Iterated Function System Consisting of Kannan Contraction in Complete b-Metric Space

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Abstract
The present study is to originate theKannan Iterated Function System in complete b-metric space and we demonstrate several uniqueness and existence of theattractor. Our results deduce and expand several modern consequence correlated with Hutchinson-Barnsley operator and KIFS in complete b-metric space. To obtain our result we using some basic concepts and properties given in the literature. As an application, we formulate Collage theorem for KIFS in complete b-metric spacewhich can be used to solve inverse problems of reconstruction the fractal objects. We also discuss well-posednessfor Kannan mapping in complete b-metric spacewith illustrative example.

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1. Introduction

Firstly, B. Mandelbrot [1] presented the idea of fractal analysis in 1975. We know that, an union of a few self-similar pieces indicates the terminology fractals acquire the similar features in distinct measures. The fractal analysis provide us many research possibilities in engineering, computer science, biology, astronomy and astrophysics, quantum mechanics, pure and applied science etc.

In the nature, fractals found as self- similar images of objects might beconstructed by utilizing IFS. The fundamental definition of IFS was firstly proposed by Hutchinson [2]. The theorywas popularized by Barnsley and Demko[3] and many others. Hutchinson-Barnsley theory (HB theory) initiated and developed by the Hutchinson [2] and Barnsley[4]. Hutichinson[2] designed IFS as a collection of finite set of contraction mappings and presented HB operator.They also construct an attractor of such a system originate through the IFS adopting the Banach fixed point principle. Furthermore, there has been a lot of progress in the investigation of IFS by many researchers. In 2010,Sahu et al.[5] investigated the KIFS based on kannan mappings and established the Collage theorem in the setting of complete metric space. In 2011, S.C. Shrivastava and Padmavati [6] established the collage theorem for IFS under the contraction condition in two mappings. They reported the extension of Hutchinson's classical framework for commuting mapping. Recently, S. C. Shrivastava and Padmavati [7] introduced D-iterated function system they designed IFS in D-metric space and established collage theorem in D-metric space. In 2012, S. C. Shrivastava and Padmavati [8] introduced an iterated function system due to Reich. For years, many researchers studied IFS and they published the research papers of IFS to construct attractors for different spaces and different mappings.

b- metric space is another important development of metric space.Backhtin [9] gave an idea of b-metric spaceadopting that thoughtCzerwik[10] introduced a speculation of the Banach fixed point hypothesis in the framework of b-metric space in 1993. They provided a new concepts and theory for mathematicians. Numerous mathematicians have studied and extended the fixed point theorem inthe same setting. Boriceanu et al.[11] expanded the idea of fractal operator theory for multivalued operators in the same setting. They considered multi- function system and established a fewexistence and uniqueness outcomes in b-metric space for multi-function system and they also considered the well-posedness of the self-similarity problems in the same framework of the system.In 2013, R. Uthaykumar et al. [12] presented Reich IFS and utilizethe HB operator to reconstruct an attractor by applying contractions and they also discussed well-posedness of fixed point problem in the same setting.S. Phiangsungnoen and P.Kumam [13] presented the unique hypothesisof multivalued fuzzy contraction mappings in the same frameworkin 2015 and established the existence of α-fuzzy fixed point theorems in the same framework. They also provided illustrative examples to support their outcome. In 2016, Naziret. al. [14] introduced fractals of generalized F-HBoperator in the same spaceand they employ commutativity assumption on the maps. Recently,Hung et.al. [15] studied fractal theory and introduced multi fractals of generalized MIFS in the same setting. They defined some basic concepts and notion concerning b-metric space, IFS and generalized multi-Iterated Function System. They also demonstrated several existence and uniqueness results and Collage theorem for multifractals in the same framework.

The intension of this study to findKIFS in complete b-metric space. For this purpose we divide our paper into 5 sections. In the section 2, we give some basic concepts, results and hypothesis concerning b-metric spaces & KIFS which is useful for our outcome. In section 3, we verified a few existence and uniqueness results of attractors for KIFS in complete b-metric space. In section 4, we discussed well-posedness problem for kannanmapping in the same setting with illustrative example. In section 5, we conclude our outcome.

