe several levels of mental models in understanding the concept of quadrilaterals.

2.2 Research Subject

The subjects in this study were 7th grade, totaling 34 students. The subjects of this study were selected based on several criteria set by the researcher to achieve the research objectives. Among them are the subjects in this study are students who are and have obtained material about quadrilaterals and the subject has good communication skills so that the disclosure of understanding of the quadrilateral concept at each level of the mental model can be carried out properly.

2.3 Data Collection Technique

In this case the researcher acts as the main instrument because he is directly involved in the research process and as a data collector and analyzer (Moleong, 2006). The auxiliary instruments in this research are questions related to quadrilateral understanding. The questions consist of 3 items which contain an understanding of the quadrilateral whose material has been obtained by students as a subject. Data collection techniques in this study were observation, interviews and documentation so that the data obtained were in the form of test results and interview transcripts with research subjects. Students are given a question then from all students will be selected several students who meet the criteria in leveling mental models.

2.4 Data Analysis Technique

The data analysis process begins by examining the results of all existing data from various sources, namely the results of written tests and transcripts of interviews with students. Then the students' answers will be given a score

for the quality of students' answers according to the mental model leveling assessment indicators. so that the subjects in each level of the mental model are obtained.

3.Results

Data collection begins with sorting out answers from students according to the mental model level assessment indicators. To make it easier to analyze the data, the initials are used, namely S1 for subject 1, S2 for subject 2, S3 for subject 3, S4 for subject 4 and S5 for subject 5. The assessment indicators used as a reference are as follows.

Table 1. Mental Model Assessment Indicators According to Park & Light (2009)

Type	Assesment Criteria
1(Early Mental Model)	Students are unable to answer with the concepts that exist within themselves or cannot describe them and are scientifically unacceptable because they do not have any concept
2(Intermediate 1 Mental Model)	Students are able to answer correctly with the concepts that are in themselves with models that are starting to be formed and the explanations become scientific closer to the truth
3(Intermediate 2 Mental Model)	Students are able to answer correctly with the concepts that exist within themselves with models that are starting to be formed where the explanations are partially correct and become scientifically closer to the truth
4(Intermediate 3 Mental Model)	Students are able to answer correctly and provide an explanation correctly, in addition to being able to give scientifically correct answers, they can be accepted scientifically closer to the truth
5(Target Mental Model)	Students are able to answer correctly and provide an explanation correctly, in addition to being able to give correct answers, the concept of a quadrilateral is scientifically acceptable and the structure described is also correct

From the assessment indicators, 5 students' answers were selected according to the level of the mental model. The mental model levels used are initial (initial mental model), transition I (intermediate mental model 1), synthetic (intermediate mental model 2), transition II (intermediate mental model 3) and formal (target mental model). Each level of the mental model is represented by 1 subject, so that S1 is an initial level subject, S2 is a transition level I subject, S3 is a synthesis level subject, S4 is a transitional level II subject and S5 is a formal level subject.

The answer given by S1 which is included in the initial subject illustrates that this student is not able to describe the concepts that exist in him, because he does not have or forgets the concept of a quadrilateral. The answers for S1 can be seen in Figure 1 below.

