A Comparative Study and Statistical Analysis of Classical SCA and Hybrid Genetic Sine Cosine Algorithm

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Abstract:

This paper puts forward the comparative study of the Classical Sine Cosine Algorithm and the newly introduced Hybrid Genetic Sine Cosine Algorithm. Though the existing literature proves that Sine Cosine Algorithm has sufficient capacity to explore the region of search space; however similar to other algorithms, it encounters a few complications like the stagnation of local optima, a less convergence rate with missing out of exact solutions. Thus, an advanced version of classical SCA is presented and is described as a Hybrid Genetic Sine Cosine Algorithm (HGSCA). In the proposed algorithm, the local state mechanism is hybridized with the global best state in the search equations to decide the region of search space around the global best position of a solution. In the search equation, global best is also combined with random steps to provide the statistics of the best position preserved in the memory of candidate solutions. A greedy selection mechanism and crossover with the personal best state reduce the overflow of diversity. An experimental setup that is established to execute both the algorithms on a benchmark Himmelblau function and the statistical analysis done proves the supremacy of HGSCA over Classical SCA.

Keywords: Optimization, Hybrid Genetic Algorithm, Evolutionary operators, Non parametric tests, Statistical analysis

1. **Introduction:**

Genetic algorithms focus on the simulation of the natural evolutionary process of nature which can surpass standard optimization techniques in certain real-world problems. GAs are computerized algorithms for optimization problems that are focused on the mechanism of biological evolution. These algorithms proceed on the basis of biological inheritances on a random population of string structures that represent the variables of the problem. As a global search technique, Genetic algorithms work effectively. However, they could usually take a somewhat extensive duration to arrive at global optimal value. Local search methods can be integrated with Genetic Algorithms for their effortless functioning and accomplishment. The hill climbing and heuristic crossover operators can be combined with GA's for resolving many problems. El-Mihoub et al manifested the outcome of the probability of local search on the population size of Genetic Algorithms. There exists a probability of replacing the genetic operators i.e., reproduction, crossover, and mutation with other search techniques. Headar and Fukushima used a simplex crossover instead of a standard crossover. Leng developed an algorithm in which penalties from the guided local search are utilized in fitness functions for enhancing the execution of GAs.

Sine cosine algorithm (SCA) is a newly established population-related method for solving global optimization problems. This algorithm incorporates the features of trigonometric sine and cosine functions in the process of search. In a number of experiments performed, this algorithm has revealed its competence with regard to exploration and exploitation.

In classical SCA, the entire algorithm simply contracts into premature convergence when the globally optimal solutions get trapped into the local optimum. Thus, to condense the probability of the algorithm falling into the local optimum, the leading part of the improved individual solutions probably prevailing close to the optimum value must be employed. The random individuals which are nearer to the optimal solution are utilized to substitute the recent optimum to lead the search of the algorithm. Thus, to alleviate the shortcomings, the Classical Sine-Cosine algorithm is hybridized with Genetic Algorithm operators, and a local search technique is employed which is termed as Hybrid Genetic Sine-Cosine Algorithm. restricts the likelihood of the algorithm skipping out of the local optimal value.

The remaining part of the paper is organized as follows: Section-2 aims to discuss the hybrid genetic algorithm. Section-3 delivers a short reference to the classical Sine Cosine Algorithm. In Section-4, we will discuss our newly introduced Hybrid Genetic Sine-Cosine Algorithm, evolutionary operators, and local search mechanism employed. In Section-5, the Experimental study is conducted to compare HGSCA with Classical SCA by testing on solving the Himmelblau function optimization problem. In Section-6, a statistical analysis is carried out to prove the supremacy of HGSCA over Classical SCA. Section-7 concludes the work of the paper.