2. Preliminaries

A. b- Metric Space

This segmentdefine some essential concepts and results which is valuable for demonstrating our outcome.

Definition 2.1 [16]:Consider X is a non-empty set together with $t \in R,\ where\ t \geq 1$.Let $d: X \times X \to R^+$ is said to be a b-metricif and only iffor every $u, v, w \in X$.If it satisfies the following axioms:

$[b_1] d(u, v) = 0\ iff\ u = v.$
$[b_2] d(u, v) = d(v, u).$
$[b_3] d(u, w) \leq t[d(u, v) + d(v, w)].$

The pair (X, d) is said to be a $b - metric\, space.$

Remark 2.1:

I. If $t = 1$. Then the $b - metric\, space$ can be compressed in metric space.
II. Definition (2.1) represents each metric space be a b-metric space on the hand generally the opposite does not possible.

We discuss some illustration which define$b$-metric spaces,

Example 2.1[17]:Consider$X = \{0,1,2\}$ and $d(2,0) = d(0,2) = m \geq 2$;
$d(0,1) = d(1,2) = d(1,0) = d(2,1) = 1$ and $d(0,0) = d(1,1) = d(2,2) = 0$.

$$d(u,w) \leq \frac{n}{2}[d(u,v) + (v,w)] \qquad \forall\, u,v,w \in X.$$

For $n > 2$, thetriangle inequalitydoes not possible.

Remark 2.2: We note that example (2.1)defined$b$-metric on X but it is not necessary to have the metric on$X$.

Remark 2.3: Usuallyeach metric is a continuous functional in the pair of factors although ab-metricnecessarilyoccupythis axioms, that is ab-metric space but it is not necessary to have the continuous.

Example 2.2[18]: Consider a set $l_p(R)$ along$0 < p < 1$,where$l_p(R) = \{(x_n) \subset R: \sum_{n=1}^{\infty}|x_n|^p < \infty\}$,togetherwith the function $d: l_p(R) \times l_p(R) \to R$,

$$d(x,y) = (\sum_{n=1}^{\infty}|x_n - y_n|^p)^{\frac{1}{p}}$$

Where; $x = \{x_n\},\ y = \{y_n\} \in l_p(R)$ be a b-metric space with $t = 2^{\frac{1}{p}} > 1$.
By an elementary calculation we get,

$$d(x,z) \leq 2^{\frac{1}{p}}[d(x,y) + d(y,z)].$$

Remark 2.4: The above example(2.2) consider the familiar condition$l_p(X)\ with\ 0 < p < 1$, where X be a Banachspace.

Example2.3[16]:Let $l_p[0,1]$ be a space of all real functions$x(u), u \in [0,1]$ such that $\int_0^1|x(u)|^p\,dt < \infty$,together with the function

$$d(x,y) = (\int_0^1|x(u) - y(u)|^p)^{\frac{1}{p}}, \qquad \text{for each } x,y \in L_p[0,1]$$

be ab-metric space with $t = 2^{\frac{1}{p}}$.

Definition 2.2 ([10],[16]): A sequence $\{x_n\}_{n\in N}$ in a b-metric space (X,d) is said to be:

i. Convergent ifffor every$\varepsilon > 0$ and $n \in N\ \exists\ x \in X$ such that $d(x_n,x) < \varepsilon$ i.e. $d(x_n,x) \to 0$as $n \to \infty$. Consider$x$be the limit of the sequence can be expressed as $lim_{n\to\infty} x_n = x$.
ii. Cauchy if and only if for each $\varepsilon > 0$ and $n,m \in N \exists\ x \in X$ such that $d(x_n,x_m) < \varepsilon\ \forall\, n,m \geq N$ i.e. $d(x_n,x_m) \to 0$ as $n,m \to \infty$.

Remark 2.5: Suppose(X,d)be a metric space holds the following properties:

i. If a sequence is convergent then it has a unique limit point.
ii. Every convergent sequence be a Cauchy sequence.

