MHD Third Grade Thin Film Fluid Flow Over a Vertical Belt in the Presence of Thermal Radiation and Mass Transfer under Slip boundary conditions.

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Abstract

In the present paper, we have discussed the electrical conducting thin film flow of third grade fluid due to vertical belt in appearance of thermal radiation and mass transfer. The MHD and heat transfer analysis are taken into account. The non-linear conservative equations of presents problems are solved analytically under the appropriate boundary conditions by using traditional perturbation techniques. The results obtained are displayed graphically with respect to different pertinent parameters for flow velocity, temperature variation, profile of skin friction and concentration distribution.

Keywords:

Third grade fluid, Lifting, Drainage, Magnetohydrodynamics(MHD), Heat transfer, Thin film, Thermal radiation, Chemical reaction, Perturbation method.

1 Introduction

Recently, non-Newtonian fluids have gained considerable most importance because of their applications in mathematics, chemical engineering and production of industry. Examples of non-Newtonian fluids includes lubricants performance, transpiration cooling, plastic manufacturing, processing of food, fiber and wire coating, biological fluids movement, heat pipes, gaseous diffusion etc. There are different subdivision of non-Newtonian fluids, especially important classification of third grade fluid equations based on the powerful theoretical foundation. In literature, the survey of third order fluid flow through numerous geometrical planes has accepted huge concentration from scientists. Following, Schowalter (1) developed the different types of non-Newtonian fluid in theory of fluid mechanics. Rajagopal et.al., (2) investigated the exact solution for non-Newtonian fluid flow with infinite porous plate. Farooq et.al., (3) discussed the combined effects of slip boundary conditions and magnetic field on the thin layer liquid flow on moving vertically to solved velocity exactly. Nasir et.al., (4) analyzed thin layer flow third order fluid on a oscillating perpendicular belt under the effect of magnetohydrodynamics using different method. Hammed et.al., (5) interpreated MHD flow of an electrically conducting non-Newtonian fluid on a moving perpendicular belt with uniform properties. Siddiqui et.al., (6) evaluated non-Newtonian thin layer flow of a moving perpendicular belt with two types of fluid is taken as constant. Laminar flow of a third grade liquid through a channel of flat permeable is assumed, the rate of injection of the fluid at one boundary is equal to the rate of suction at the other boundary is estabilished by Ariel (7). Khan et.al., (8) investigated slip influence condition on a thin layer flow of a third order fluid with two types of moving channel under suitable differential method. Effects of slip velocity and thermal jump boundary conditions on non-Newtonian flow and transfer of heat in the channel have been analyzed by Jingzhu (9). Ajadi et.al., (10) studied flow and transfer of heat of power law fluid over a flate plate with thermal convective and slip boundary conditions using similarity techniques transformation. Ellahi et.al., (11) contemplated the effects of slip boundray

conditions on non-Newtonian flows of an oldroyd fluid developing a model with flow speed of horizontal channel. Mahmoud et.al., [12] discussed surface effects of slip and absorption generation heat on the flow of non-Newtonian power law fluid transfer heat on a moving continuously have been examined. Entrained flow and non-Newtonian third order fluid with transfer of heat due to a linearly surface with slip partial is sustained by Sahoo et.al., [13] .
Different flow types of situations such as third order fluid is a subdivision of non-Newtonian various types of fluids have been studied successfully many researchers. Keimanesh et.al., [14] developed to third order fluid flow of non-Newtonian with double parallel plate using differential transform method. Various analytical method are find out to derived differential equations. The basic fundamental of perturbation approach, which is used for more variety of problems in research process is exhibited in [15,16]

Our purpose motive of the present study to analytical result for MHD heat transfer thin film flow of non-Newtonian third order fluid on vertical belt using perturbation techniques. let us assume two types of thin layer flows (i) lifting flow, (ii) drainage flow. The portrayal of the mathematical model as well as the fundamental governing equations are clarify to attain velocity, thermal field, skin friction coefficient and profile concentration with and without chemical reactions both flow of lift and drainage problems. Consequence of different pertinent variables are exhibited graphically. Analogy study, our perturbation outcomes are comprehended with Adomain decomposition solutions for MHD thin film flow analysis of third grade fluid on a vertical belt for both lift and drainage with slip boundary conditions by Taza Gul et.al., [17] . The numerical outputs and absolute error are calculated in tables.

2 Mathematical formulation

(i)Lifting flow problem:

Consider a flat thin belt moves vertically at unchanged speed V in the direction of uphill, through an enormous shower of third order liquid as shown in Fig. 1. The vertical belt conveys with in a layer of liquid at unchanged thickness δ, for examination the framework arrangement is preferred, in which $y-$ axis is chosen perpendicular to the belt and $x-$ axis corresponding to belt of the surface. Magnetic induction is uniformly applied transitionally to the belt. Assume the flow to be steady, incompressible and laminar flow behind a little partition over the fluid surface. The outside atmospheric pressure is all over.

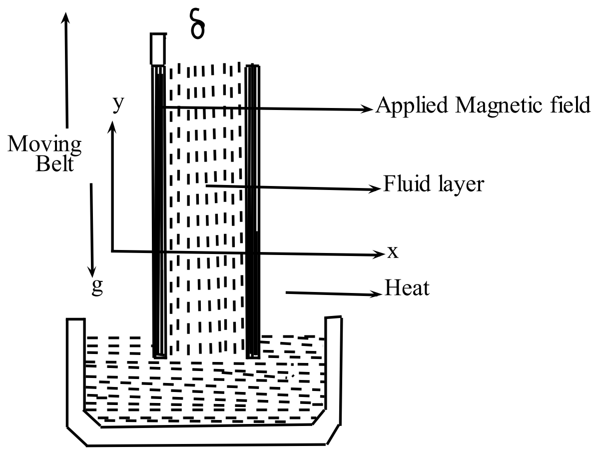

Fig.1 Physical configuration of the lifting flow problem:

The constitutive equation of an incompressible and magneto hydrodynamics third order fluid is represents by,

Conservation of mass

$$\nabla.\mathbf{V} = \mathbf{0} \tag{1}$$

where, $\mathbf{V}$ denoted as fluid of the velocity vector.

Conservation of momentum

$$\rho\frac{D\mathbf{V}}{Dt} = \nabla.\mathbf{T} + \rho\mathbf{g} + \mathbf{J}\times\mathbf{B} \tag{2}$$

where, $\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{V}.\nabla)$ describes as material time derivative, ρ be a constant density, $\mathbf{T}$ be a shear stress, $\mathbf{J}$ denoted as current density, $\mathbf{B}$ represents as magnetic induction and $\mathbf{g}$ indicates as gravitational force.

Conservation of energy

$$\rho c_p\frac{D\mathbf{\Theta}}{Dt} = k\nabla^{\mathbf{2}}\mathbf{\Theta} + tr(\boldsymbol{\tau}.\mathbf{L}) - \nabla.q_r \tag{3}$$

where, $\mathbf{L}=\nabla\mathbf{V}$, k represents as thermal conductivity, c_p denoted as specific heat, $\boldsymbol{\tau}$ be a stress of cauchy's tensor, $\boldsymbol{\Theta}$ describes as thermal.

Conservation of species

$$\frac{D\mathbf{C}}{Dt} = D\nabla^{\mathbf{2}}\mathbf{C} - K\mathbf{C} \tag{4}$$

Where, $\mathbf{C}$ represents as third order concentration fluid, K indicates the chemical reaction, D describes as mass diffusivity coefficient.

The magnetic induction $\mathbf{B} = [0, B_0, 0]$ is transversely applied to belt and external body force is denoted by,

$$\mathbf{J} \times \mathbf{B} = [0, \sigma B_0^2 v(x), 0] \tag{5}$$

Stress shear tensor $\boldsymbol{T}$ is given by,

$$\mathbf{T} = -p\mathbf{I} + \boldsymbol{\tau}, \tag{6}$$

In which $-p\mathbf{I}$ denotes shear spherical stress and $\boldsymbol{\tau}$ is defined by,

$$\boldsymbol{\tau} = \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1) + \beta_3(tr\mathbf{A}_1^2)\mathbf{A}_1. \tag{7}$$

Here, kinematic tensor are $\mathbf{A}_1, \mathbf{A}_2$ and $\mathbf{A}_3$ and material constant moduli are α_i and β_j represents by,

$$\left.\begin{aligned} &\mathbf{A}_1 = (\nabla\mathbf{V}) + (\nabla\mathbf{V})^T \\ &\mathbf{A}_n = \tfrac{D\mathbf{A}_{n-1}}{Dt} + \mathbf{A}_{n-1}(\nabla\mathbf{V}) + (\nabla\mathbf{V})^T\mathbf{A}_{n-1}, n \geq 1 \end{aligned}\right\} \tag{8}$$

$$\mu \geq 0, \alpha_1 \geq 0, |\alpha_1 + \alpha_2| \leq \sqrt{24\mu\beta_3}, \beta_3 \geq 0 \tag{9}$$

Let us consider the motion of an steady incompressible third grade fluid in which velocity field and thermal profile are,

$$\mathbf{V} = [0, v(x), 0] \quad and \quad \Theta = \Theta(x). \tag{10}$$

Inserting the velocity field from equation (10) in continuity (1), momentum, energy equations (2), (4) and in (6) - (8), the continuity equation identically satisfied and then equation (6) reduced to the following component of stress tensor as:

$$\left.\begin{aligned} T_{xx} &= -p+(2\alpha_1+\alpha_2)(\tfrac{dv}{dx})^2, \\ T_{xy} &= \mu\tfrac{dv}{dx}+(2\beta_2+\beta_3)(\tfrac{dv}{dx})^3, \\ T_{yy} &= -p+\alpha_2(\tfrac{dv}{dx}), \\ T_{zz} &= -p, \\ T_{xz} &= T_{yz}=0. \end{aligned}\right\} \tag{11}$$

Making use of equations (11) in equations (2) and (3), the momentum and energy equations with all assumptions are reduced to,

$$\left.\begin{aligned} 0 &= \mu\tfrac{d^2v}{dx^2}+6(\beta_2+\beta_3)(\tfrac{dv}{dx})^2(\tfrac{d^2v}{dx^2})-\rho g-\sigma B_0^2 v(x), \\ 0 &= k\tfrac{d^2\Theta}{dx^2}+\mu(\tfrac{dv}{dx})^2+(2\beta_2+\beta_3)(\tfrac{dv}{dx})^4, \end{aligned}\right\} \tag{12}$$

With relevant boundary conditions on velocity and temperature are introduced, to solve the above equations (12),

$$\left.\begin{aligned} v &= U_0-\gamma T_{xy}\ at\ x = 0 \\ \tfrac{dv}{dx} &= 0\ at\ x = \delta \\ \Theta &= \Theta_0\ at\ x = 0\ and\ \Theta = \Theta_1\ at\ x = \delta \end{aligned}\right\} \tag{13}$$

2.1 Solution Method

(i) Determination of Velocity and Temperature:

Following dimensionless variables are introduced:

$$\left.\begin{aligned} &\overline{v}=\tfrac{\delta}{\nu}v,\ \overline{x}=\tfrac{x}{\delta},\ \overline{\Theta}=\tfrac{\Theta-\Theta_0}{\Theta_1-\Theta_0},\ \lambda=\tfrac{\mu\nu^2}{k(\Theta_1-\Theta_0)\delta^2},\ m=\tfrac{\delta^3 g}{\nu^2}, \\ &\overline{\gamma}=\tfrac{\mu\gamma}{\delta},\ \wedge=\tfrac{\nu\gamma}{\delta},\ \alpha=\tfrac{\delta U_0}{\nu},\ \nu=\tfrac{\mu}{\rho},\ R_e=\tfrac{U\delta}{\nu}, \\ &M=\tfrac{\sigma B_0^2\delta^2}{\mu},\ \beta=\tfrac{(\beta_2+\beta_3)\nu^2}{\mu\delta^4},\ N=\tfrac{16\sigma\Theta_0^3}{3\alpha k} \end{aligned}\right\} \tag{14}$$

Substitute non-dimensional variables in equations (12) and neglecting bars we get,

$$\frac{d^2v}{dx^2}+6\beta(\frac{dv}{dx})^2(\frac{d^2v}{dx^2})-m-Mv(x)=0 \tag{15}$$

$$(1+N)\frac{d^2\Theta}{dx^2}+\lambda\left\{(\frac{dv}{dx})^2+2\beta(\frac{dv}{dx})^4\right\}=0, \tag{16}$$

where, M is a magnetic parameter, β is non-Newtonian effect, λ be a dimensionless number and m is the gravitational parameter, N is denoted by thermal radiation.