2. Hybrid Genetic Algorithms

Genetic algorithms are based on the biological theory of evolution devised by Charles Darwin in the value of the objective function determination and evaluate the fitness of each member of an individual class for a specific optimization problem. Many other optimization techniques can be smoothly hybridized with Genetic Algorithms to upgrade their execution. Initially, Moscato invented Hybrid Genetic Algorithms which are generally referred to as population-based Genetic Algorithms. The principal primacy of hybridized GA as compared with other methods is its superior competence and assurance of feasible and optimal solutions. Many local search algorithms such as Memetic algorithm, Lamarkian evolutionary algorithms, Baldwinian algorithms, and genetic local search were integrated with GAs to resolve the problem of sampling potential. The basic methodology of HGA is as follows:

1. Define the objective or fitness function and genetic operators i.e., population size, reproduction operator, the probability of crossover, and probability of mutation.
2. Randomly generate the preliminary population as the present parent population.
3. Assess the objective function for every discrete value in the premier population.
4. Create a next-generation population with the help of genetic operators stated above.
5. Calculate the objective function for every individual in the newly generated population.
6. Perform a local search technique on each and every individual of the newly generated population.
7. Evaluate the fitness function of each newly obtained local solution and replace the offspring if there exists an improved local solution.
8. The newly generated and improved offspring/parent population replaces the individuals from the current population.

3. Sine Cosine Algorithm

Sine Cosine Algorithm (SCA) is a population-based meta-heuristic algorithm designed by Mirjalili in 2015 for the purpose of solving optimization problems. In this algorithm, a mathematical model constructed on the basis of trigonometric sine and cosine functions generates numerous original random solutions and involves those results to oscillate towards or away from the elite solution. Several arbitrary and adaptive variables are also unified with this algorithm to underline exploration and exploitation of the search space in diverse stages of optimization. SCA begins with a collection of random candidate solutions, thereafter each solution restores its position by using the following equations –

$$x_{i,t+1} = x_{i,t} + A \sin(r_1)\left|Cx_{best} - x_{i,t}\right| \quad r < 0.5$$

$$x_{i,t+1} = x_{i,t} + A \cos(r_1)\left|Cx_{best} - x_{i,t}\right| \quad otherwise$$

where $x_{i,t}$ and $x_{i,t+1}$ signifies the ith solution vector at t^{th} and $(t+1)^{th}$ iteration respectively. x_{best}

represents the fittest solution from a given set of candidate solutions and r is a uniformly distributed random number in the interval (0, 1). r_1 is a vector which rules the path of the movement of the current solution that may be in the direction of x_{Best} or away from x_{Best}. The vector C gives weight to x_{Best} that stresses upon exploitation ($C < 1$) and exploration ($C > 1$) and also assists in escaping the premature convergence at the final stages of generations. The purpose of vector r is to assist in shifting from sine to cosine functions and vice versa. The parameter A is a random vector that chooses the part of the search space around the current solution. This section of search space may lie inside x_{Best} and $x_{i,t}$ or outside them. The parameter A helps in exploration and exploitation of the region of search domain and also in sustaining an appropriate equilibrium among them. Mathematically, the vector A can be stated as below –

$$A = 2e^{1/2}$$

where, t represents the iteration count and T signifies the upper limit on the number of iterations which is a pre-established concluding criterion for SCA.

The stages of the Sine Cosine Algorithm are given below:

Classical Sine Cosine Algorithm (SCA)

1. Generate the initial set of random uniformly distributed solutions inside the search domain
2. Compute the fitness of each solution vector
3. Initialize the parameters A and T (upper limit on the number of iterations)
4. Select the fittest solution x_{Best} from the set of solutions
5. Initialize the iteration enumeration $t = 0$
6. **while** $t < T$
7.   Update each solution vector with the help of the above equations
8.   Compute the fitness of each updated solution vector
9.   Update the best solution x_{Best}
10.   Update the parameter A
11. $t = t + 1$

12. **end of while**

13. Return the fittest solution x_{Best}

4. Hybrid Genetic Sine Cosine Algorithm

Even though the Sine Cosine algorithm investigates the search domain very proficiently, it occasionally encounters the problem of an excess of multiplicity. In several instances, it suffers from some major complications as missing of accurate results and stagnation of local optima and consequently, an up-gradation is essential in the exploration approach of basic SCA. The excess of multiplicity escapes the exact solutions of the problem if an appropriate equilibrium among exploration and exploitation is not formulated in the algorithm. Since coefficient A contributes to the exploration of the domain of the search area, thus the solutions are moved away from the current position. Similarly, the weighing vector C supports the exploration all through the search sequence. Thus, throughout the course of the search, in every iteration, the results miss their individual characteristics and constantly move to a new position. The excess of diversity i.e., exploration and missing out of characteristics of the obtained results outflows the exact solutions. But these escaped results might have a probability to offer improved locations in the subsequent iterations by exploiting the domain in the neighborhood of the current state of the solution. Hence, to decrease the aforementioned problems from basic SCA, a few alterations are proposed in the algorithm HGSCA to continue with the search. Here, the Classical Sine-Cosine Algorithm is hybridized with evolutionary operators i.e. Crossover and Mutation along with the local search mechanism.