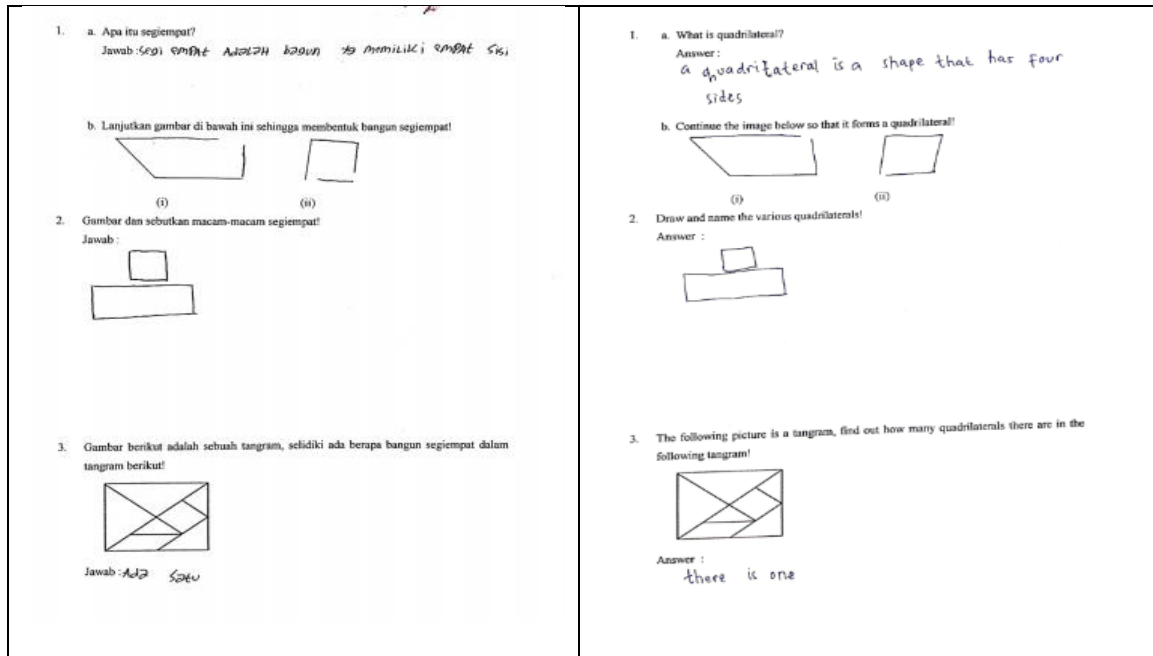


Figure 1. Answer S1

Based on the picture above, the results of the S1 answers on the questions about quadrilaterals were given by the researcher. From the results, it can be seen that S1 does not have a quadrilateral concept on him, the concept he once got can't be displayed again. It was also shown during the interview process that S1 stated that a quadrilateral is only a shape that has 4 sides. It can also be seen in question no 2, that in the S1 concept, the example of a quadrilateral is only a square and a rectangle. So that when answering question no 3, S1 can only see 1 quadrilateral contained in the tangram.

Furthermore, the answer for S2 which represents the subject with transition level I can be seen from Figure 2 below.

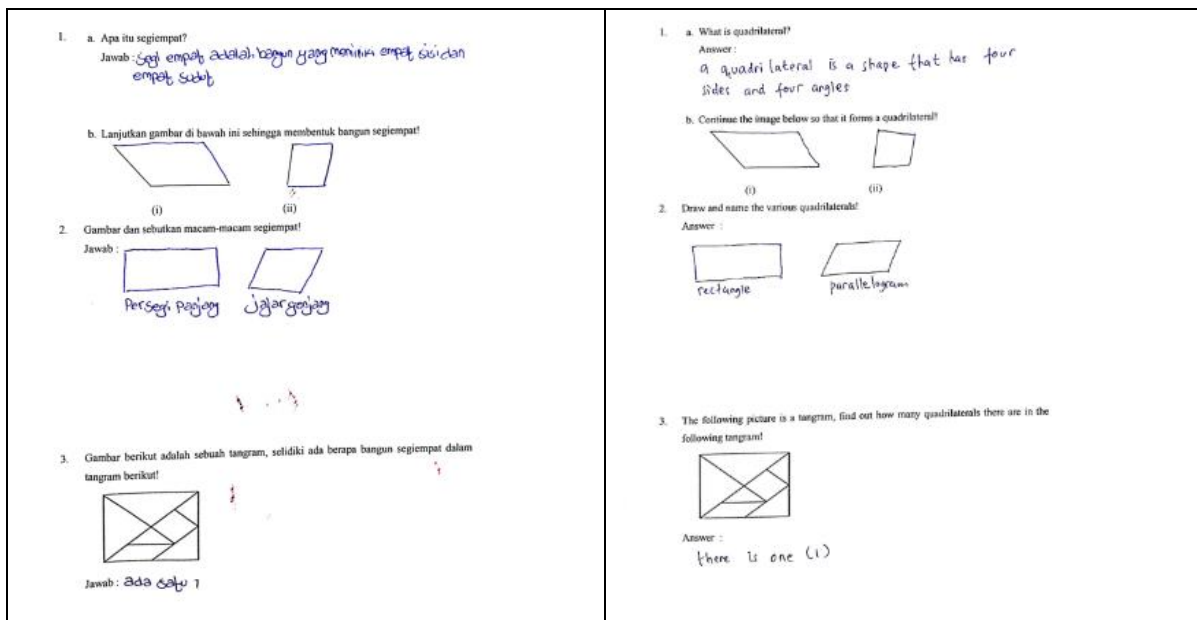


Figure 2. Answer S2

Based on the picture above, the results of the S1 answers on the questions about quadrilaterals were given by the researcher. From the results above, it can be seen that S2 is starting to be able to answer correctly even though not all of the answers from S2 are perfect. For question no. 1, S1 was able to answer correctly about what a quadrilateral is. When the interview was conducted, S2 also seemed to be able to show his concept of a quadrilateral, namely that a quadrilateral is a shape that has 4 sides and 4 angles. For question no 2, S2 describes two quadrilaterals, namely a rectangle and a parallelogram. When the interview was conducted, why did S2 only

write down two shapes, S2 answered that he only re-described the shapes in question number 1. For question no. 3, in the S2 concept, there was only one quadrilateral in the tangram, namely the outside.