From equation (13), the non-dimensional are:

$$\left.\begin{array}{l} v=\alpha-\wedge\left\{(\frac{dv}{dx})+2\beta(\frac{dv}{dx})^3\right\}\ at\ x=0 \\ \frac{dv}{dx}=0\ at\ x=1 \\ \Theta=0\ at\ x=0\ and\ \Theta=1\ at\ x=1. \end{array}\right\} \tag{17}$$

Equations (15) and (16) can be solved, using perturbation technique. The velocity and temperature are represented as,

$$\left.\begin{array}{l} v(x)=v_0(x)+\beta v_1(x)+0(\beta^2). \\ \Theta(x)=\Theta_0(x)+\beta\Theta_1(x)+0(\beta^2). \end{array}\right\} \tag{18}$$

Base part of lifting problem:

$$\beta^0:\frac{d^2v_0}{dx^2}-m-Mv_0=0$$

Solve the above equation, we get

$$v_0(x)=A_1e^{-\sqrt{M}x}+A_2e^{\sqrt{M}x}-\frac{m}{M}, \tag{19}$$

$$v_0(0)=\alpha-\wedge(\frac{dv_0}{dx}),\frac{dv_0}{dx}(1)=0, \tag{20}$$

Where, $c_5=\alpha-\wedge(\frac{dv_0}{dx})$

Using the boundary conditions (20) in equation (19), the base part is,

$$v_0(x)=\left\{(c_5+\frac{m}{M})(1-\frac{e^{-\sqrt{M}}}{2})\right\}e^{-\sqrt{M}x}+\left\{(c_5+\frac{m}{M})\frac{e^{-\sqrt{M}}}{2}\right\}e^{\sqrt{M}x}-\frac{m}{M} \tag{21}$$

Perturbation part of lifting problem:

$$\beta^1 : \frac{d^2v_1}{dx^2} + 6(\frac{dv_0}{dx})^2\frac{d^2v_0}{dx^2} - Mv_1 = 0$$

Solving the above perturb part is,

$$v_1(x) = A_3e^{-\sqrt{M}x} + A_4e^{\sqrt{M}x} + \frac{c_1e^{-3\sqrt{M}x}}{8M} + \frac{c_2e^{3\sqrt{M}x}}{8M} + \frac{c_3xe^{\sqrt{M}x}}{2\sqrt{M}}$$

$$-\frac{c_4xe^{-\sqrt{M}x}}{2\sqrt{M}}, \tag{22}$$

$$v_1(1) = \alpha - \wedge(\frac{dv_0}{dx} + 2(\frac{dv_0}{dx})^3),\ \frac{dv_1}{dx}(1) = 0, \tag{23}$$

Where, $c_6 = \alpha - \wedge(\frac{dv_0}{dx} + 2(\frac{dv_0}{dx})^3)$

Using the boundary conditions (23) in equation (22), the perturbation part is,

$$v_1(x) = \left\{c_6(1 - \frac{e^{-\sqrt{M}}}{2}) + (\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5)\right\}e^{-\sqrt{M}x} + \left\{c_6\frac{e^{-\sqrt{M}}}{2}\right.$$

$$\left. -(\eta_1 + \eta_2 + \eta_3 + \eta_4 +)\right\}e^{\sqrt{M}x} + \frac{c_1e^{-3\sqrt{M}x}}{8M} + \frac{c_2e^{3\sqrt{M}x}}{8M}$$

$$+\frac{c_3xe^{\sqrt{M}x}}{2\sqrt{M}} - \frac{c_4xe^{-\sqrt{M}x}}{2\sqrt{M}}, \tag{24}$$

Combined equations (21) and (24) into equation (18), neglecting the higher order terms (β^2), gives the velocity distribution of third grade fluid(lifting)as,

$$v(x) = A_1e^{-\sqrt{M}x} + A_2e^{\sqrt{M}x} - \frac{m}{M} + \beta\left\{A_3e^{-\sqrt{M}x} + A_4e^{\sqrt{M}x} + \frac{c_1e^{-3\sqrt{M}x}}{8M}\right.$$

$$\left.+\frac{c_2e^{3\sqrt{M}x}}{8M} + \frac{c_3xe^{\sqrt{M}x}}{2\sqrt{M}} - \frac{c_4xe^{-\sqrt{M}x}}{2\sqrt{M}}\right\} \tag{25}$$

2.2 Temperature analysis of lifting flow problem:

Base part of lifting problem:

$$\beta^0 : (1+N)\frac{d^2\Theta_0}{dx^2} + \lambda(\frac{dv_0}{dx})^2 = 0$$

solve the above equation as become,

$$\Theta_0(x) = A_5x + A_6 - \frac{\lambda}{(1+N)}\left\{\frac{A_1^2e^{-2\sqrt{M}x}}{4} + \frac{A_2^2e^{2\sqrt{M}x}}{4} - MA_1A_2x^2\right\} \quad (26)$$

$$\Theta_0(0) = 0, \Theta_0(1) = 1. \quad (27)$$

Using the boundary conditions (27) in equation (26), the base part solution is,

$$\Theta_0(x) = \left\{1 + \frac{\lambda}{(1+N)}(\frac{A_1^2}{4}(e^{-2\sqrt{M}x} - 1) + \frac{A_2^2}{4}(e^{2\sqrt{M}x} - 1) - MA_1A_2)\right\}x$$

$$+\frac{\lambda}{(1+N)}\left\{\frac{A_1^2}{4} + \frac{A_2^2}{4} - \frac{A_1^2e^{-2\sqrt{M}x}}{4} + \frac{A_2^2e^{2\sqrt{M}x}}{4} - MA_1A_2x^2\right\} \quad (28)$$

Perturbation part of lifting problem:

$$\beta^1 : (1+N)\frac{d^2\Theta_1}{dx^2} + 2\lambda\left\{(\frac{dv_0}{dx})(\frac{dv_1}{dx}) + (\frac{dv_0}{dx})^4\right\} = 0$$

The solution of above equation becomes,

$$\Theta_1(x) = A_7x + A_8 - \frac{\lambda}{(1+N)}\left\{p_1\frac{e^{-2\sqrt{M}x}}{2M} + p_2\frac{e^{2\sqrt{M}x}}{2M} + p_3\frac{e^{-4\sqrt{M}x}}{8M} + p_4\frac{e^{2\sqrt{M}x}}{8M}\right.$$

$$+p_5\frac{x^2}{2} + p_6\frac{x^3}{3} + p_7(\frac{xe^{-2\sqrt{M}x}}{2M} + \frac{e^{-2\sqrt{M}x}}{4M^{3/2}}) + p_8(\frac{xe^{2\sqrt{M}x}}{2M} - \frac{e^{2\sqrt{M}x}}{4M^{3/2}}) + n_1\frac{e^{-4\sqrt{M}x}}{16M}$$

$$\left.+n_2\frac{e^{4\sqrt{M}x}}{16M} + n_3\frac{x^2}{2} + n_4\frac{e^{-2\sqrt{M}x}}{16M} + n_5\frac{e^{2\sqrt{M}x}}{16M}\right\} \quad (29)$$

$$\Theta_1(0) = 0, \Theta_1(1) = 1. \quad (30)$$

Using the boundary conditions (30) in equation (29), the perturbation part solution is,

$$\Theta_1(x) = 1 + \frac{\lambda}{(1+N)}\Bigg\{(\frac{p_1}{2M}(e^{-2\sqrt{M}}-1) + \frac{p_2}{2M}(e^{2\sqrt{M}}-1) + \frac{p_3}{2M}(e^{-4\sqrt{M}}-1)$$

$$+\frac{p_4}{2M}(e^{4\sqrt{M}}-1) + \frac{p_5}{2} + \frac{p_6}{3} + p_7(\frac{e^{-2\sqrt{M}}}{2M} + \frac{1}{4M^{3/2}}(e^{-2\sqrt{M}}-1))$$

$$+p_8(\frac{e^{2\sqrt{M}}}{2M} - \frac{1}{4M^{3/2}}(e^{-2\sqrt{M}}-1)) + \frac{n_1}{8M}(e^{-4\sqrt{M}}-1) + \frac{n_2}{8M}(e^{4\sqrt{M}}-1)$$

$$+\frac{n_3}{2M} + \frac{n_4}{8M}(e^{-2\sqrt{M}}-1) + \frac{n_5}{8M}(e^{2\sqrt{M}}-1))x + (\frac{p_1}{2M} + \frac{p_2}{2M} + \frac{p_3}{8M} + \frac{p_4}{8M}$$

$$+\frac{p_7}{4M^{3/2}} - \frac{p_8}{4M^{3/2}} + \frac{n_1}{8M} + \frac{n_2}{8M} + \frac{n_4}{2M} + \frac{n_5}{2M}) - (p_1\frac{e^{-2\sqrt{M}x}}{2M} + p_2\frac{e^{2\sqrt{M}x}}{2M}$$

$$+p_3\frac{e^{-4\sqrt{M}x}}{8M} + p_4\frac{e^{2\sqrt{M}x}}{8M} + p_5\frac{x^2}{2} + p_6\frac{x^3}{3} + p_7(\frac{xe^{-2\sqrt{M}x}}{2M} + \frac{e^{-2\sqrt{M}x}}{4M^{3/2}}) + n_1\frac{e^{-4\sqrt{M}x}}{16M}$$

$$+p_8(\frac{xe^{2\sqrt{M}x}}{2M} - \frac{e^{2\sqrt{M}x}}{4M^{3/2}}) + n_2\frac{e^{4\sqrt{M}x}}{16M} + n_3\frac{x^2}{2} + n_4\frac{e^{-2\sqrt{M}x}}{16M} + n_5\frac{e^{2\sqrt{M}x}}{16M})\Bigg\} \quad (31)$$

Using equation (28) and (31) into equation (18), neglecting the higher order terms (β^2), gives the temperature profile of third grade fluid(lifting)as

$$\Theta(x) = \frac{\lambda}{(1+N)}\Bigg\{(r_1+\beta r_4)e^{-2\sqrt{M}x} + (r_2+\beta r_5)e^{2\sqrt{M}x} + (r_3+\beta r_{13})$$

$$+\beta(r_6e^{-4\sqrt{M}x} + r_7e^{4\sqrt{M}x} + r_8xe^{-2\sqrt{M}x} + r_9xe^{2\sqrt{M}x} + r_{10}x^2 + r_{11}x^3 + r_{12}x)\Bigg\} \quad (32)$$

2.3 Skin friction coefficient for lifting flow problem:

The dimensionless form of skin friction become,

$$C_f(0) = \frac{\tau_{xy}|_{x=0}}{\frac{1}{2}\rho u^2} = \frac{2}{R_e^2}\left\{\frac{dv}{dx} + 2\beta(\frac{dv}{dx})^3\right\} \quad (33)$$

Using Eqn (25) in (33), the skin friction as,

$$C_f(0) = \frac{2\sqrt{M}}{R_e^2}\Bigg\{(A_2+\beta A_4) - (A_1+\beta A_3) + \frac{3\beta}{8M}(c_2-c_1) + \frac{3\beta}{2M}(c_3-c_4)$$

$$+2\beta M\left\{(A_2+\beta A_4) - (A_1+\beta A_3) + \frac{3\beta}{8M^2}(c_2-c_1) + \frac{3\beta}{2M^2}(c_3-c_4)\right\}^3\Bigg\} \quad (34)$$

2.4 Average velocity of lifting flow problem:

The average velocity $\overline{v}$ is given by,

$$\overline{v} = \int_0^1 v dx = \frac{(A_1 + \beta A_3)}{\sqrt{M}}(1 - e^{-\sqrt{M}}) + \frac{(A_2 + \beta A_4)}{\sqrt{M}}(e^{\sqrt{M}} - 1)$$

$$-\frac{m}{M} + \beta\Bigg\{\frac{c_1}{24M^{3/2}}(1 - e^{-3\sqrt{M}}) + \frac{c_2}{24M^{3/2}}(e^{3\sqrt{M}} - 1) + \frac{c_3}{2M\sqrt{M}}$$

$$(e^{-\sqrt{M}}(\sqrt{M} + 1) - 1) + \frac{c_4}{2M\sqrt{M}}(e^{\sqrt{M}}(\sqrt{M} - 1) + 1)\Bigg\} \tag{35}$$

Relative velocity V is represented by,

$$V = v - \overline{v} = (A_1 + \beta A_3)e^{-\sqrt{M}x} + (A_2 + \beta A_4)e^{\sqrt{M}x} + \beta\Bigg\{\frac{c_1 e^{-3\sqrt{M}x}}{8M}$$

$$+\frac{c_2 e^{3\sqrt{M}x}}{8M} - \frac{c_3 x e^{\sqrt{M}x}}{2\sqrt{M}} + \frac{c_4 x e^{-\sqrt{M}x}}{2\sqrt{M}}\Bigg\} + f_1 \tag{36}$$

3 (ii)Drainage flow problem:

Geometry and cartesian coordinates is same as previous case. Non-Newtonian thin layer third order fluid draining down to the belt due to gravity and belt is stationary as sketched in Fig. 2 . Let us assumed flow is steady, incompressible, laminar and shear forces and external pressure neglected, Thickness of the thin layer and gravity balance remain constant.