The crossover operator applied here is **Logistic Crossover.** The logistic distribution has extensive tails than a normal distribution consequently it is more reliable with the fundamental statistics and delivers an improved perception into the probability of happening of extreme events. To use Logistic Distribution, two parents p_1 and p_2 are taken to produce two offsprings y_1 and y_2 in the following equations:

$$y_1 = p_1 * \log(x) + p_2 * (1-\log(x))$$
$$y_2 = p_2 * \log(x) + p_1 * (1-\log(x))$$

The mutation operator, namely **Position-based Exponential Mutation (PEM)** has been applied in our current study. The name of the proposed mutation operator recommends that it is directed by the positional information of the variables.

By employing the recommended process, the following steps are depicted for locating the desired information for the variables:

Step 1: At the beginning of each iteration, the average value of all the variables existing in the population is evaluated.

Step 2: At this point, the evaluated average value of the variable is coordinated with its current value already existing in the population solution. At this instant, if the average value of the variable is found to be greater than the value of the parent solution, then the mutated solution proceeds positively in the direction of the parent solution. Else, this information leads to its negative counterpart.

We compute two different mutation perturbation values ($\beta_1 and \ \beta_2$) using the following equations:

$$\beta_1 = e^{r^2} \times e^{\left(r - 2/r\right)} \ldots\ldots (1)$$

$$\beta_2 = e^{\left(r - r^2\right)} \times e^{\left(r - 2/r\right)} \ldots (2)$$

where r is a non-zero uniformly distributed random number in the interval (0,1).

A mutated solution y_m is generated from a parent solution y_p depending upon the nature of positional information as follows:

$$y_m = y_p \pm \left(\beta_1 \text{ or } \beta_2\right)\left|y_{pmean} - y_p\right|$$

To hybridize the algorithm with local search, the **Nelder-Mead Simplex** technique is employed.
The sine-cosine algorithm approach in the vicinity of the optimum solution is

$$x_{i,t+1} = x_{i,t} + A \sin\left(r_1\right)\left|Cx_{best}\left(\lambda x_i + \left(1 - \lambda\right)x_{i+1}\right) - x_{i,t}\right| \quad r < 0.5$$

$$x_{i,t+1} = x_{i,t} + A \cos\left(r_1\right)\left|Cx_{best}\left(\lambda x_i + \left(1 - \lambda\right)x_{i+1}\right) - x_{i,t}\right| \quad otherwise$$

The variations which have been projected in HGSCA are as follows

Algorithm:

1. Generate the initial set of uniformly distributed random solutions inside the search arena
2. Compute the objective function value of each solution vector
3. Apply the Current Optimum Opposition-based Learning Mechanism on the initial population within the search space.
4. Select the fittest solution points by taking the union of all the population points.
5. Initialize the parameters T and A
6. Choose the fittest individual x_{Best} from the population of individuals
7. Initialize the iteration count $t = 0$
8. **while** $t<T$
9. **for** each individual solution
10.   Update the position by using the above-updated HGSCA equations
11.   Employ the crossover operator between the best individuals and update the HGSA solution as defined in the crossover process
12.   Compute the objective function value of the updated solution vector
13.   Apply mutation operator as described in mutation equation
14.   Perform the greedy selection mechanism
15.   Update the best solution x_{Best}
16.   Apply Nelder Mead Simplex Method for the local search.
17. **end of for**
18. $t = t + 1$
19. **end of while**
20. Return the fittest solution x_{Best}

5. Experimental setup

We execute the above-stated hybrid algorithm HGSCA and Classical SCA on **Himmelblau Function** and study the comparative performance of both the algorithms.