Definition 2.3[16]: Suppose b-metric space(X,d) is complete if every Cauchy sequence in X converges.

Definition 2.4[16]: Suppose(X,d) is a b-metric space.Then a subset Y of X is said to be:

i. Closed iff for every sequence$\{x_n\}_{n\in N}$in Y which converges to an element $x \in Y(i.e.;\ Y = \bar{Y})$.

ii. Compact ifffor every sequence in Yhas a convergent subsequence in$Y$.

iii. Bounded iff$\delta(Y) = sup\{d(a,b): a,b \in Y\} < \infty$.

Definition 2.5 [14]: Let Xbe a complete b-metric space and $H(X)$denotes the family of all non-empty compactsubsets of X.Let $a,b \in X$ and $A,B \in H(X)$. Then the Hausdorff metric is expressed by;

$$h_d(A,B) = max \quad \{ sup_{a\in A} d(a,B) \ , sup_{b\in B}\, d(b,A)\}$$

Where; $d(a,B) = inf\{d(a,b): b \in B\}$. The pair $(H(X), h_d)$ is called Hausdorff metric space (or Fractal space).

Lemma 2.1[16]:Consider(X,d) be a b-metric space.Then for all $A,B,C \in H(X)$, we have

$$h_d(A,C) \leq t[h_d(A,B) + h_d(B,C)].$$

Theorem 2.1[16]: If (X,d) be a complete b-metric space.Then$(H(X), h_d)$ is also a complete b-metric space.

Theorem 2.2(Banach contraction principle [19]): Consider(X,d) be a complete b-metric space with constant $t \geq 1$, such that b-metric is a continuous functional. Suppose that $T: X \to X$ be a contraction mapping with contractivity ratio$s \in [0,1)$ in a complete b-metric space X. Then T possesses exactly one fixed point $x^* \in X$. Moreover, for eachpoint $x \in X$, the sequence $\{T^n(x): n = 0,1,2,3,----\}$ converges to x^*. That is $lim_{n\to\infty} T^n(x) = x^*$, for each $x \in X$.

B. K-Iterated Function Systemin metric space

Kannan[20] introduced a new mapping in 1969, which was defined as follows:

Definition 2.6 [20]: Let X be a metric space with distance function d and T be a self- mapping from X into itself. Then T is said to be a Kannan mapping if there exists a K-contractivity factor s, $0 < s < \frac{1}{2}$, such that for every$x,y \in X$,

$$d\big(T(x),T(y)\big) \leq s[d\big(x,T(x)\big) + d\big(y,T(y)\big)]$$

Sahuet. al.[5] established the concept of$K$-IFS in complete metric space based onIterated Function System provided by Barnsley[4], which was defined as follows:

Definition 2.7[5]:A K-IFS consistingof a complete matric space(X,d) together with a definite collection of contraction mappings $T_n: X \to X$, with K-contractivity factor s_n, for all$n \in N$.

Theorem 2.3[5]: Let $\{X: T_n, n = 1,----,N\}$ bea K-IFS with K-contractivity factor s. Then the mappingW:H(X)→H(X) represented by $W(B) = \cup_{n=1}^{N} T_n(B)$for every $B \in H(X)$ is a continuous Kannan mapping on the complete metric space $(H(X), h)$ with contractivity ratio s. Its unique $A \in H(X)$, exists and is given by $A = lim_{n\to\infty} W^{on}(B)$ for any $B \in H(X)$.

Definition 2.8[5]: The fixed point $A \in H(X)$ of the HB operator W describe in Theorem(2.3)is said to be attractor of the KIFS in complete metric space.