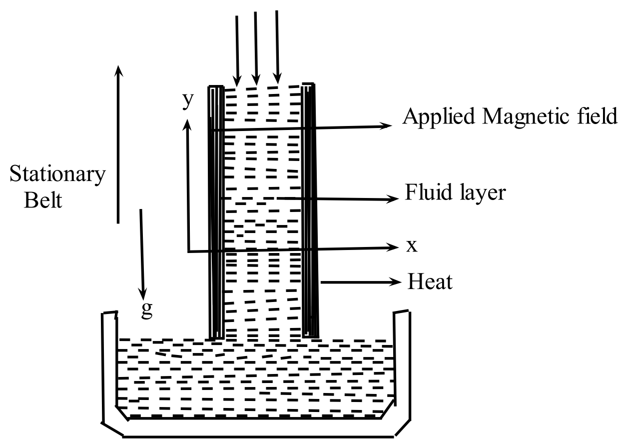

Fig.2 Physical configuration of the drainage flow problem:

Using the component of stress tensor equations (11) and with all assumptions in (2) and (3) of drainage flow reduce to the momentum and energy equations made by,

$$\left.\begin{aligned} 0 &= \mu\tfrac{d^2v}{dx^2} + 6(\beta_2+\beta_3)(\tfrac{dv}{dx})^2(\tfrac{d^2v}{dx^2}) + \rho g - \sigma B_0^2 v(x), \\ 0 &= k\tfrac{d^2\Theta}{dx^2} + \mu(\tfrac{dv}{dx})^2 + (2\beta_2+\beta_3)(\tfrac{dv}{dx})^4, \end{aligned}\right\} \tag{37}$$

The following Boundary conditions on velocity and temperature are introduced, to solve the above equations (37),

$$\left.\begin{aligned} v &= -\gamma T_{xy} \; at \; x = 0 \\ \tfrac{dv}{dx} &= 0 \; at \; x = \delta \\ \Theta &= \Theta_0 \; at \; x = 0 \; and \; \Theta = \Theta_1 \; at \; x = \delta \end{aligned}\right\} \tag{38}$$

Using non-dimensional quantities from (14) in equations (37) and (38) and neglecting bars we obtain

$$\frac{d^2v}{dx^2} + 6\beta(\frac{dv}{dx})^2(\frac{d^2v}{dx^2}) + m - Mv(x) = 0 \tag{39}$$

$$\frac{d^2\Theta}{dx^2} + \lambda\left\{(\frac{dv}{dx})^2 + 2\beta(\frac{dv}{dx})^4\right\} = 0, \tag{40}$$

$$\left.\begin{aligned} & v = -\wedge\left\{(\tfrac{dv}{dx}) + 2\beta(\tfrac{dv}{dx})^3\right\} \; at \; x = 0 \\ & \tfrac{dv}{dx} = 0 \; at \; x = 1 \\ & \Theta = 0 \; at \; x = 0 \; and \; \Theta = 1 \; at \; x = 1. \end{aligned}\right\} \tag{41}$$

Using perturbation technique from equation (18) in equation (39) and (40) can be solved analytically, The velocity and temperature are described by,

Base part of Drainage problem:

$$\beta^0 : \frac{d^2v_0}{dx^2} + m - Mv_0 = 0$$

$$v_0(x) = A_1' e^{-\sqrt{M}x} + A_2' e^{\sqrt{M}x} + \frac{m}{M}, \tag{42}$$

$$v_0(0) = - \wedge (\frac{dv_0}{dx}), \ \frac{dv_0}{dx}(1) = 0, \tag{43}$$

Using the boundary conditions (43) in equation (42), the base part solution is,

$$v_0(x) = \left\{ (c_7 - \frac{m}{M})(1 - \frac{e^{-\sqrt{M}}}{2}) \right\} + \left\{ (c_7 - \frac{m}{M})(1 - \frac{e^{-\sqrt{M}}}{2}) \right\} + \frac{m}{M} \tag{44}$$

Perturbation part of Drainage problem:

$$\beta^1 : \frac{d^2 v_1}{dx^2} + 6(\frac{dv_0}{dx})^2 \frac{d^2 v_0}{dx^2} - M v_1 = 0$$

$$v_1(x) = A_3' e^{-\sqrt{M}x} + A_4' e^{\sqrt{M}x} + \frac{c_1' e^{-3\sqrt{M}x}}{8M} + \frac{c_2' e^{3\sqrt{M}x}}{8M} - \frac{c_3' x e^{\sqrt{M}x}}{2\sqrt{M}}$$

$$+ \frac{c_4' x e^{-\sqrt{M}x}}{2\sqrt{M}}, \tag{45}$$

$$v_1(1) = - \wedge (\frac{dv_0}{dx} + 2(\frac{dv_0}{dx})^3), \ \frac{dv_1}{dx}(1) = 0, \tag{46}$$

Using the boundary conditions (46) in equation (45), the perturbation solution is,

$$v_1(x) = \left\{ c_8(1 - \frac{e^{-\sqrt{M}x}}{2})(\eta_1' + \eta_2' + \eta_3' + \eta_4' + \eta_5') + (\frac{c_8 e^{-\sqrt{M}x}}{2}) - (\eta_1' + \eta_2' \right.$$

$$\left. + \eta_3' + \eta_4') + \frac{c_1' e^{-3\sqrt{M}x}}{8M} + \frac{c_2' e^{3\sqrt{M}x}}{8M} - \frac{c_3' x e^{\sqrt{M}x}}{2\sqrt{M}} + \frac{c_4' x e^{-\sqrt{M}x}}{2\sqrt{M}} \right\}, \tag{47}$$

By using equations (44) and (47) into equation (18), neglecting the higher order terms (β^2), gives the velocity distribution of third grade fluid as,

$$v(x) = A_1' e^{-\sqrt{M}x} + A_2' e^{\sqrt{M}x} + \frac{m}{M} + \beta \left\{ A_3' e^{-\sqrt{M}x} + A_4' e^{\sqrt{M}x} + \frac{c_1' e^{-3\sqrt{M}x}}{8M} \right.$$

$$\left. + \frac{c_2' e^{3\sqrt{M}x}}{8M} - \frac{c_3' x e^{\sqrt{M}x}}{2\sqrt{M}} + \frac{c_4' x e^{-\sqrt{M}x}}{2\sqrt{M}} \right\} \tag{48}$$

3.1 Temperature analysis of drainage flow problem:

Base part of Drainage problem:

$$\beta^0 : (1+N)\frac{d^2\Theta_0}{dx^2} + \lambda(\frac{dv_0}{dx})^2 = 0$$

solve the above equation as become,

$$\Theta_0(x) = A_5^{'}x + A_6^{'} - \frac{\lambda}{(1+N)}\left\{\frac{(A_1^{'})^2 e^{-2\sqrt{M}x}}{4} + \frac{(A_2^{'})^2 e^{2\sqrt{M}x}}{4} - MA_1^{'}A_2^{'}x^2\right\} \quad (49)$$

$$\Theta_0(0) = 0, \Theta_0(1) = 1. \quad (50)$$

Using the boundary conditions (27) in equation (26) , the base part solution is,

$$\Theta_0(x) = \left\{1 + \frac{\lambda}{(1+N)}(\frac{(A_1^{'})^2}{4}(e^{-2\sqrt{M}x} - 1) + \frac{(A_2^{'})^2}{4}(e^{2\sqrt{M}x} - 1) - MA_1^{'}A_2^{'})\right\}x$$

$$+\frac{\lambda}{(1+N)}\left\{\frac{(A_1^{'})^2}{4} + \frac{(A_2^{'})^2}{4} - \frac{(A_1^{'})^2 e^{-2\sqrt{M}x}}{4} + \frac{(A_2^{'})^2 e^{2\sqrt{M}x}}{4} - MA_1^{'}A_2^{'}x^2\right\} \quad (51)$$

Perturbation part of lifting problem:

$$\beta^1 : (1+N)\frac{d^2\Theta_1}{dx^2} + 2\lambda\left\{(\frac{dv_0}{dx})(\frac{dv_1}{dx}) + (\frac{dv_0}{dx})^4\right\} = 0$$

The solution of above equation becomes,

$$\Theta_1(x) = A_7^{'}x + A_8^{'} - \frac{\lambda}{(1+N)}\left\{p_1^{'}\frac{e^{-2\sqrt{M}x}}{2M} + p_2^{'}\frac{e^{2\sqrt{M}x}}{2M} + p_3^{'}\frac{e^{-4\sqrt{M}x}}{8M} + p_4^{'}\frac{e^{2\sqrt{M}x}}{8M}\right.$$

$$+p_5^{'}\frac{x^2}{2} + p_6^{'}\frac{x^3}{3} + p_7^{'}(\frac{xe^{-2\sqrt{M}x}}{2M} + \frac{e^{-2\sqrt{M}x}}{4M^{3/2}}) + p_8^{'}(\frac{xe^{2\sqrt{M}x}}{2M} - \frac{e^{2\sqrt{M}x}}{4M^{3/2}}) + n_1^{'}\frac{e^{-4\sqrt{M}x}}{16M}$$

$$\left.+n_2^{'}\frac{e^{4\sqrt{M}x}}{16M} + n_3^{'}\frac{x^2}{2} + n_4^{'}\frac{e^{-2\sqrt{M}x}}{16M} + n_5^{'}\frac{e^{2\sqrt{M}x}}{16M}\right\} \quad (52)$$

$$\Theta_1(0) = 0, \Theta_1(1) = 1. \quad (53)$$

Using the boundary conditions (30) in equation (29), the perturbation part solution is,

$$\Theta_1(x) = 1 + \frac{\lambda}{(1+N)}\Bigg\{(\frac{p_1'}{2M}(e^{-2\sqrt{M}}-1) + \frac{p_2'}{2M}(e^{2\sqrt{M}}-1) + \frac{p_3'}{2M}(e^{-4\sqrt{M}}-1)$$

$$+\frac{p_4'}{2M}(e^{4\sqrt{M}}-1) + \frac{p_5'}{2} + \frac{p_6'}{3} + p_7'(\frac{e^{-2\sqrt{M}}}{2M} + \frac{1}{4M^{3/2}}(e^{-2\sqrt{M}}-1))$$

$$+p_8'(\frac{e^{2\sqrt{M}}}{2M} - \frac{1}{4M^{3/2}}(e^{-2\sqrt{M}}-1)) + \frac{n_1'}{8M}(e^{-4\sqrt{M}}-1) + \frac{n_2'}{8M}(e^{4\sqrt{M}}-1)$$

$$+\frac{n_3'}{2M} + \frac{n_4'}{8M}(e^{-2\sqrt{M}}-1) + \frac{n_5'}{8M}(e^{2\sqrt{M}}-1))x + (\frac{p_1'}{2M} + \frac{p_2'}{2M} + \frac{p_3'}{8M} + \frac{p_4'}{8M}$$

$$+\frac{p_7'}{4M^{3/2}} - \frac{p_8'}{4M^{3/2}} + \frac{n_1'}{8M} + \frac{n_2'}{8M} + \frac{n_4'}{2M} + \frac{n_5'}{2M}) - (p_1'\frac{e^{-2\sqrt{M}x}}{2M} + p_2'\frac{e^{2\sqrt{M}x}}{2M}$$

$$+p_3'\frac{e^{-4\sqrt{M}x}}{8M} + p_4'\frac{e^{2\sqrt{M}x}}{8M} + p_5'\frac{x^2}{2} + p_6'\frac{x^3}{3} + p_7'(\frac{xe^{-2\sqrt{M}x}}{2M} + \frac{e^{-2\sqrt{M}x}}{4M^{3/2}}) + n_1'\frac{e^{-4\sqrt{M}x}}{16M}$$

$$+p_8'(\frac{xe^{2\sqrt{M}x}}{2M} - \frac{e^{2\sqrt{M}x}}{4M^{3/2}}) + n_2'\frac{e^{4\sqrt{M}x}}{16M} + n_3'\frac{x^2}{2} + n_4'\frac{e^{-2\sqrt{M}x}}{16M} + n_5'\frac{e^{2\sqrt{M}x}}{16M})\Bigg\} \quad (54)$$