For subjects representing the synthetic level, it can be seen in the S3 answer in Figure 3 below.

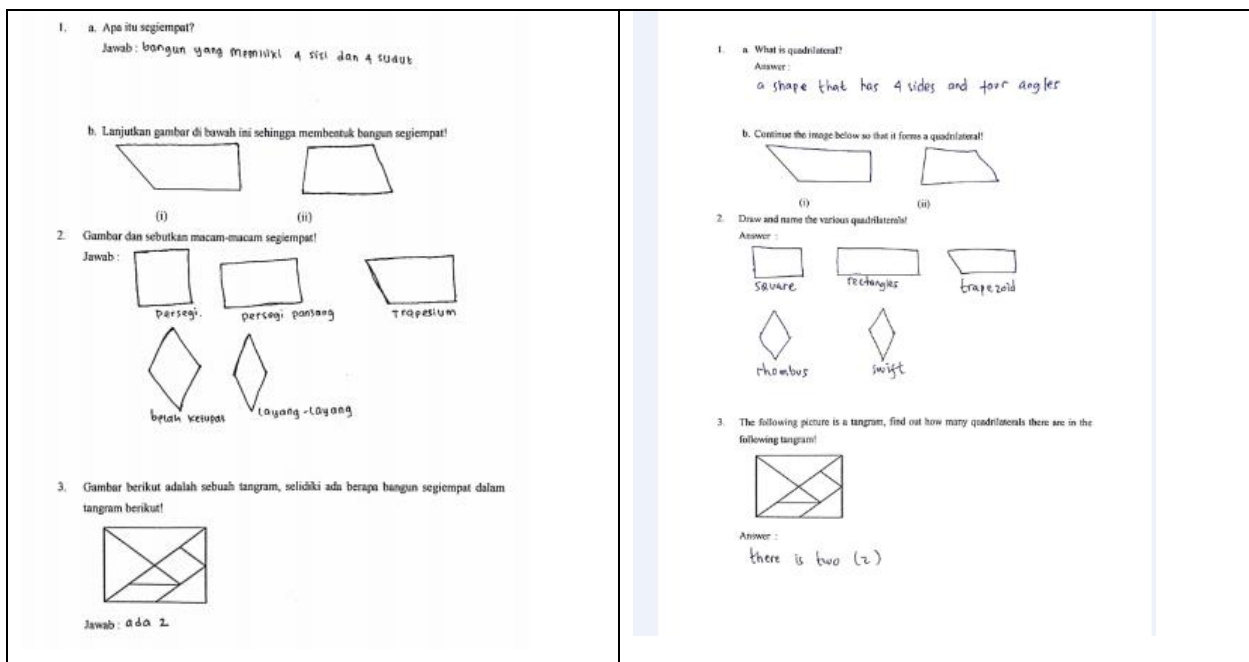


Figure 3. Answer S3

From S3 answer, it can be seen that the students have started to approach the truth of the answer. Students have started to form the model that is in the concept so that what he writes is almost close to the truth. This can be seen when answering question no. 1, S3 was able to answer correctly. The results of the interview with S3 also stated that he knew that the quadrilateral was a shape consisting of 4 sides and 4 angles. For question no 2, S3 was also able to describe several quadrilaterals, although he did not describe all of them. When asked in the doctoral interview process, he only remembers 5 quadrilaterals. In question no 3, S3 answered that there were two quadrilaterals in the tangram. When asked during the interview process why there were only 2 buildings, S3 answered that what he saw was only the outer shape and the one inside was in the shape of a trapezoid.

Furthermore, the answer S4 as a subject that represents transition level II can be seen in Figure 4 below.