$$\text{Minimize } f(x_1, x_2) = \left(x_1^2 + x_2 - 11\right)^2 + \left(x_1 + x_2^2 - 7\right)^2 ; 0 \le x_1, x_2 \le 6$$

Classical SCA

Iteration 1:

x$_1$	x$_2$	f(x)
1.6889	0.0396	93.9310
1.5022	0.4779	96.0851
3.4854	0.0871	13.8246
3.026	0.6372	14.1852
1.5992	1.3439	63.3130
0.2278	2.0567	85.5198
2.0133	0.3635	66.9043
2.3605	3.1621	33.8574
2.537	0/3869	36.0497
2.2015	1.4345	29.7794
2.2179	2.7871	19.7644

Mean = 50.2922

Median = 36.0497
Standard Deviation = 30.3721
Best = 13.8246
Worst = 96.0851
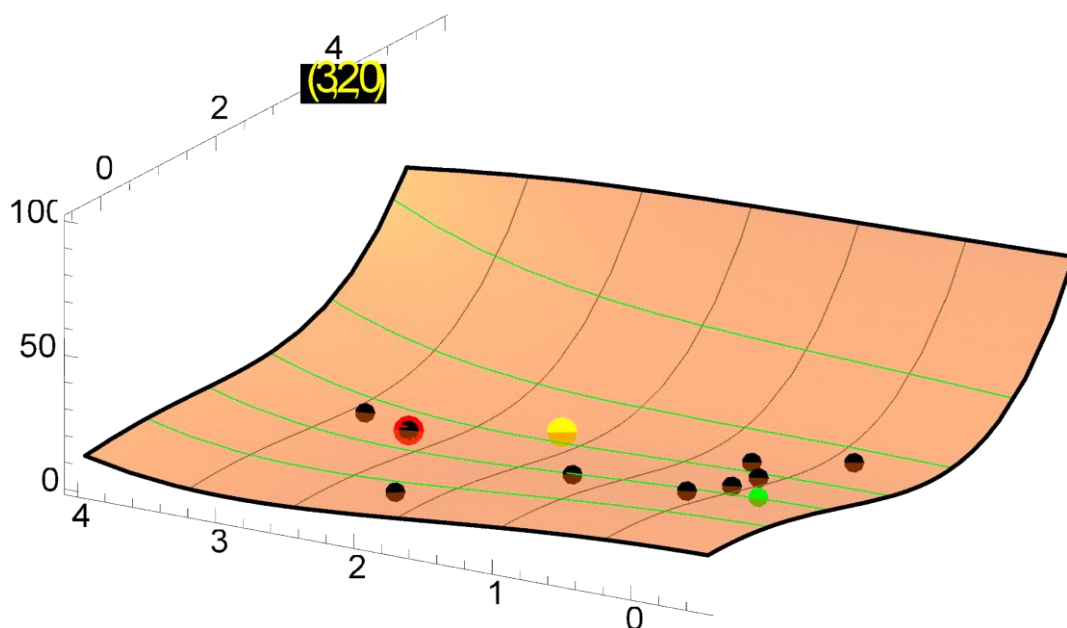
Iteration 2:

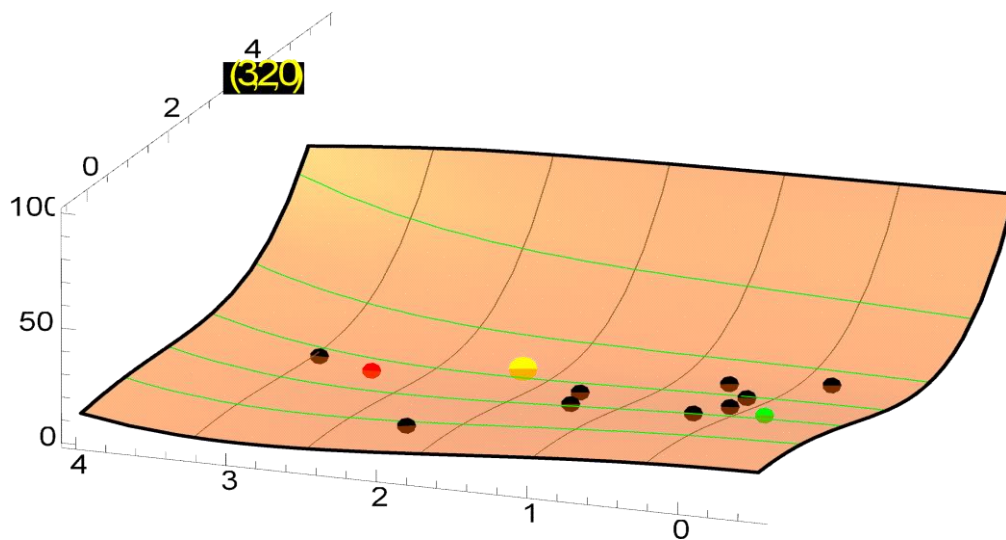| x_1	x_2	f(x)
1.826	0.078	84.2809
1.6449	0.5031	86.7332
3.4025	0.0502	13.3173
3.1228	0.6576	12.2151
1.739	1.3448	55.8910
0.4089	2.0791	81.7731
2.1407	0.3921	58.4466
2.4774	3.2178	36.7129
2.6486	0.4148	30.2126
2.3232	1.4381	24.1494
2.3391	2.8315	18.5404