Using equation (51) and (54) into equation (18), neglecting the higher order terms (β^2), gives the temperature profile of third grade fluid(lifting)as

$$\Theta(x) = \frac{\lambda}{(1+N)}\Bigg\{(r_1' + \beta r_4')e^{-2\sqrt{M}x} + (r_2' + \beta r_5')e^{2\sqrt{M}x} + (r_3' + \beta r_{13}')$$

$$+\beta(r_6'e^{-4\sqrt{M}x} + r_7'e^{4\sqrt{M}x} + r_8'xe^{-2\sqrt{M}x} + r_9'xe^{2\sqrt{M}x} + r_{10}'x^2 + r_{11}'x^3 + r_{12}'x)\Bigg\} \quad (55)$$

3.2 Skin friction coefficient for lifting flow problem:

The dimensionless form of skin friction become,

$$C_f'(0) = \frac{\tau_{xy}|_{x=0}}{\frac{1}{2}\rho u^2} = \frac{2}{R_e^2}\Bigg\{\frac{dv}{dx} + 2\beta(\frac{dv}{dx})^3\Bigg\} \quad (56)$$

Using Eqn (48) in (55), the skin friction as,

$$C_f'(0) = \frac{2\sqrt{M}}{R_e^2}\Bigg\{(A_2' + \beta A_4') - (A_1' + \beta A_3') + \frac{3\beta}{8M}(c_2' - c_1') + \frac{3\beta}{2M}(c_3' - c_4')$$

$$+2\beta M\Bigg\{(A_2' + \beta A_4') - (A_1' + \beta A_3') + \frac{3\beta}{8M^2}(c_2' - c_1') + \frac{3\beta}{2M^2}(c_3' - c_4')\Bigg\}^3\Bigg\} \quad (57)$$

3.3 Average velocity of drainage flow problem:

The average velocity $\overline{v}$ is given by,

$$\overline{v} = \int_0^1 v dx = \frac{(A_1^{'} + \beta A_3^{'})}{\sqrt{M}}(1 - e^{-\sqrt{M}}) + \frac{(A_2^{'} + \beta A_4^{'})}{\sqrt{M}}(e^{\sqrt{M}} - 1)$$

$$-\frac{m}{M} + \beta\Bigg\{\frac{c_1^{'}}{24M^{3/2}}(1 - e^{-3\sqrt{M}}) + \frac{c_2^{'}}{24M^{3/2}}(e^{3\sqrt{M}} - 1) + \frac{c_3^{'}}{2M\sqrt{M}}$$

$$(e^{-\sqrt{M}}(\sqrt{M} + 1) - 1) + \frac{c_4^{'}}{2M\sqrt{M}}(e^{\sqrt{M}}(\sqrt{M} - 1) + 1)\Bigg\} \tag{58}$$

Relative velocity V is represented by,

$$V = v - \overline{v} = (A_1^{'} + \beta A_3^{'})e^{-\sqrt{M}x} + (A_2^{'} + \beta A_4^{'})e^{\sqrt{M}x} + \beta\Bigg\{\frac{c_1^{'}e^{-3\sqrt{M}x}}{8M}$$

$$+\frac{c_2^{'}e^{3\sqrt{M}x}}{8M} - \frac{c_3^{'}xe^{\sqrt{M}x}}{2\sqrt{M}} + \frac{c_4^{'}xe^{-\sqrt{M}x}}{2\sqrt{M}}\Bigg\} + f_1 \tag{59}$$

4 Concentration of Third grade fluid

Third grade concentration fluid $C(t, x, y)$ satisfies the convection unsteady diffusivity equation reduced from (4)

$$\frac{\partial C}{\partial t} + V(x)\frac{\partial C}{\partial y} = D\Bigg\{\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\Bigg\} - KC \tag{60}$$

where D denoted as diffusion molecular, which is treated by a constant, C represented by species of the concentration, K indicates as reaction rate chemical parameter.

Transverse diffusion is high extremely comparable to longitudinal diffusion, so that $\frac{\partial^2 C}{\partial y^2} << \frac{\partial^2 C}{\partial x^2}$ Eq. (60) becomes,

$$\frac{\partial C}{\partial t} + V(x)\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial x^2} - KC \tag{61}$$

Assume the chemical reaction rate (homogeneous) is taking the center place of vertical belt and equation (61) are clarify the fluid concentration with boundary condition,

$$\left.\begin{array}{l} C = 0 \ at \ x = 0 \\ \\ \frac{\partial C}{\partial x} = 0 \ at \ x = \delta \end{array}\right\} \tag{62}$$

Introducing the following non-dimensionless variables

$$C' = \frac{C}{C_0}; t' = \frac{t}{t_0}; x' = \frac{x}{\delta}; y' = \frac{y}{\delta}; \xi = \frac{y - \overline{v_0}t}{L}; \gamma^2 = \frac{\delta^2 K}{D}. \tag{63}$$

Along, the fluid flow direction L described as characteristic length, α be a chemical parameter. Applying dimensionless parameters in (61) and (62) are,

$$\frac{\partial^2 C}{\partial x^2} = QV + \gamma^2 C \tag{64}$$

Where $Q = \frac{\delta^2}{DL}\frac{\partial C}{\partial \xi}$

$$\left.\begin{array}{l} C = 0 \ at \ x = 0 \\ \\ \frac{\partial C}{\partial x} = 0 \ at \ x = 1 \end{array}\right\} \tag{65}$$

Let us assume two parts of with and without chemical for lifting and drainage flow of the vertical belt.

4.1 Concentration of third grade fluid with chemical reaction for lift and drainage problem ($\gamma \neq 0$):

Substituting the velocity lift equation (25) in (64) with boundary conditions (65), The result of lift concentration is,

$$C = \frac{Q}{(1 - \frac{\gamma^2 x^2}{2} + \gamma^2 x)}\left\{\frac{(A_1 + \beta A_3)}{M}(e^{-\sqrt{M}x}) + \frac{(A_2 + \beta A_4)}{M}(e^{\sqrt{M}x})\right.$$

$$+\beta\left\{\frac{c_1}{72M^2}(e^{-3\sqrt{M}x}) + \frac{c_2}{72M^2}(e^{3\sqrt{M}x}) + \frac{c_3}{2M^{3/2}}(xe^{\sqrt{M}x} - \frac{2e^{\sqrt{M}x}}{\sqrt{M}})\right.$$

$$-\frac{c_4}{2M^{3/2}}(xe^{-\sqrt{M}x}+\frac{2e^{-\sqrt{M}x}}{\sqrt{M}})\Bigg\}+\frac{f_1x^2}{2}+f_2x+f_3\Bigg\} \tag{66}$$

Applying the drainage velocity equation (48) in (64) with bc_s (65), the drainage concentration solution as,

$$C_1=\frac{Q}{(1-\frac{\gamma^2x^2}{2}+\gamma^2x)}\Bigg\{\frac{(A_1'+\beta A_3')}{M}(e^{-\sqrt{M}x})+\frac{(A_2'+\beta A_4')}{M}(e^{\sqrt{M}x})$$

$$+\beta\Bigg\{\frac{c_1'}{72M^2}(e^{-3\sqrt{M}x})+\frac{c_2'}{72M^2}(e^{3\sqrt{M}x})+\frac{c_3'}{2M^{3/2}}(xe^{\sqrt{M}x}-\frac{2e^{\sqrt{M}x}}{\sqrt{M}})$$

$$-\frac{c_4'}{2M^{3/2}}(xe^{-\sqrt{M}x}+\frac{2e^{-\sqrt{M}x}}{\sqrt{M}})\Bigg\}+\frac{f_1'x^2}{2}+f_2'x+f_3'\Bigg\} \tag{67}$$

4.2 Concentration of third grade fluid with out chemical reaction for lift and drainage problem ($\gamma=0$):

Consider a chemical reaction part of the position do not chosen in the assumed vertical channel, then chemical reaction become as zero. Let C^* represents as third grade concentration fluid in with out chemical reaction, then equation (64) reduced by,

$$\frac{\partial^2C^*}{\partial x^2}=QV \tag{68}$$

The lift concentration result (C^*) from equation (25) are,

$$C^*=Q\Bigg\{\frac{(A_1+\beta A_3)}{M}(e^{-\sqrt{M}x})+\frac{(A_2+\beta A_4)}{M}(e^{\sqrt{M}x})$$

$$+\beta\Bigg\{\frac{c_1}{72M^2}(e^{-3\sqrt{M}x})+\frac{c_2}{72M^2}(e^{3\sqrt{M}x})+\frac{c_3}{2M^{3/2}}(xe^{\sqrt{M}x}-\frac{2e^{\sqrt{M}x}}{\sqrt{M}})$$

$$-\frac{c_4}{2M^{3/2}}(xe^{-\sqrt{M}x}+\frac{2e^{-\sqrt{M}x}}{\sqrt{M}})\Bigg\}+\frac{f_1x^2}{2}+f_2x+f_3\Bigg\} \tag{69}$$

Similarly, drainage concentration problem become as,

$$C_1^*=Q\Bigg\{\frac{(A_1'+\beta A_3')}{M}(e^{-\sqrt{M}x})+\frac{(A_2'+\beta A_4')}{M}(e^{\sqrt{M}x})$$

$$+\beta\left\{\frac{c_1^{'}}{72M^2}(e^{-3\sqrt{M}x})+\frac{c_2^{'}}{72M^2}(e^{3\sqrt{M}x})+\frac{c_3^{'}}{2M^{3/2}}(xe^{\sqrt{M}x}-\frac{2e^{\sqrt{M}x}}{\sqrt{M}})\right.$$

$$\left.-\frac{c_4^{'}}{2M^{3/2}}(xe^{-\sqrt{M}x}+\frac{2e^{-\sqrt{M}x}}{\sqrt{M}})\right\}+\frac{f_1^{'}x^2}{2}+f_2^{'}x+f_3^{'}\Bigg\} \tag{70}$$

The coefficients of concentration for lifting (C, C^*) and drainage (C_1, C_1^*) are evaluated and outcomes are analyzed from the investigation in sec. 5

Table:1 Comparison of ADM and Perturbation method for lift flow velocity distribution

x	ADM	PM	Absolute Error
0.0	0.10647	0.106974	0.504×10^{-3}
0.1	0.0629797	0.0628091	0.171×10^{-3}
0.2	0.0238868	0.0237237	0.163×10^{-3}
0.3	-0.0108672	-0.0104578	0.409×10^{-3}
0.4	-0.0412621	-0.040359	0.903×10^{-2}
0.5	-0.0672262	-0.0659299	0.131×10^{-2}
0.6	-0.0886591	-0.0871146	0.155×10^{-2}
0.7	-0.105452	-0.103789	0.166×10^{-2}
0.8	-0.117503	-0.115807	0.170×10^{-2}
0.9	-0.124729	-0.123038	0.169×10^{-2}
1.0	-0.127078	-0.125393	0.168×10^{-2}

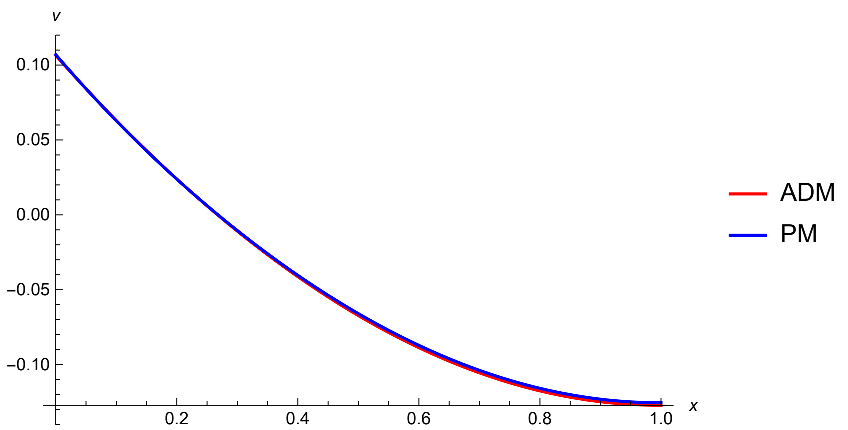

Figure-3:Comparison of ADM and Perturbation method on axial velocity profile ”$v(x)$” for lifting problem.