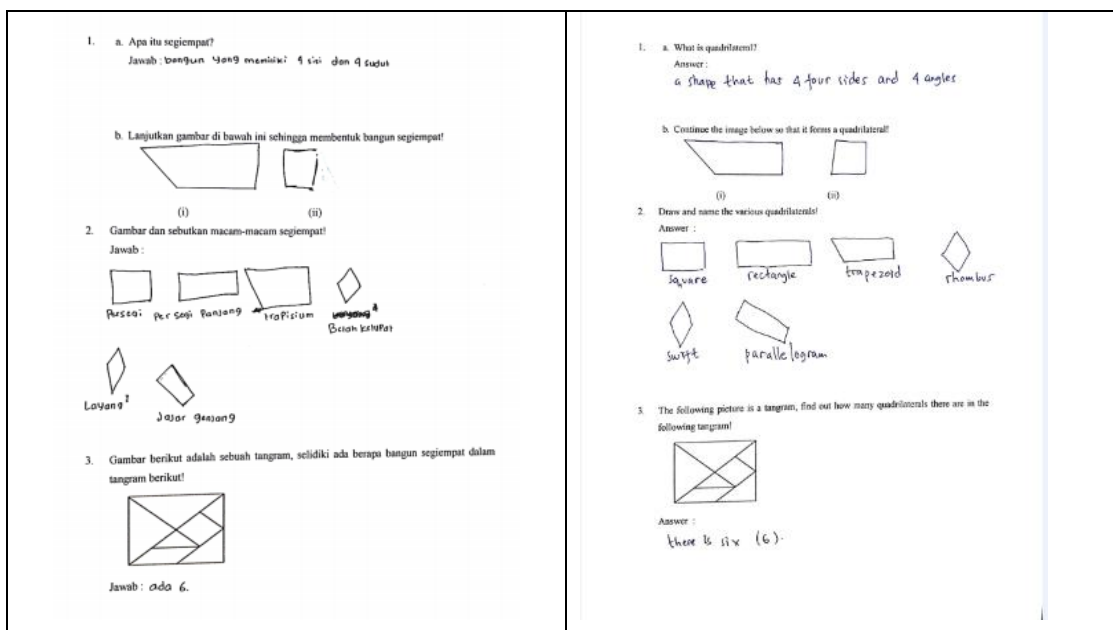


Figure 4. Answer S4

From the S4 answers, it can be seen that students are starting to approach the truth in expressing the concepts that exist within themselves. For question no. 1, the same as previous S4 students had the same answer. During the

interview process, S4 could answer that a quadrilateral is a shape that has 4 sides and 4 angles. For question no 2, S4 has been able to describe various kinds of quadrilaterals from square to parallelogram, and the results of the interview stated that S4 has started to express the concept of a quadrilateral through the pictures. There are 6 quadrilaterals in the tangram. During the interview, S4 stated that he only saw 6 rectangles in the tangram.

Furthermore, the answer S5 as a subject that represents the formal level can be seen in Figure 5 below.