Mean = 45.6611
Median = 36.7129
Standard Deviation = 27.6872
Best = 12.2151
Worst = 86.7332

Hybrid Genetic Sine-Cosine Algorithm
Iteration 1:

x_1	x_2	f(x)
2.8307	2.1979	1.0604
1.9921	0.6633	61.4205
3.6713	0.3364	7.9233
3.120	0.657	12.2615
2.0394	1.4093	38.3492
1.6135	2.4247	35.9065
2.2389	0.5828	48.758

2.4002	2.5479	10.8218
3.1583	0.6025	12.2799
2.5029	1.5289	14.9459
2.3323	2.2876	11.0308

Mean = 23.1598
Median = 12.2799
Standard Deviation = 18.6763
Best = 1.0604
Worst = 61.4205
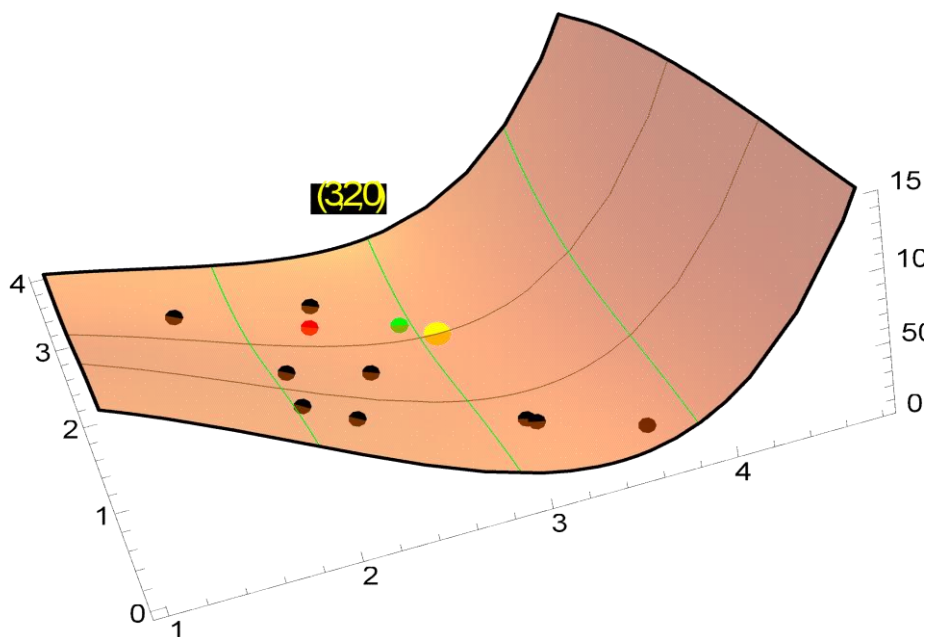
Iteration 2:

x_1	x_2	$f(x)$
2.9947	1.9645	0.0258
2.8485	1.6109	4.0487
3.0033	0.9345	10.8492
3.1255	1.1103	6.9934
2.6258	1.6467	8.8083
2.4267	2.2720	8.4071
2.7201	0.9988	17.5452
2.7935	2.0587	1.2953
3.1433	1.0739	7.3107
2.8894	1.8586	1.0591
2.7629	2.1913	1.6997