Table-2 Comparison of ADM and Perturbation method for drainage flow velocity distribution

x	ADM	PM	Absolute Error
0.0	0.0000632524	0.0000869721	0.237×10^{-4}
0.1	0.00901985	0.00890304	0.117×10^{-3}
0.2	0.0163739	0.0161673	0.211×10^{-3}
0.3	0.022645	0.0223583	0.287×10^{-3}
0.4	0.0278879	0.027526	0.362×10^{-3}
0.5	0.0321262	0.0316922	0.434×10^{-3}
0.6	0.0353733	0.034869	0.504×10^{-3}
0.7	0.037638	0.0370645	0.574×10^{-3}
0.8	0.0389262	0.0382839	0.642×10^{-3}
0.9	0.0392417	0.0385307	0.711×10^{-3}
1.0	0.0385871	0.0378072	0.780×10^{-3}

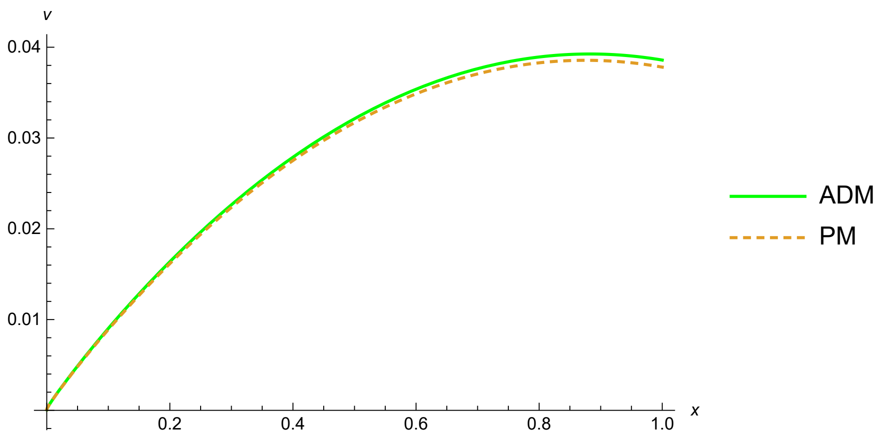

Figure-4:Comparison of ADM and Perturbation method on axial velocity field "$v(x)$" for drainage problem.

5 Results and Discussion

The effects of non-dimensional parameters, such as non-Newtonian parameter (β), Magnetic parameter (M), gravitational parameter (m) thermal radiation (N), dimensionless variable (λ), slip parameter $(\wedge)$ on these physical quantities are visualized graphically. Figures (5) and (10) illustrates the influence of non-Newtonian effect (β) increases with an enhances the flow of speed. For smaller values of (β), the speed distribution differs slightly from the Newtonian. However as (β) rises, these profiles become more flat, which results in a thinner effect of the cut. Figures (6) and (11) exhibits the effect of the magnetic parameter (M) increases in both problems as declined in lifting as well as raises in drainage respectively. since, we can see that the boundary layer thickness is relative to the cross magnetic induction and the velocity descends as the fluid progresses towards the surface. Hence, the speed of the fluid flow is minimum at the surface and maximum at the belt of the surface. Figures (7) and (12) displays the gravitational parameter (m) increases as velocity decreases in lift flow and increases in drainage flow. The effect of gravity due to friction force seems a little near the belt. The approximately speed of the fluid becomes same for various values of (m) in the domain. On rising the gravitational parameter as decreases the speed for lifting flow and increases the speed for drainage flow behind this point due to inconsequential friction. Some values of slip effect parameter $(\wedge)$ for the velocity profile are displayed in figures (8) and (13). It is perceived, velocity of the fluid enhances as slip parameter also enhances for beside the belt is outstanding on the surface. Temperature distribution for different values of dimensionless parameter and thermal radiation are presented in figures (9) and (14). Non-dimensional variable and thermal radiation are increases temperature reduces in (λ) increases in (N) for both flow analysis. Figures (15) and (16) represents the coefficient of skin friction for various values of Reynolds number and non-Newtonian effect. Skin friction coefficient reduces in (R_e) and enhances in (β) for fixed gravity induction.

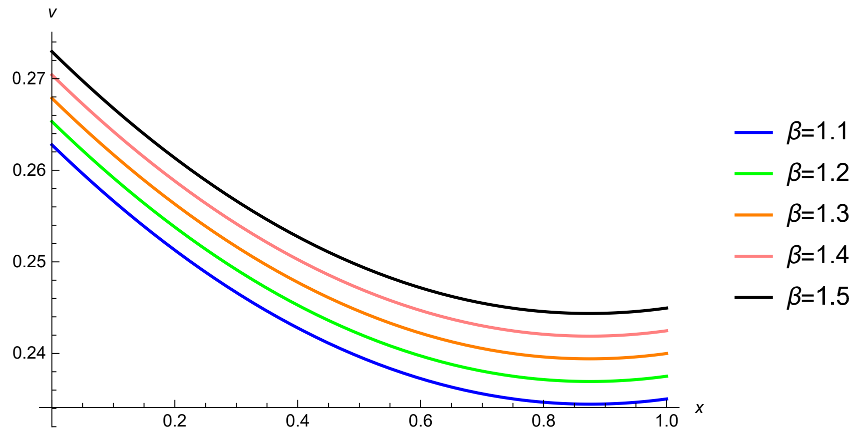

Figure-5: Variation of lifting problem "v" with the variation of axis "x" for different "β".

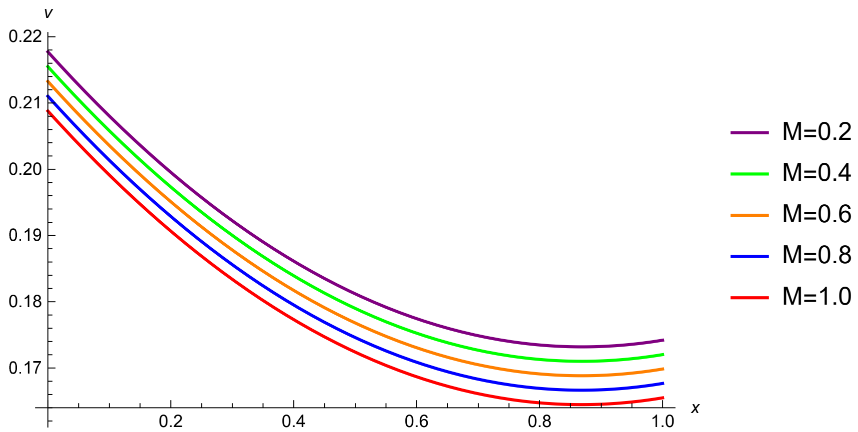

Figure-6: Variation of lifting problem "v" with the variation of axis "x" for different "M".

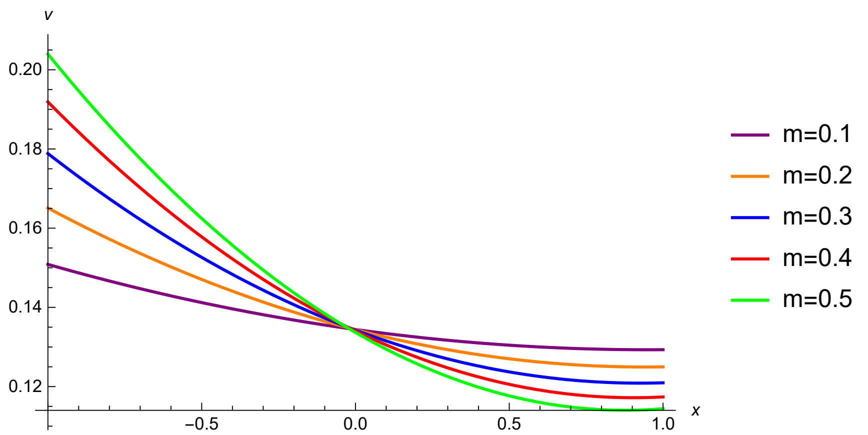

Figure-7:Variation of lifting problem "v" with the variation of axis "x" for different "m".

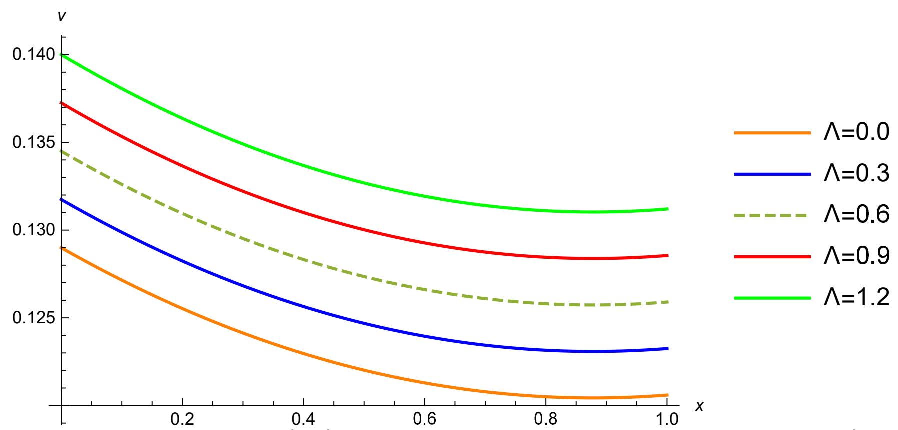

Figure-8: Variation of lifting problem "v" with the variation of axis "x" for different "$\wedge$".

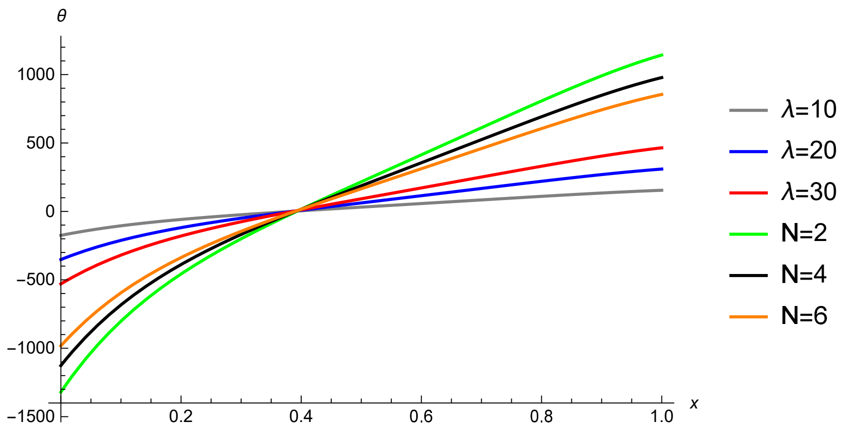

Figure-9: Influence effect of lifting Non-Newtonian parameter "λ, N" in the temperature distribution"$\theta(x)$".

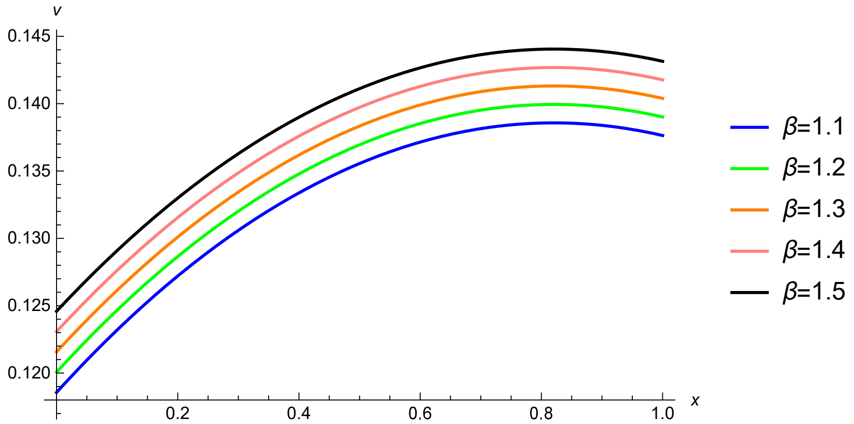

Figure-10: Variation of drainage problem "v" with the variation of axis "x" for different "β".

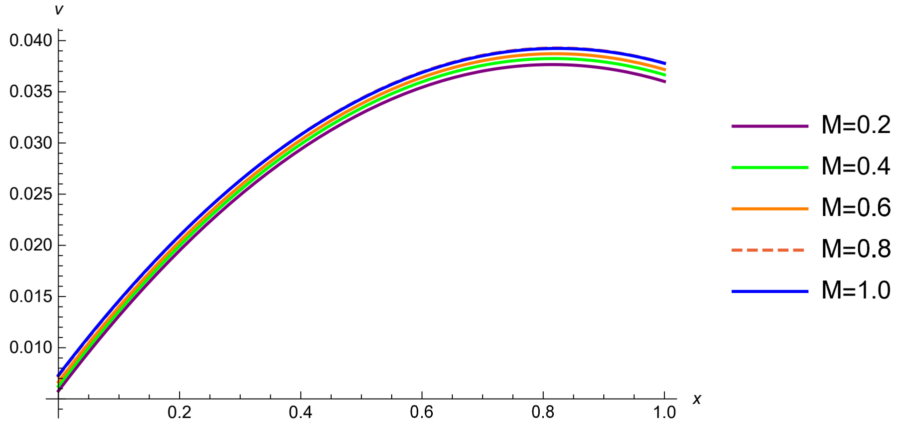

Figure-11: Variation of drainage problem "v" with the variation of axis "x" for different "M".

