Some more properties onsemipre- Regular Space(SP-T_3)

Suhad K.Hameed, Shahad Jasim
University of Diyala, Collage of Sciences
suhadkareem@uodiyala.edu.iq

Abstract

The aim of this research is to give more properties for the semipre-regular topological space which introduced byGovindappa Navalagi[1]. The property of weakly semipre- Hausdorff was considered and the relationamong(SP-T_3) spaces and other type of a topological space,werediscussed. Furthermore the notation of strongly (SP-T_3) define,as well as somecertain featuresabout the space wasproved.

Keywords : semipre-(closed) sets, g- closed sets, simepre- closure of a set, semipre- Hausdorff space, S-openmapping, semipre-open mapping.

Introduction and Preliminaries

Firstly the symbols of closure of a subset A is *cl(A)*, the interior is *int (A)* , and the complement is represent by A^cin a topological space (X,τ)**.** For the subset with respected to relative topology $\tau_A$we will use the symbols *cl* τ_A and *int* τA as the closure and the interior operators for a subspace $(A, \tau A)$ of a topological space (X, τ). A semipre-open set was introduce by D.Anderijevie [2],A subset A of a space X is called semipre-open if $A \subset cl$ (int (cl(A))),while the complement of semipre-openis semipre-closed. The concept semipre-regular space define by Govindappa Navalagi[1] as a generalize of the concept regular space.

1.Semipre - regular spaces(SP-T_3)

Definition 1.1[1]: The space $X$is said to be a semipre-regular spaceif for every semipre-closed set F,$x \in X$-Fthere is semipre-open sets U , $V$with$F \subset U$,as $x \in V$.
Lemma 1.2:[3] Let (E, τE) be a subspace of a topological space X, assume thatCis a subset of A with C is a semipre-closed subset and $E \subset X$is a semipre-closed. Then C is semipre-closed.
Theorem 1.3: Let, (E , τ) be a closed subspace of (X , τ) ,then every semipre open subset of X is semipre open set onτ.
Proof: Let ,$C \subset E \subset X$, and$E$ is closed set in X ,Csemipre- open in X. Then$C$ is semipre-open in X iff for each semipre-closed set F in X with$F \subset B$,that implies$F \subset int(cl(B))$. Let $E \subset C$,E is semipre-closed set, as$E$ is closed thenits semipre-closed from lemma 1.2. E is semipre-closed on$X$, but Csemipre open on$X$, then$E \subset$ int τ(cl $\tau(C)$) $\subset$ int τ(cl $\tau(C)$)$\cap E \subset$ int τ(cl $\tau(C)$)$\subset$ int τ (cl $\tau(C \cap E)$)$\subset$ int τ((cl $\tau(C) \cap$ cl $\tau(E)$)$\subset$ int τ((*cl* $\tau(C) \cap (E)$)$\subset$ *int* τ((*cl* $\tau(C)$) therefore C is semipre open on τ.

Definition1.4:[2] The semipre- closure of a set B, represented by $cl_{sp}(B)$, is the intersection of all semipre closed sets containing B.

Theorem 1.5: Let X be a topological space , X is semipre-regular space iff for each $x \in X$ and each open set Ucontain in a finite base $\beta$with$x \in U$, there issemipre open set E with$x \in V$,*cl(int(E))*$\subset U$.
Proof : $x \in U$, so $x \notin U^c$,U^c is closed, therefore there is a disjoint semipre open setE_1,$E_2$with $x \in E_1$ and $U^c \subset E_2$, we get $E_1 \subset (E_2)^c \subset U$. Moreover, $(E_2)^c$is semipre closed set ,U is open set therefore U is

g-open set, sofrom thecharacterization of semipre closed subset we get cl(int$(E_1)) \subseteq$ cl $(\text{int}(E_2)^c) \subset U$. For substantiationsuppose $x \in X, G$ be Closed subset of X with$x \notin G$nowclaim that U_1, $U_2, \ldots \ldots U_k \in \beta$ such that $x \in \bigcap_i^k = 1 \ \ Ui \subset G^c$. Then there is a semipre open set $E_i$with $x \in E_i$, cl(int$(E_i)) \subset U_i$,andi =1 ,2 ,…,n . So that semipre open set $E_1 = \bigcap_{i=1}^n$ Ui ,$U_2 = (\bigcap_{i=1}^n \ cl \ \ int(\text{E}i))^c$ are separated sets with$x \in E_1, G \subset (\bigcap_{i=1}^n \text{U}i)^c \subset (\bigcap_{i=1}^n cl\, int(\text{Ei}))^c = E_2$.