Mean = 6.1857
Median = 6.9934
Standard Deviation = 5.0213
Best = 0.0258
Worst = 17.5452
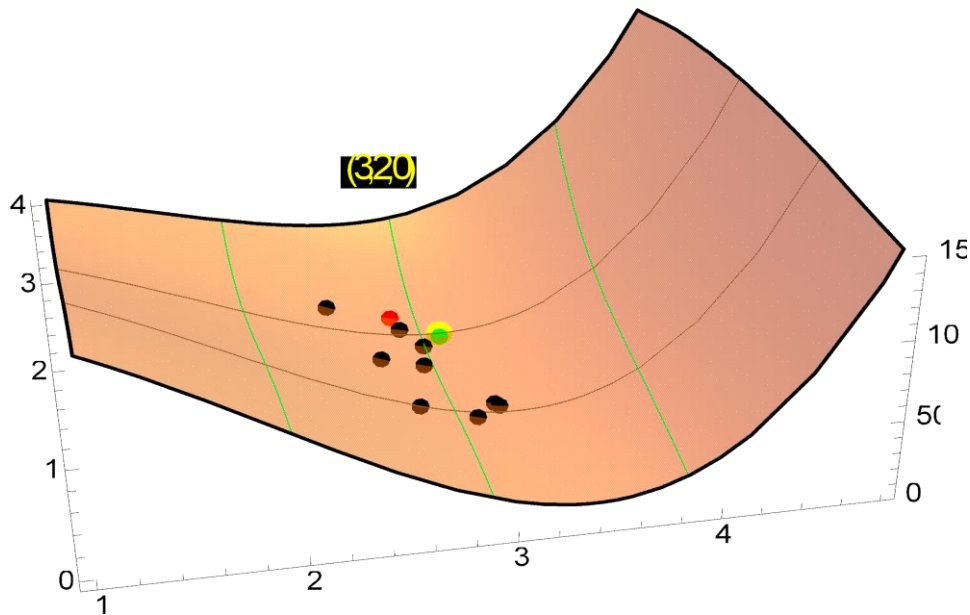
6. Statistical Analysis of results:

The statistical analysis of the results plays a vital role in comparing the specified algorithms. Thus, in this section, a non-parametric test is employed between classical SCA and HGSCA to observe the upgradation in HGSCA. A non-parametric statistical test is selected since it does not involve statistics about the distribution of the data set. Further, in the non-parametric tests, the measure of central tendency is median. To appraise the operation of any search algorithm, the median value is a relatively superior statistical measure. The test has been performed at a 2.5% (one-tailed) level of significance and the obtained results are reported below. The non-parametric test used in our study is Mann-Whitney U-test.

Mann and Whitney U-test

Mann and Whitney's U-test or Wilcoxon rank-sum test is the non-parametric statistical hypothesis test which helps in analyzing the deviations between two independent samples of a categorical and statistical data type where the variables have ordered categories and the distances between the categories are not known. For applying this test in our study, we have specified two randomly drawn samples and we want to validate if these two samples are from the same population or not.

Steps for Performing the Mann Whitney U test:

1. Collect two samples i.e. sample 1 and sample 2.

2. Consider the first observation from sample 1 and relate it with observations in sample 2. Calculate the number of observations in Sample 2 that are smaller than that and equal to it. For, example, if 5 observations in sample 2 are smaller than the first observation in sample 1, then the rank for this sample point is 6.

3. Reiterate Step 2 for all observations in sample 1

4. Add together all the values obtained from Steps 2 and 3 individually for both the samples which is termed as Rank sum

5. Compue the value of U statistics by means of the following formula

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

where:

n_1: number of points in sample 1

n_2: number of points in sample 2

R_1: Rank sum of sample 1

R_2: Rank sum of sample 2

Define the test statistic U= Min. (U_1, U_2)

Next, observe the critical values in the table with reference to the sample points n_1 and n_2.

If calculated U $\leq$ the tabulated U, reject the null hypothesis. Else, accept the alternative hypothesis.

Results:

We test the significance of our algorithm by using the above demonstrated Mann- Whitney U-test and compare the results of f(x) obtained from both the Sine-Cosine Algorithm and Hybrid Genetic Sine-Cosine Algorithm for both the iterations.