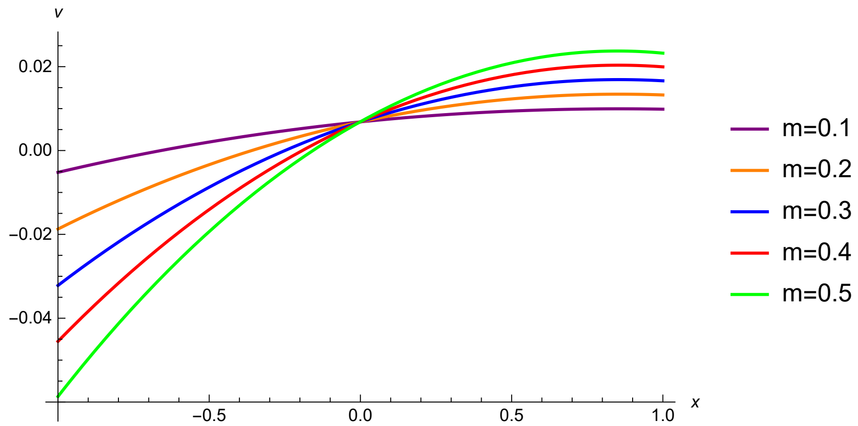

Figure-12:Variation of drainage problem "v" with the variation of axis "x" for different "m".

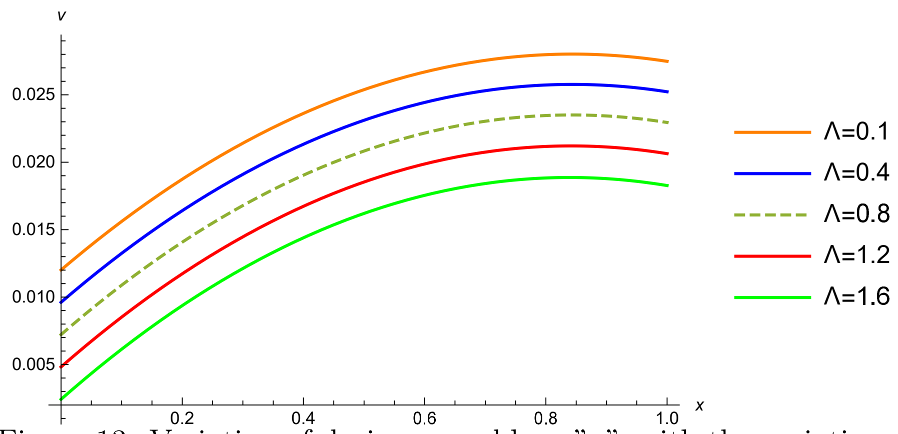

Figure-13: Variation of drainage problem "v" with the variation of axis "x" for different "$\wedge$".

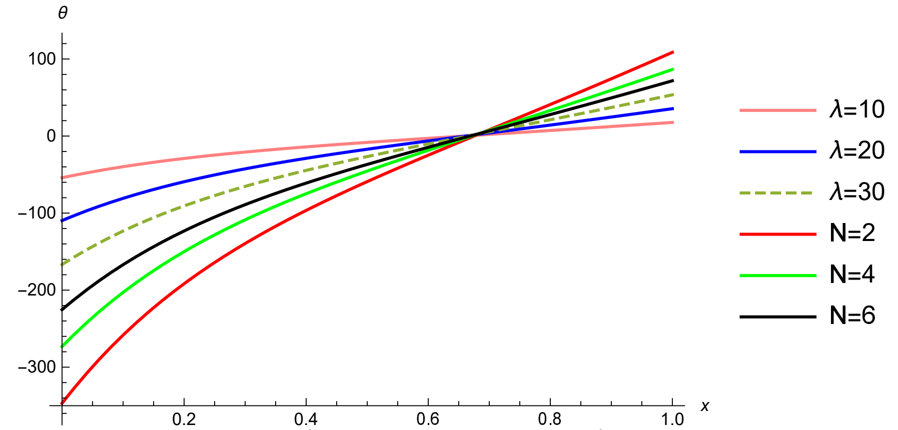

Figure-14: Variation of drainage problem "θ" with the variation of axis "x" for different values "λ, N".

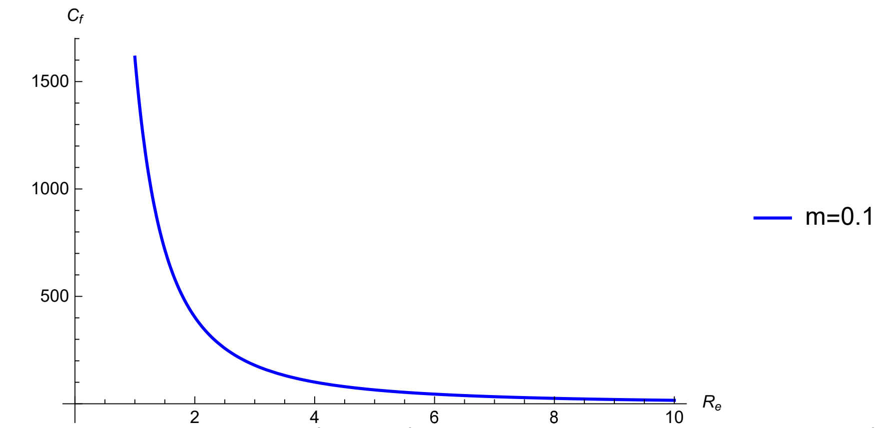

Figure-15: Variation of skin friction versus Reynolds number for fixed value of "m".

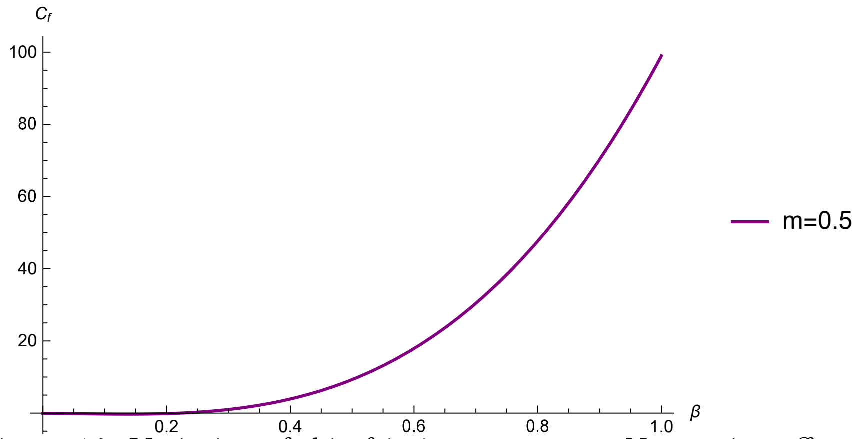

Figure-16: Variation of skin friction versus non-Newtonian effect "β" for fixed value of "m".

5.1 Effects of concentration coefficients in the presence of chemical reaction for both lift and drainage flow:

Figures $(17, 19, 20)$ depicts the coefficient of concentration for both lifting and drainage. Raises for different values of slip parameter $(\wedge)$, non-Newtonian effect (β), magnetic parameter (M) with increases lift concentration while decreases drainage concentration. The reaction rate chemical parameter (γ), gravitational variable (m) are rises in Figs. (18) and (21) which is falling lift problem and enhances drainage problem.

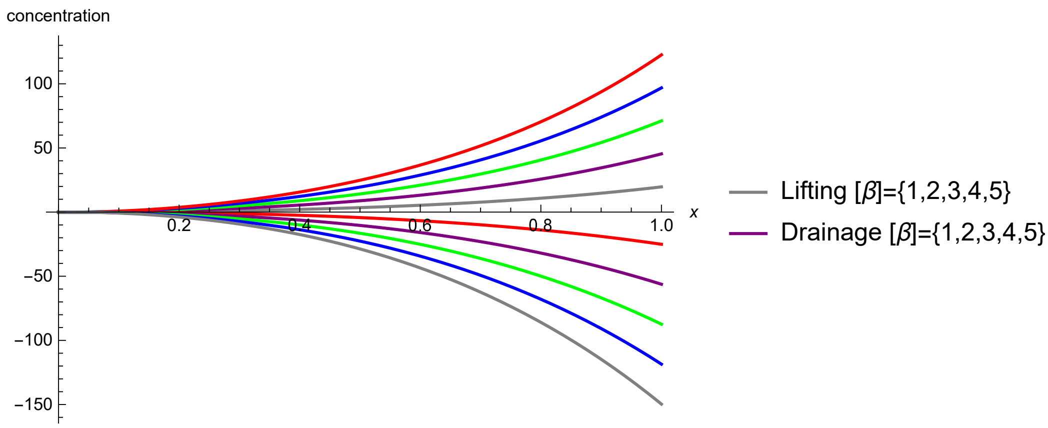

Figure-17: Chemical reaction variation of ("c","c^*") with the variation of axis "x" for different values "β".

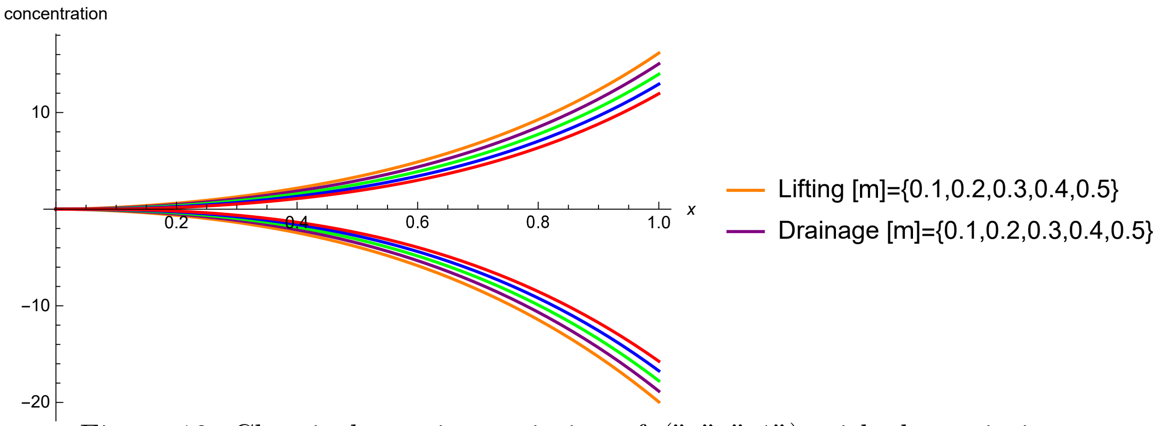

Figure-18: Chemical reaction variation of ("c","c^*") with the variation of axis "x" for different values "m".

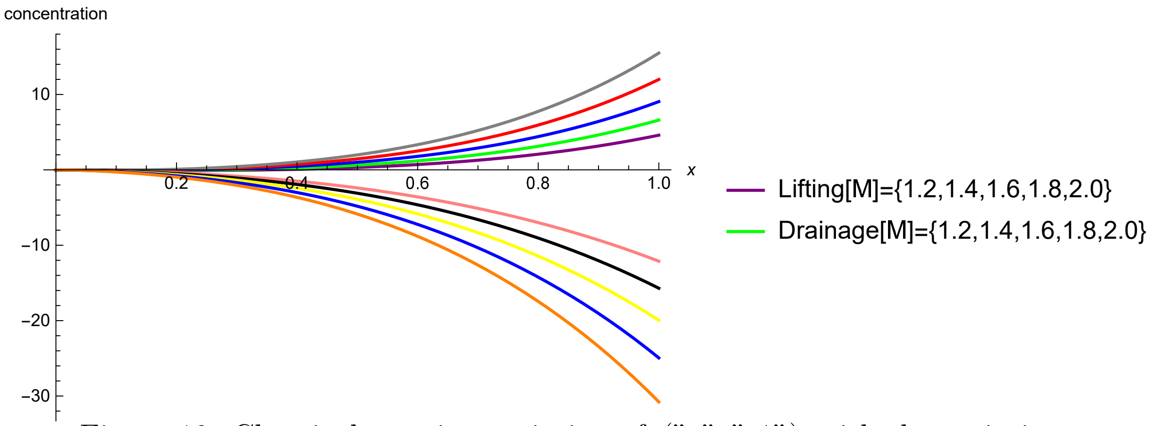

Figure-19: Chemical reaction variation of ("c","c^*") with the variation of axis "x" for different values "M".