Definition1.6:A space X is said to be weaklly semipre- Hausdorff ,if for everyvarious points $a, b \in X$ with$a \notin cl\ (Ub)$,with Ubsemipre open set withb $\in U$b,so there is semipredisjoint open subsets U, $W$with$a \in U$,and $b \in W$.

Theorem1.7: Every semipre- regular is weakly semipre- Hausdorff .
Proof: suppose X is(SP$-T_3$) , $x, y \in X$with $x \neq y$,let $x \notin \text{cl}(U_y)$ withU_y is asemipre open set consisting$x$. We have$X$ is semipre regular ,so that there is semipredisjoint open sets V_x , $V_y$where $x \in V_x$, $y \in \text{cl}(U_y) \subset V_y$,then$X$ be a weakly – Hausdorff space . In case $y \notin \text{cl}(U_x)$ it's the same Way of proof .

Theorem 1.8:A semipre-T_1, and semipre-regular is semipre-T_2 .
Proof: Assume that X is semipre-T_1,semipre-regular, then every singleton set$\{x\}$ is sp-closed , $\forall x \in X$, and$\{x\}$ is semipre-closed subset of X , let a be a point on $X/\{x\}$, so that $x \neq a$. From the fact of regularity there exist a disjoint semipre-open sets E_1, E_2 with $\{x\} \subset E_1$,$a \in E_2$, that mean $x \in E$, $a \in E_2$, thus $X$is semipre-T_2.

Definition1.9: [4] A space X is said to be semipre- Hausdorff if for each points $x, y \in X$, $x \neq y$there is semipre-open disjoint sets V_x and V_y containing x and y respectively .
We will give an example explain that the quotient topology of the semipre regular space could be semipre Hausdorff space.

Example 1.11: Let,X is a(SP-T_3),G is a closed subset of X. Declare$\Re$ as relation on a space X ,in wich x$\Re$y iff either x,ybelong to$G$ or $x,y \notin G$,in this case$x=y$. Its clear that $\Re$ is an equivalents relation, in order toexplain the set$X/\Re$ with quotients topology is semipre Hausdorff claim $[x],[y] \in$ X /$\Re$where$[x] \neq [y]$,its obvious either$x$ or $y \in G$. let $x \in G$; y $\notin G$, while$G$ is closed semipre regular space X thus there is semipredisjoint open subsets U and $V$where$[x] \subset G \subset U$ also, $[y] \subset V$and that meant X is semipre Hausdorff.

The example below explain that the quotientt space of semipre regular space doesn't necessary a semipre regularly.

Example 1.12: Taking the real number onthe usual topology ,define Q: R $\rightarrow$F, withF ={a,b,c} declareby following : $Q(x)=a$ if x >0 , $Q(x)=b$ if $x<0$,and $Q(x)=c$ if $x=0$ .Therefore the quotient topology on Aisτ={ϕ,{a},{b},{a,b},{a,b,c}} then Fona topology τdoesn't semipre regular space while R is a semipre regular.

Definition 1.13:[5] A mapping$f: X \rightarrow Y$ is said to be semipre irresolute if the inverse image of each semipre open set in Y is semipre open set in X.

Theorem1.14:A function$f$: $X \rightarrow Y$is a closed semipre irresolute injective if$Y$ is semipre regular space then X is semipre regular space.