Theorem 2.4(Collage Theorem for KIFS in Metric space [5]): Consider(X,d) be a complete metric space.Suppose$L \in H(X)$and $\varepsilon \geq 0$ be given. ChooseKIFS$\{X: (T_o), T_1, T_2, ------, T_N\}$, where T_0 is the condensation mapping with K-contractivity ratio s, such that

$$h_d\left(L\,,\bigcup_{n=0,n=1}^{N} T_n(L)\right) \leq \varepsilon$$

Then

$h_d(L,A^*) \leq \varepsilon\dfrac{1-s}{1-2s}$

The notation A^* is called attractor of the KIFS. Equivalently,

$$h_d(L,A^*)) \leq \left(\frac{1-s}{1-2s}\right)h_d\left(L\,,\bigcup_{n=0,n=1}^{N} T_n(L)\right)$$

For all $L \in H(X)$.

3. KIFS in b-metric space

Throughout this portion, we will attempt to research the chance of progress in KIFS by supplantingcomplete metric space by a complete b-metric space.

Now, first we present the fixed point theorem for Kannan mapping in b-metric space.

Theorem 3.1: Assume that (X,d)be a complete b-metric space. Consider$T:X \to X$ be a Kannan contractionmapping satisfying$d\big(T(x),T(y)\big) \leq s\big[d\big(x,T(x)\big)+d\big(y,T(y)\big)\big]$for all $x,y \in X$on a metric space(X,d),where$s$is a K-contractivity factor$s$and satisfying$0 < s < \frac{1}{2}$. Then T has a exactly one fixed point.

Proof: -Now, the verificationof above statementwill be possibleby two steps.

Step I: Firstly, we have to show that the existence of the fixed point.

$d(T^n x,T^{n+1}x) = d(T(T^{n-1}x),T(T^n x))$
$\leq s[d(T^{n-1}x,T^n x)+d(T^n x,T^{n+1}x)]$
$\leq sd(T^{n-1}x,T^n x)+sd(T^n x,T^{n+1}x)$
$(1-s)d(T^n x,T^{n+1}x) \leq sd(T^{n-1}x,T^n x)$
$d(T^n x,T^{n+1}x) \leq \left(\dfrac{s}{1-s}\right)d(T^{n-1}x,T^n x)$
$d(T^n x,T^{n+1}x) \leq kd(T^{n-1}x,T^n x)$
Where,$k = \left(\frac{s}{1-s}\right)$, $0 < s < \frac{1}{2}$.
$\leq k[kd(T^{n-2}x,T^{n-1}x)]$
$\leq k^2[d(T^{n-2}x,T^{n-1}x)]$
$\leq k^3[d(T^{n-3}x,T^{n-2}x)]$

$d(T^n x,T^{n+1}x) \leq k^n[d(x,Tx)]$

Presently, we will demonstrate the sequence$\{x_n\}$be a Cauchy sequence in$X$.

We know that,
$d(T^n x,T^{n+m}x) \leq t[d(T^n x,T^{n+1}x)+d(T^{n+1}x,T^{n+m}x)]$
$\leq td(T^n x,T^{n+1}x)+t\{t[d(T^{n+1}x,T^{n+2}x)+d(T^{n+2}x,T^{n+m}x)]\}$
$\leq td(T^n x,T^{n+1}x)+t^2 d(T^{n+1}x,T^{n+2}x)+ ------+t^m d(T^{n+m-1}x,T^{n+m}x)$
$\leq tk^n d(x,Tx)+t^2k^{n+1}d(x,Tx)+ --------+t^m k^{n+m-1}d(x,Tx)$

$\leq tk^n[1 + tk + t^2k^2 + ------------ + t^{-1}k^{m-1}]d(x,Tx)$
$\leq tk^n(1 - tk)^{-1}d(x,Tx)$
$\leq \frac{tk^n}{(1-tk)} d(x,Tx)$ $\forall\ n,m \geq N.$
Taking $n \to \infty$, $since\ k^n \to 0.$
It follows that; $lim_{n\to\infty} d(T^n x, T^{n+m}x) = 0$ $since,\ \frac{s}{1-s} < 1.$
Hence, $\{T^n x\}$ be a Cauchy sequence.
Since, X be a complete b-metric space, $\exists\ p \in X$ s.t. $T^n(x) \to p$.