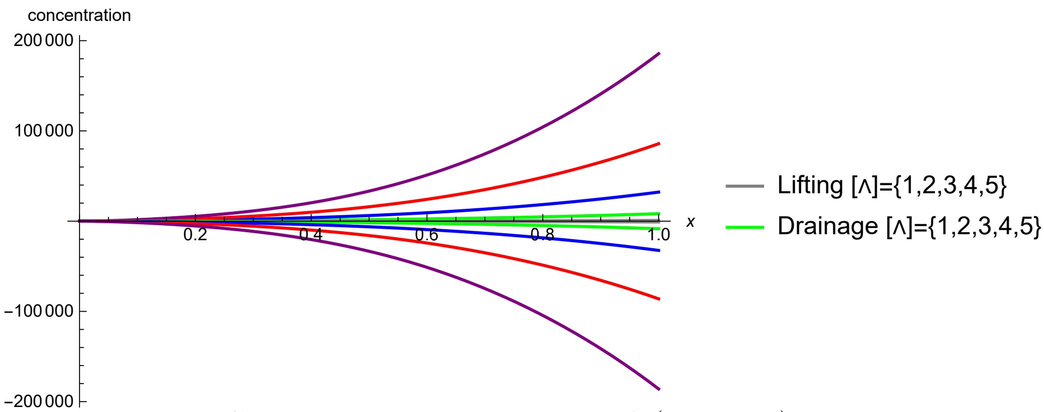

Figure-20: Chemical reaction variation of ("c","c^*") with the variation of axis "x" for different values "$\wedge$".

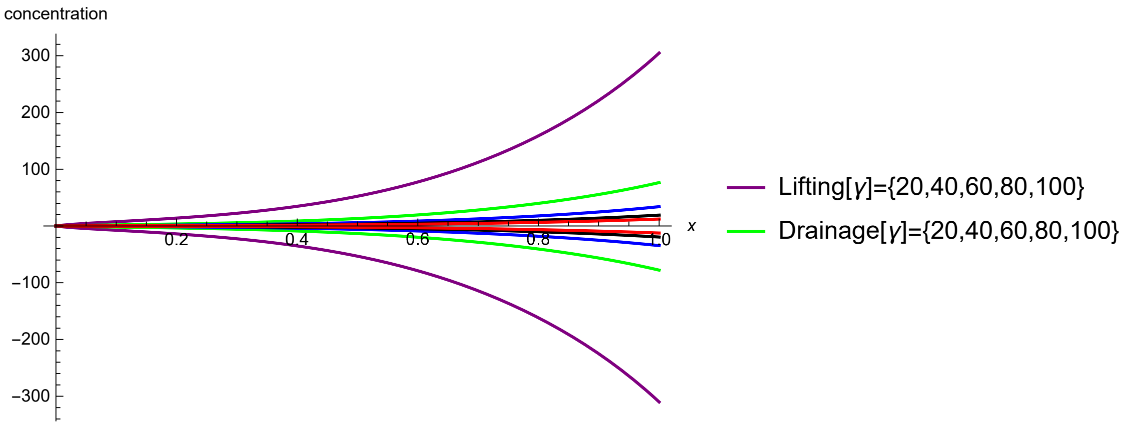

Figure-21: Chemical reaction variation of ("c","c^*") with the variation of axis "x" for different values "γ".

5.2 Effects of concentration coefficients in the absence of chemical reaction for both lift and drainage flow:

Figures (22), (24) and (25) presented, the concentration profile for both flow problems. Rises various values of Magnetic field (M), slip parameter $(\wedge)$, non-Newtonian effect (β), with enhances lift concentration while declined drainage concentration. Gravitational variable (m) are rises in figure (23) which is escalation lift problem and raises drainage problem.

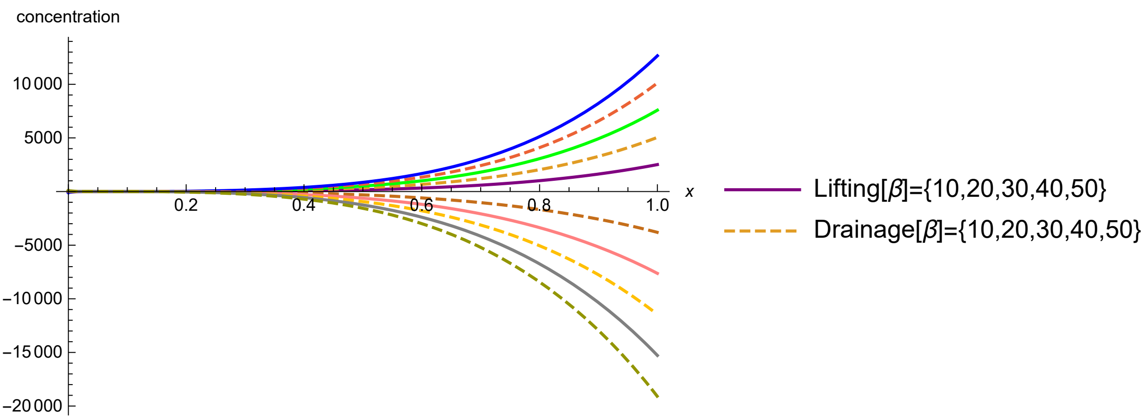

Figure-22:Concentration distribution for "c_1","c_1^*" versus "x" axis for different values of "β".

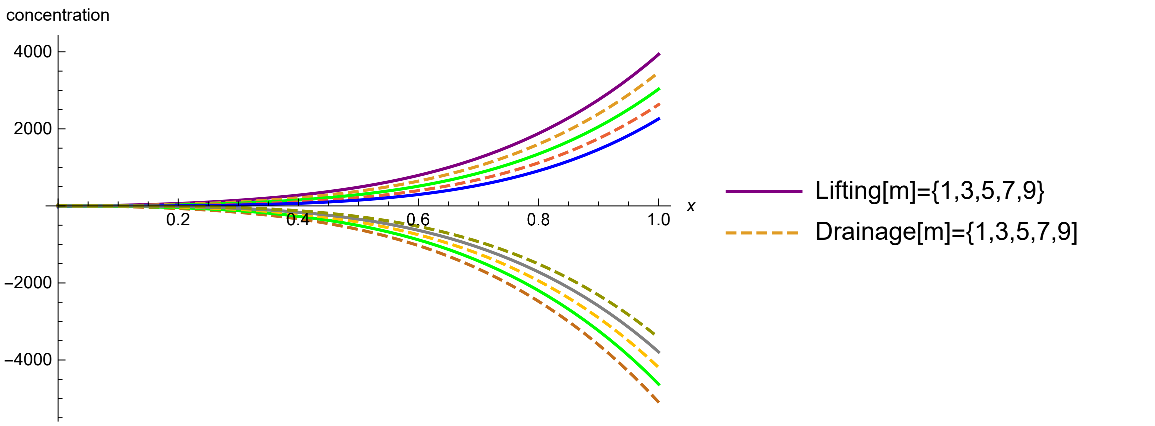

Figure-23:Concentration distribution for "c_1","c_1^*" versus "x" axis for different values of Non-Newtonian effects "m".

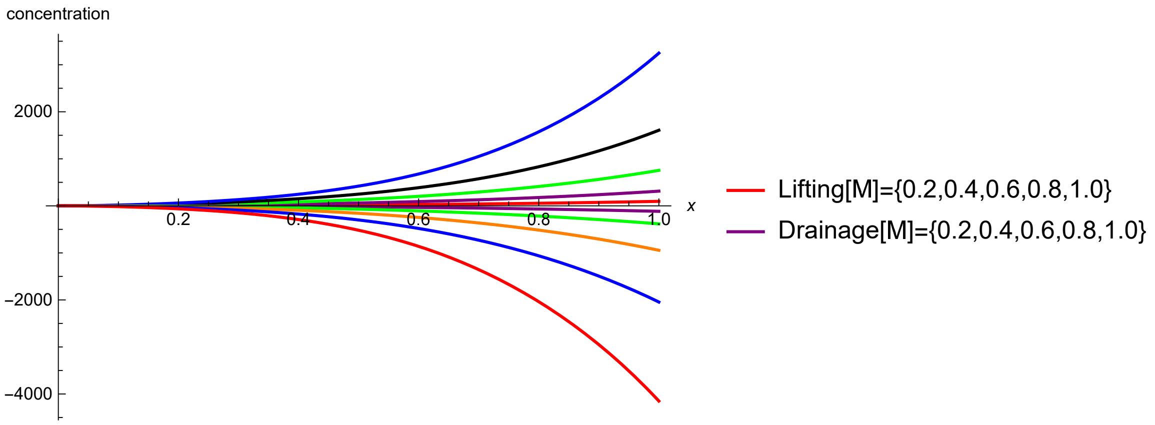

Figure-24:Concentration distribution for "c_1","c_1^*" versus "x" axis for different values of "M".

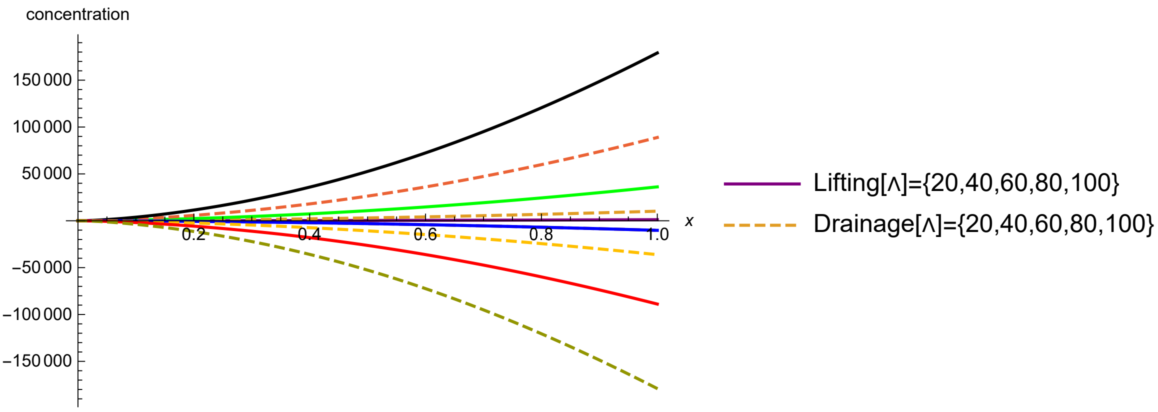

Figure-25:Concentration distribution for "c_1","c_1^*" versus "x" axis for different values of "$\wedge$".

Its conclude that, Increases gravitational parameter (m) and chemical reaction rate (γ) which is falling lift concentration and enhances drainage concentration for with and with out chemical reaction.

6 Conclusion

The problem of MHD, heat and mass transfer of a thin film third grade fluid for lifting and drainage flow in a vertical belt was analyzed analytically by using regular perturbation method. Expression for velocity profile, temperature field, concentration distribution and skin friction has been derived and sketched. The speed of flow enhances as the non-Newtonian parameter (β), slip parameter $(\wedge)$ increases in both flow types. Its most dominant coefficient for this type of problems. An increases in chemical reaction parameter due to an decreased concentration profile for lifting while increased concentration field for drainage. The effects of different embedded flow parameters are discussed and results are displayed in tables and plots.