First, we set Null Hypothesis H_0: there is no significant difference between the two samples against the alternative hypothesis

H_1: Sample 2 obtained from HGSCA gives better results than sample 1 obtained from SCA as is significant from the median values

For Iteration 1:

SCA	HGSCA	Rank(SCA)	Rank(HGSCA)
93.9310	1.0604	21	1
96.0851	61.4205	22	17
13.8246	7.9233	7	2
14.1852	12.2615	8	5
63.3130	38.3492	18	15
85.5198	35.9065	20	13
66.9043	48.758	19	16
33.8574	10.8218	12	3
36.0497	12.2799	14	6
29.7794	14.9459	11	9
19.7644	11.0308	10	4

Here, $U_1 = 121 + \frac{11*12}{2} - 162 = 121 + 66 - 162 = 25$

$U_2 = 121 + 66 - 91 = 96$

U= Min $(U_1, U_2) = 25$

From the statistical table (given below), we can see that the value of U at 0.025(one-tailed) level of significance is 30.

Here, the calculated value of U i.e.25 is less than the tabulated value i.e.30

Therefore, we reject null hypothesis and accept alternative hypothesis that Sample 2 obtained from HGSCA gives better results than sample 1 obtained from SCA as is significant from the median values.

For Iteration 2:

SCA	HGSCA	Rank(SCA)	Rank(HGSCA)
84.2809	0.0258	21	1
86.7332	4.0487	22	5
13.3173	10.8492	12	10
12.2151	6.9934	11	6
55.8910	8.8083	18	9
81.7731	8.4071	20	8
58.4466	17.5452	19	13
36.7129	1.2953	17	3
30.2126	7.3107	16	7
24.1494	1.0591	15	2
18.5404	1.6997	14	4

Here, $U_1 = 121 + \frac{11*12}{2} - 185 = 2$

$U_2 = 119$

$U = \text{Min} (U_1, U_2) = 2$

From the statistical table (given below), we can see that the value of U at 0.025 (one-tailed) level of significance is 30.

Here, the calculated value of U i.e. 2 is less than the tabulated value i.e. 30

Therefore, we reject the null hypothesis and accept the alternative hypothesis that Sample 2 obtained from HGSCA gives better results than sample 1 obtained from SCA as is significant from the median values.

										n_2										
n_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	0	0	0	0	1	1	1	1	1	2	2	2	2
3	-	-	-	-	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4	-	-	-	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5	-	-	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6	-	-	1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7	-	-	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8	-	0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9	-	0	2	4	7	10	12	15	17	21	23	26	28	31	34	37	39	42	45	48
10	-	0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11	-	0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12	-	1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13	-	1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76

14	-	1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15	-	1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16	-	1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17	-	2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	
18	-	2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	
19	-	2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	
20	-	2	8	14	20	27	34	41	48	55	62	69	76	83	90	98	103	112	119	

Statistical Table (one-tailed at .025; two-tailed at .05) Critical one- and two-tailed values of U for a Mann–Whitney Independent Groups test, where n_1 and n_2 are the number of entries in two samples. (U is significant if it is less than or equal to the table value)

7. **Conclusion:**

To maintain the leading attributes of the solution and to exploit the regions of the search area, crossover and mutation operators are performed in Classical SCA. To counter the surplus diverseness, a greedy selection is employed between the preceding and recent population of solutions. A local search mechanism called Nelder-Mead Simplex is employed to upgrade the execution of the algorithm. The introduced alterations in the procedure of the optimal search approach decrease the complications of the basic Sine Cosine Algorithm. This new hybrid form of SCA with Evolutionary operators improves the exploitation capability of results and lessens the excess of exploration existing in the search equations of classical SCA. Both the algorithms are executed on testing the minima of benchmark Himmelblau function and the obtained results show substantial improvement in the value of the function in the case of HGSCA and its convergence rate. The corresponding 3-D figures show how the points get clustered around the true minimum of the function as the iteration count proceeds further in HGSCA as compared with Classical SCA. The statistical analysis of the results conducted in the paper, recommends that the proposed algorithm can be used over classical SCA as it has outperformed classical SCA.

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