$d(p,Tp) \leq t[d(p,T^{n+1}p) + d(T^{n+1}p,Tp)]$
$\leq t[d(p,T^{n+1}p) + d(T(T^n p),Tp)]$
$\leq t\{d(p,T^{n+1}p) + s[d(T^n p.T^{n+1}p) + d(p,Tp)]\}$
$\leq td(p,T^{n+1}p) + ts\, d(T^n p,T^{n+1}p) + ts\, d(p,Tp)$
$(1 - st)d(p,Tp) \leq td(p,T^{n+1}p) + ts\, d(T^n p.T^{n+1}p)$
$d(p,Tp) \leq td(p,T^{n+1}p) + ts\, d(T^n p,T^{n+1}p)$
$d(p,Tp) \leq (\frac{1}{1-st})[td(p,T^{n+1}p) + tsk^n d(x,Tx)]$
Taking $n \to \infty$, which implies that
$d(p,Tp) = 0$
Implies that, p is the unique fixed point of T.
Hence, $p = Tp$.

Step II: Now, we will prove that the uniqueness of the fixed point.
We consider,

p^* be the unique fixed point of T.
$i.e.,$ $p^* = Tp^*$.
Suppose that $\exists$ one more fixed point $q^* \in X$ s.t. $q^* = Tq^*$.
$d(p^*,q^*) = d(Tp^*,Tq^*)$
$\leq s[d(p^*,Tp^*) + d(q^*,Tq^*)]$
$\leq s[d(p^*,p^*) + d(q^*,q^*)]$
$d(p^*,q^*) = 0.$
Therefore; $p^* = q^*$.

We need the verification of subsequent lemmas to establishment of KIFS in the new framework.

Lemma 3.1: Suppose that X be a complete b-metric space with metric d. Consider the mapping $T: X \to X$ be a continuous Kannan mapping satisfying the inequality,

$$d(T(x),T(y)) \leq s[d(x,T(x)) + d(y,T(y))]$$

for all $x, y \in X$, where s is a K-contractivity factor and satisfying $0 < s < \frac{1}{2}$.
Then $T: H(X) \to H(X)$ defined by $T(B) = \{T(x): x \in B\}$ is also a Kannan contraction mapping satisfies the inequality,

$$h_d(T(B),T(C)) \leq s[h_d(B,T(B)) + h_d(C,T(C))]$$

for each $B, C \in H(X)$ with k-contractivity factor s.

Proof: by definition of K –Iterated Function System;

$d(T(B),T(C)) = s[d(B,T(B)) + d(C,T(C))]$ $\quad \forall\ B,C \in H(X)$
and $d(T(C),T(B)) = s[d(C,T(C)) + d(B,T(B))]$ $\quad \forall\ B,C \in H(X)$

Let $B,C \in H(X)$, Then

$$h_d(T(B),T(C)) = d(T(B),T(C)) \vee d(T(C),T(B))$$
$$\leq s[d(B,T(B)) + d(C,T(C))] \vee s[d(C,T(C)) + d(B,T(B))]$$
$$\leq s\{[d(B,T(B)) + d(C,T(C))] \vee [d(C,T(C)) + d(B,T(B))]\}$$
$$= s[d(B,T(B)) + d(C,T(C))]$$
$$h_d(T(B),T(C)) \leq s[h_d(B,T(B)) + h_d(C,T(C))]$$

Lemma 3.2: Let (X,d) is a complete b-metric space. Consider $\{T_n : n = 1, - - - -, N\}$ be continuous Kannan mappings which maps $(H(X), h_d)$ into $(H(X), h_d)$. Let the K-contractivity factor for T_n be denoted by s_n for each n. Define $W: H(X) \to H(X)$ by $W(B) = T_1(B) \cup T_2(B) \cup - - - - - \cup T_n(B) = \cup_{n=1}^{N} T_n(B)$ for every $B \in H(X)$. Then T also satisfying

$$h_d(W(B),W(C)) \leq s[h_d(B,W(B)) + h_d(C,W(C))]$$

for all $B,C \in H(X)$, with K-contractivity factor $s = max\{s_n : n = 1,2,3, - - - - - -, N\}$.