Proof. Suppose that $x \in X$,F be any closed subset of $X$with x$\notin F$, then $f(F)$ is closed in $Y$where$f(x) \notin f(F)$ and since Y is semipre regular space, then there exists semipre- disjoint open sets W,U with $f(F) \subset W$ and $f(x) \in U$. implies that $F \subset f^{-1}(f(F)) \subset f^{-1}(W)$ and x$\in f^{-1}(U)$ furthermore$f^{-1}(W) \cap f^{-1}(U) = \phi$, but the function$f$ is semipre irresolute thus $f^{-1}(W)$, $f^{-1}(U)$ are semipre open subsets of X.

Definition 1.15. [5**]** A function $f: X \to Y$ is said to beS- open if the image of everysemipre-open set in X is semipre open set in Y.

Theorem1.16. Letthe mapping f: $X \to Y$is a bijective, S- open and continuous mapping, and X is semipre regular space, thus$Y$ is semipre- regular space.

Proof. Suppose that $A$is a closed subset in Y with y $\notin A$, then $f^{-1}(A) \subset X$ and f^{-1}(y) $\notin f^{-1}(A)$. Since $f^{-1}(A)$ is closed in X, X is semipre regular space therefore there exists semipredisjoint open sets W and U such that $f^{-1}(A) \subset W$ and $f^{-1}(y) \in W$, but f is S – open thus it is clear that $f(W)$ and $f(U)$ are semipre with$f(W) \cap f(U) = \emptyset$ in Y containing A and y respectively.

2. Strongly semipre regular

Definition 2.1:A space is said to be stronglysemipre-regular space if for each $y \notin S$, Ssemipre closed set then there is two open sets W and $U$with $y \in W$ and $S \subset U$, and $W \cap U = \emptyset$.

every strong semipre-regular is mildly-regular but the convers need not necessarily true as shown in the example below .

Example 2.2: Determine X={a,b,c} with topology τ={X,ϕ,{a},{b,c}} is semipre-regular but not strong semipre-regular

Theorem.2.3: For a topological space(X,τ) the followingsstatement are equivalently:

(i) Xbe a strongsemipreregular space.

(ii) For every$y \in W$, Wisa semipre open, there is an open set $U$with $y \in U \subset cl(U) \subset W$

(iii) For every$y \in X$,semipre closed set W, with $y \notin W$, there is an open set $U$with $y \in U$ and $cl(U) \subset W^c$.

Proof: (i)$\to$(ii) we have U^c is semipre closed and y$\notin W^c$ and then there exist U and $G$with$y \in U$ and $W^c \subset G$ where G and U are disjoint open sets and it clear that $cl(U) \subset G^c$thus cl(U)$\cap W^c \subset cl(U) \cap G = \phi$, thus cl $(U) \subset W$.

(ii) $\to$(iii) We use (ii) on y and W^c to find open set $U$with $y \in U \subset cl(U) \subset W^c$

(iii) $\to$(i) let $y \in X$, and W any semipre closed subset of $X$with$y \notin W$, then its easy to find two disjoint open sets U ,and (cl $(U))^c$with$y \in U$ and $W \subset$(cl $(U))^c$.

Definition2.4: [6] A mapping f:$X \to Y$ is calledS- closed if each image of semipre closed subset in X is semi closed in Y.

Theorem 2.5: If $f: X \to Y$ is injective, continuous, S- closed fmapping and Ybe a strongelysemipre regular then X is strongelysemipre regular.

Lemma 2.6:[3**]** Let $A \subset B \subset X$, and A is semipre closed subset with respect to relative B,and B is g-open and semipre closed set with respect to relative topology X, then A is semipre closed relatively to X.

Theorem 2.7: For a semipre regular space$X$,the subset B is a g-open and semipre close subspace off Xso that$B$ is semipre regular space.

Proof: Suppose*FB* be a semipre close set relatively to *B*,$x \in B$with $x \notin F_B$. From lemma (2.6)F_B. is semipre close set relatively to *X*,then there isan open sets *W*,*U*with$y \in U$,$F_B \subset W$ where ($W \cap U=\emptyset$). We getthatthe sets $B \cap W$, and $B \cap U$ are disjoint open sets relatively to *B*such that $B \cap W$containing F_B,and $B \cap U$ containing *x*.

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