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Appendix:

$A_1 = (c_5 + \frac{m}{M})(1 - \frac{e^{-\sqrt{M}}}{2})\,; A_2 = (c_5 + \frac{m}{M})\frac{e^{-\sqrt{M}}}{2}\,;$

$A_3 = c_6(1 - \frac{e^{-\sqrt{M}}}{2}) + (m_1 + m_2 + m_3 + m_4 + m_5)\,;\ A_4 = c_6(\frac{e^{-\sqrt{M}}}{2}) - (m_1 + m_2 + m_3 + m_4)\,;$

$A_5 = 1 + \frac{\lambda}{4(1+N)}(MA_1A_2 + \frac{A_1^2}{4}(e^{-2\sqrt{M}} - 1) + \frac{A_2^2}{4}(e^{2\sqrt{M}} - 1))\,; A_6 = \frac{\lambda}{4(1+N)}(\frac{A_1^2}{4} + \frac{A_2^2}{4})\,;$

$A_7 = 1 + \frac{\lambda}{(1+N)}\left\{\frac{p_1e^{-2\sqrt{M}}}{2M} + \frac{p_2e^{2\sqrt{M}}}{2M} + \frac{p_3e^{-4\sqrt{M}}}{8M} + \frac{p_4e^{4\sqrt{M}}}{8M} + \frac{p_5e^{-2\sqrt{M}}}{2M}(1 + \frac{1}{2\sqrt{M}}) + \frac{p_6e^{2\sqrt{M}}}{2M}(1 - \frac{1}{2\sqrt{M}}) + \frac{p_7}{3} + p_8 + p_9 + \frac{n_1e^{-4\sqrt{M}}}{8M} + \frac{n_2e^{4\sqrt{M}}}{8M} + n_3 - \frac{p_1}{2M} - \frac{p_2}{2M} - \frac{p_3}{8M} - \frac{p_4}{8M} - \frac{p_5}{4M\sqrt{M}} + \frac{p_6}{4M\sqrt{M}} - \frac{n_1}{8M} - \frac{n_2}{8M}\right\};$

$A_8 = \frac{\lambda}{(1+N)}\left\{\frac{p_1}{2M} - \frac{p_2}{2M} - \frac{p_3}{8M} - \frac{p_4}{8M} - \frac{p_5}{4M\sqrt{M}} + \frac{p_6}{4M\sqrt{M}} + \frac{n_1}{8M} + \frac{n_2}{8M} + \frac{n_4}{2M} + \frac{n_5}{2M}\right\};$

$m_1 = \frac{c_1}{16M}(e^{-\sqrt{M}} - 3e^{-3\sqrt{M}})\,;\ m_2 = \frac{c_2}{16M}(e^{-\sqrt{M}} + 3e^{-3\sqrt{M}})\,;$

$m_3 = \frac{c_3e^{-\sqrt{M}}}{4\sqrt{M}}(1 - \frac{1}{\sqrt{M}})\,;\ m_4 = \frac{c_4e^{\sqrt{M}}}{4\sqrt{M}}(1 + \frac{1}{\sqrt{M}})\,;\ m_5 = \frac{-1}{8M}(c_1 + c_2)\,;$

$c_1 = -6M^2A_1^3\,; c_2 = -6M^2A_2^3\,;\ c_3 = 6M^2A_2^2A_1\,; c_4 = 6M^2A_1^2A_2\,;$

$p_1 = (MA_1A_3 + \frac{c_4A_1}{2} - \frac{3A_2c_1}{8})\,; p_2 = (MA_2A_4 + \frac{c_4A_2}{2} - \frac{3A_1c_2}{8})\,;$

$p_3 = -\frac{3c_1A_1}{8}\,; p_4 = \frac{3c_2A_2}{8}\,;\ p_5 = -\frac{\sqrt{M}A_1c_4}{2}\,; p_6 = \frac{\sqrt{M}A_2c_3}{2}\,;$

$p_7 = \frac{\sqrt{M}}{2}(A_2c_4 - A_1c_3)\,; p_8 = -\frac{1}{2}(A_2c_4 + A_1c_3)\,;$

$p_9 = -(MA_1A_4 + MA_2A_4)\,; n_1 = (M^2A_1^4 - 4M^2A_1^3A_2)\,;$

$n_2 = (M^2A_2^4 - 4M^2A_2^3A_1)\,;\; n_3 = 6M^2A_1^2A_2^2\,;$

$r_1 = -\frac{A_1^2}{4}\,; r_2 = -\frac{A_2^2}{4}\,; r_3 = -(\frac{p_1}{2M} + \frac{p_5}{4M\sqrt{M}})\,;\; r_4 = -(\frac{p_2}{2M} + \frac{p_6}{4M\sqrt{M}})\,;$

$r_5 = -(\frac{p_3}{8M} + \frac{p_1}{8M})\,; r_6 = -(\frac{p_4}{8M} + \frac{p_2}{8M})\,;\; r_7 = -\frac{p_5}{2M}\,; r_8 = -\frac{p_6}{2M}\,;$

$r_9 = -(p_8 + p_9 + p_3)\,; r_{10} = \beta A_7 + A_8\,;\; r_{11} = -MA_1A_2\,; r_{12} = \beta A_6 + A_7\,; r_{13} = -\frac{p_7}{3}\,;$

$$f_1 = -\left\{(A_1+\beta A_3)(\frac{1-e^{-\sqrt{M}}}{\sqrt{M}})+(A_2+\beta A_4)(\frac{e^{\sqrt{M}}-1}{\sqrt{M}})+\beta\left\{\frac{c_1(1-e^{-3\sqrt{M}})}{24M^{\frac{3}{2}}}+\frac{c_2(e^{3\sqrt{M}}-1)}{24M^{\frac{3}{2}}}+\frac{c_3}{2\sqrt{M}}(\frac{e^{\sqrt{M}}}{\sqrt{M}}(1-\frac{1}{\sqrt{M}})+\frac{1}{M})+\frac{c_4}{2\sqrt{M}}(\frac{e^{-\sqrt{M}}}{\sqrt{M}}(1+\frac{1}{\sqrt{M}})-\frac{1}{M})\right\}\right\};$$

$$f_2 = -\left\{(A_2+\beta A_4)(\frac{e^{\sqrt{M}}}{\sqrt{M}})-(A_1+\beta A_3)(\frac{-e^{\sqrt{M}}}{\sqrt{M}})+\beta(-\frac{c_1e^{-3\sqrt{M}}}{24M^{\frac{3}{2}}}+\frac{c_2e^{3\sqrt{M}}}{24M^{\frac{3}{2}}}+\frac{c_3e^{\sqrt{M}}}{2M}(1-\frac{1}{\sqrt{M}})+\frac{c_4e^{-\sqrt{M}}}{2M}(1+\frac{1}{\sqrt{M}}))\right\};$$

$$f_3 = -\left\{\frac{A_1}{M}+\frac{A_2}{M}+\beta(\frac{A_3}{M}+\frac{A_4}{M}+\frac{c_1}{72M^2}+\frac{c_2}{72M^2}-\frac{c_3}{M^2}-\frac{c_4}{M^2})\right\};$$

$A_1' = (c_5' + \frac{m}{M})(1 - \frac{e^{-\sqrt{M}}}{2})\,; A_2' = (c_5' + \frac{m}{M})\frac{e^{-\sqrt{M}}}{2}\,;$

$A_3' = c_6'(1 - \frac{e^{-\sqrt{M}}}{2}) + (m_1' + m_2' + m_3' + m_4' + m_5')\,;\; A_4' = c_6'(\frac{e^{-\sqrt{M}}}{2}) - (m_1' + m_2' + m_3' + m_4')\,;$

$A_5' = 1+\frac{\lambda}{4(1+N)}(MA_1'A_2'+\frac{A_1^2}{4}(e^{-2\sqrt{M}}-1)+\frac{A_2^2}{4}(e^{2\sqrt{M}}-1))\,; A_6' = \frac{\lambda}{4(1+N)}(\frac{A_1^2}{4}+\frac{A_2^2}{4})\,;$

$$A_7' = 1 + \frac{\lambda}{(1+N)}\left\{\frac{p_1'e^{-2\sqrt{M}}}{2M} + \frac{p_2'e^{2\sqrt{M}}}{2M} + \frac{p_3'e^{-4\sqrt{M}}}{8M} + \frac{p_4'e^{4\sqrt{M}}}{8M} + \frac{p_5'e^{-2\sqrt{M}}}{2M}(1 + \frac{1}{2\sqrt{M}}) + \frac{p_6'e^{2\sqrt{M}}}{2M}(1 - \frac{1}{2\sqrt{M}}) + \frac{p_7'}{3} + p_8' + p_9' + \frac{n_1e^{-4\sqrt{M}}}{8M} + \frac{n_2'e^{4\sqrt{M}}}{8M} + n_3' - \frac{p_1'}{2M} - \frac{p_2'}{2M} - \frac{p_3'}{8M} - \frac{p_4'}{8M} - \frac{p_5'}{4M\sqrt{M}} + \frac{p_6'}{4M\sqrt{M}} - \frac{n_1'}{8M} - \frac{n_2'}{8M}\right\};$$

$$A_8' = \frac{\lambda}{(1+N)}\left\{\frac{p_1'}{2M} - \frac{p_2'}{2M} - \frac{p_3'}{8M} - \frac{p_4'}{8M} - \frac{p_5'}{4M\sqrt{M}} + \frac{p_6'}{4M\sqrt{M}} + \frac{n_1'}{8M} + \frac{n_2'}{8M} + \frac{n_4'}{2M} + \frac{n_5'}{2M}\right\};$$

$m_1' = \frac{c_1'}{16M}(e^{-\sqrt{M}} - 3e^{-3\sqrt{M}})\,;\; m_2' = \frac{c_2'}{16M}(e^{-\sqrt{M}} + 3e^{-3\sqrt{M}})\,;$

$m_3' = \frac{c_3'e^{-\sqrt{M}}}{4\sqrt{M}}(1 - \frac{1}{\sqrt{M}})\,;\; m_4' = \frac{c_4'e^{\sqrt{M}}}{4\sqrt{M}}(1 + \frac{1}{\sqrt{M}})\,;\; m_5' = \frac{-1}{8M}(c_1' + c_2')\,;$

$c_1' = -6M^2A_1^3\,; c_2' = -6M^2A_2^3\,;\; c_3' = 6M^2A_2^2A_1'\,; c_4' = 6M^2A_1^2A_2'\,;$

$p_1' = (MA_1'A_3' + \frac{c_4'A_1'}{2} - \frac{3A_2'c_1'}{8})\,; p_2' = (MA_2'A_4' + \frac{c_4'A_2'}{2} - \frac{3A_1'c_2'}{8})\,;$

$p_3' = -\frac{3c_1'A_1'}{8}\,; p_4' = \frac{3c_2'A_2'}{8}\,;\; p_5' = -\frac{\sqrt{M}A_1'c_4'}{2}\,; p_6' = \frac{\sqrt{M}A_2'c_3'}{2}\,;$

$p_7' = \frac{\sqrt{M}}{2}(A_2'c_4' - A_1'c_3')\,; p_8' = -\frac{1}{2}(A_2'c_4' + A_1'c_3')\,;\; p_9 = -(MA_1'A_4' + MA_2'A_4')\,;$

$n_1^{'} = (M^2A_1^4 - 4M^2A_1^3A_2)\,; n_2^{'} = (M^2A_2^4 - 4M^2A_2^3A_1)\,;$

$n_3^{'} = 6M^2A_1^2A_2^2\,;$

$r_1^{'} = -\frac{A_1^2}{4}\,; r_2^{'} = -\frac{A_2^2}{4}\,; r_3^{'} = -(\frac{p_1}{2M} + \frac{p_5}{4M\sqrt{M}})\,;\; r_4^{'} = -(\frac{p_2}{2M} + \frac{p_6}{4M\sqrt{M}})\,;$

$r_5^{'} = -(\frac{p_3}{8M} + \frac{p_1}{8M})\,; r_6^{'} = -(\frac{p_4}{8M} + \frac{p_2}{8M})\,;\; r_7^{'} = -\frac{p_5}{2M}\,; r_8^{'} = -\frac{p_6}{2M}\,;$

$r_9^{'} = -(p_8 + p_9 + p_3)\,; r_{10}^{'} = \beta A_7 + A_8\,;\; r_{11}^{'} = -MA_1A_2\,; r_{12}^{'} = \beta A_6 + A_7\,; r_{13}^{'} = -\frac{p_7}{3}\,;$

$f_1^{'} = -\left\{(A_1+\beta A_3)(\frac{1-e^{-\sqrt{M}}}{\sqrt{M}})+(A_2+\beta A_4)(\frac{e^{\sqrt{M}}-1}{\sqrt{M}})+\beta\left\{\frac{c_1(1-e^{-3\sqrt{M}})}{24M^{\frac{3}{2}}}+\frac{c_2(e^{3\sqrt{M}}-1)}{24M^{\frac{3}{2}}}+\frac{c_3}{2\sqrt{M}}(\frac{e^{\sqrt{M}}}{\sqrt{M}}(1-\frac{1}{\sqrt{M}})+\frac{1}{M})+\frac{c_4}{2\sqrt{M}}(\frac{e^{-\sqrt{M}}}{\sqrt{M}}(1+\frac{1}{\sqrt{M}})-\frac{1}{M})\right\}\right\};$

$f_2^{'} = -\left\{(A_2+\beta A_4)(\frac{e^{\sqrt{M}}}{\sqrt{M}})-(A_1+\beta A_3)(\frac{-e^{\sqrt{M}}}{\sqrt{M}})+\beta(-\frac{c_1e^{-3\sqrt{M}}}{24M^{\frac{3}{2}}}+\frac{c_2e^{3\sqrt{M}}}{24M^{\frac{3}{2}}}+\frac{c_3e^{\sqrt{M}}}{2M}(1-\frac{1}{\sqrt{M}})+\frac{c_4e^{-\sqrt{M}}}{2M}(1+\frac{1}{\sqrt{M}}))\right\};$

$f_3^{'} = -\left\{\frac{A_1}{M}+\frac{A_2}{M}+\beta(\frac{A_3}{M}+\frac{A_4}{M}+\frac{c_1}{72M^2}+\frac{c_2}{72M^2}-\frac{c_3}{M^2}-\frac{c_4}{M^2})\right\};$