Proof: we have to using mathematical induction method to prove of the lemma;

The lemma is obviously valid for $N = 1$.
Now, for $N = 2$, we see that

$$h_d(W(B) \cup W(C) = h_d(T_1(B) \cup T_2(B), T_1(C) \cup T_2(C))$$
$$\leq h_d(T_1(B),T_1(C)) \vee h_d(T_2(B),T_2(C))$$
$$\leq s_1[h_d(B,T_1(B)) + h_d(C,T_1(C))] \vee s_2[h_d(B,T_2(B)) + h_d(C,T_2(C))]$$
$$\leq (s_1 \vee s_2)\{[h_d(B,T_1(B)) \vee h_d(B,T_2(B))] + [h_d(C,T_1(C)) \vee h_d(C,T_2(C)]\}$$
$$\leq s[h_d(B,T_1(B) \cup T_2(B)) + h_d(C,T_1(C) \cup T_2(C))]$$

Therefore;

$$h_d(W(B),W(C)) \leq s[h_d(B,W(B)) + h_d(C,W(C))]$$

Lemma (3.2) is verified through the mathematical induction principle.

Hence, we explain KIFS with the help based on previous provided outcomes and definitions. Now, we introduce an interesting result for KIFS in the new setting.

Definition 3.1: A K-IFS consisting of a complete b −metric space (X, d) together with a definite collection of contraction mapping $T_n: X \to X$ with K-contractivity ratio s_n for $n = 1, 2, - - - -, N$.

The representation of the IFS just introduced is $\{X: T_n, n = 1,2,3 - - - -, N\}$ and its K-contractivity ratio is $s = max\{s_n : n = 1,2,3, - - - -, N\}$.

Theorem 3.2: Consider (X,d) is a complete b-metric space. Let $T_n: X \to X$ are continuous and satisfies the kannan contractive condition as
$d(T_n x, T_n y) \leq s_n[d(x, T_n x) + d(y, T_n y)]$
for all $x, y \in X$, where s_n are K −contractivity factors with $0 < s_n < \frac{1}{2}$.

Then the transformation $W: H(X) \to H(X)$ represented by $W(B) = \bigcup_{n=1}^{N} T_n(B)\, for\ all B \in H(X)$ also satisfies the kannancontractivity condition,
$h_d(W(B), W(C)) \leq s[h_d(B, W(B)) + h_d(C, W(C)]$
Where; $B, C \in H(X), s = max\{s_n: n = 1,2, - - - - -, N\}$. Its unique fixed point in $(H(X), h_d)$, which is also known as attractor, $A^* \in H(X)$ satisfying $A^* = W(A^*) = \bigcup_{n=1}^{N} T_n(A^*)$ and is given by $A^* = lim_{n\to\infty} W^{on}(B)\, for\ any B \in H(X)$, where W^{on} denotes the n −fold compostion of W.

Proof: Let (X, d) is acomplete b −metric space.
Then $(H(X), h_d)$ is a Complete Hausdorff b −metric spaceby theorem(2.1).
Also, theHB operator W is Kannan contraction mapping by lemma(3.2).
Hence; we conclude that W has a unique fixed point bykannan fixed point theorem (3.1).

Now, we develop a Collage theorem for KIFSin b −metric space.

Theorem 3.3: Consider (X, d) is a complete b–metric space. Assume that $L \in H(X)$ and $\varepsilon \geq 0$ be given. Choose KIFS $\{X: (T_0), T_1, T_2, - - - - -, T_N\}$, where T_0 is the condensation mapping with k −contractivityratio s, such that

$$h_d(L, \cup_{n=0,n=1}^{N} T_n(L)) \leq \varepsilon$$

Then;

$$h_d(L, A^*) \leq \varepsilon\left(\frac{t}{1-tk}\right)$$

The notation A^* is calledattractor of the KIFS.Equivalently;

$$h_d(L, A^*) \leq \left(\frac{t}{1-tk}\right) h_d(L, W(L))$$

Or;

$$h_d(L, A^*) \leq \left(\frac{t}{1-tk}\right) h_d(L\,, \cup_{n=1}^{N} T_n(L))$$

holds for all $L \in H(X)$.