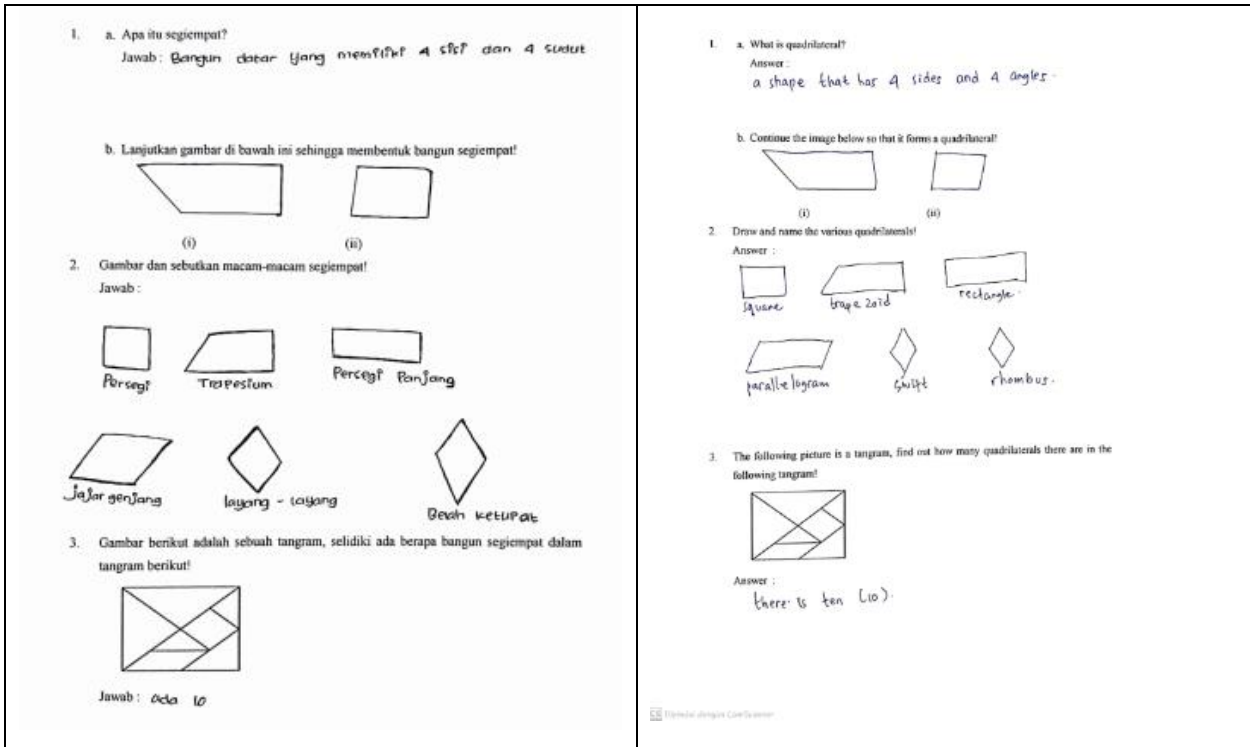


Figure 5. Answer S5

From this S5 answer, it can be seen that this student has been able to correctly answer all the questions given by the researcher. And when the interview process is carried out, S5 can also answer by providing the correct explanation because S5 can express the concept of a quadrilateral perfectly. For question number 1, S5 answered the same as the others that the quadrilateral is a shape that has 4 sides and 4 angles. For question no 2, S5 can also describe 6 quadrilaterals and mention the names of the shapes. When answering number 3, S5 can also answer correctly that there are 10 shapes in the tangram.

4. Discussion

From the answers that have been written by each student representing the mental model level, we can find that for students with initial levels, this S1 does not have a quadrilateral concept in itself or the concept that is in him cannot be recalled when he answers the questions. - questions given. So that S1 cannot clearly describe the quadrilateral concept. If the concept is not embedded very well in students' minds, then the concept will not last long in students' memory (Lunenburg, 2012).

At transition level 1, S2 is able to answer correctly using the concepts that are in him. So when describing the answer is also close to the truth. There is a slight improvement from S1 as a subject with an initial level, S2 who is at transition level 1 already has the concept of a quadrilateral in himself and when needed, S2 can remember this quadrilateral concept even though it is not perfect. Whatever is in the student's concept is related to the memory that exists in the student and the processes that occur in his brain (Gurbin, 2015). Similarly, what happened to S2, the memory was still not perfect when the quadrilateral concept was recalled.

For S3 which is at the synthesis level, this form of mental model can be seen from the S3's efforts in bringing together their assumptions with information from adults about certain concepts. S3 has begun to be able to answer correctly with a concept model that is starting to form perfectly. This synthesis level is a refinement of transition level 1, where the subject of transition 1 has an imperfectly formed quadrilateral concept. Concept information owned by S3 is almost perfect.

For S4 students with transition level II, the form of transition mental model 2 can be seen when individuals have not fully managed to reorganize their theoretical framework and accommodate new information to reflect

adult understanding of a particular concept. Slightly higher than the synthetic level, this transitional level 2 is close to the truth both in answering and describing the answers that exist in his self-concept of quadrilaterals.

Furthermore, S5 students with a formal level, the form of a formal mental model can be seen when they successfully rearrange their theoretical framework and accommodate new information to reflect an adult's understanding of a particular concept. It can be said that S5 has a perfect quadrilateral concept and can use that concept well in answering the questions given. In describing the answers that have been written are also correct and the answers given can be accepted scientifically.

5. Conclusion

In the process of learning mathematics, one thing that is interesting is when students construct a concept and connect one concept to another to build knowledge (Utami & Sa'dijah, 2018). The mental model of students is very important for us educators to know. Mental models have several functions that allow students to store data and apply strategies to obtain results. Students with initial level mental models are not able to store the concepts they have in themselves, so they cannot find strategies to get good results. Likewise with other levels, students have levels in storing the concept of a quadrilateral in each of them. The best in storing concepts are at the formal level so that students with formal levels can find strategies to get the best/maximum results.

Mental models are the stages of a person's thought process. Mental models are formed with people who understand and conceptualize events in the world. In their thinking, mental models are internal representations of the actual situation in people's minds to understand. According to (Ozlan, 2013), mental models are internal representations that act as structural analogies of situations or processes. Its role is to account for the reasoning of both when they try to understand discourse and when they try to explain and predict the behavior of the world. Thus an analysis of mental models can provide us with very valuable information, understanding students' feelings and learning processes.

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