Proof: Consider (X, d) be a complete b −metric space.
Applying definition of b −metric space, we know that

$h_d(L, W^n(L) \leq t[h_d(L, W(L)) + h_d(W(L), W^n(L))]$
$\leq th_d(L, W(L)) + th_d(W(L), W^n(L))$
$\leq th_d(L, W(L)) + t\{t[h_d(W(L), W^2(L)) + h_d(W^2(L), W^n(L))]\}$
$\leq th_d(L, W(L)) + t^2 h_d(W(L), W^2(L)) + t^2 h_d(W^2(L), W^n(L))$
Continue these process, we get
$\leq th_d(L, W(L)) + t^2 h_d(W(L), W^2(L)) + - - - - - - + t^n h_d(W(L), W^n(L))$(1)

Since W is a Kannan contraction mapping, we have

$h_d(W^n(L), W^{n+1}(L)) = h_d(W(W^{n-1}(L), W(W^n(L))$
$\leq s\{h_d(W^{n-1}(L), W^n(L)) + h_d(W^n(L), W^{n+1}(L))\}$
$\leq sh_d(W^{n-1}(L), W^n(L)) + sh_d(W^n(L), W^{n+1}(L)$
$(1 - s)h_d(W^n(L), W^{n+1}(L)) \leq sh_d(W^{n-1}(L), W^n(L))$
$h_d(W^n(L), W^{n+1}(L)) \leq \left(\frac{s}{1-s}\right) h_d(W^{n-1}(L), W^n(L))$
$h_d(W^n(L), W^{n+1}(L)) \leq k\, h_d(W^{n-1}(L), W^n(L))$

Where; $k = \frac{s}{1-s}$, $0 < s < \frac{1}{2}$.

$\le k[k\, h_d(W^{n-2}(L), W^{n-1}(L))]$
$\le k^2[h_d(W(L), W^{n-1}(L)]$

$h_d(W^n(L), W^{n+1}(L)) \le k^n h_d(L, W(L))$(2)

From equation (1) & equation (2); we get

$$h_d(L, W^n(L)) \le t h_d(L, W(L)) + t^2 k h_d(L, W(L)) + ------ + t^n k^{n-1} h_d(L, W(L))$$
$$\le t[1 + tk + -------- + t^{n-1}k^{n-1}] h_d(L, W(L)); \quad for\ \ tk < 1$$
$$\le t\left[\frac{1-(tk)^n}{1-tk}\right] h_d(L, W(L)) \quad ; \quad for\ \ tk < 1$$

Taking the limit as $n \to \infty$, *we get*

$$h_d(L, W^n(L)) \le \left(\frac{t}{1-tk}\right) h_d(L, W(L))$$

Or; $$h_d(L, A^*) \le \left(\frac{t}{1-tk}\right) h_d(L, \cup_{n=1}^{N} T_n(L))$$

holds for all$L \in H(X)$.

Remark3.1: The above theorem(3.3) shows that; the Hausdorff distance amongfractal and Collage of the picture is little. At that point the distance of the attractor of KIFS from the fractal will be little. Clearly, aKIFS has anextraordinaryattractor in complete b-metric space.

Remark 3.2: We observe that our result(3.2)is a generalization of the fundamental outcomes concerning the well- knownHutchinson IFS and the K-IFS, respectively.

4. Well-Posedness

Now, we present well-posedness concept for KIFS in the new framework.

Definition 4.1([11],[21],[22]): Consider(X, d) is acomplete b-metric space. Assume that$T: X \to X$ is an operator.Then$T$ is calledwell-posed $iff\ Fix(T) = \{p\}$ and if $\{x_n\}_{n\in N}$be a sequence in X such that $d(x_n, Tx_n) \to 0\ as\ n \to \infty$,then $d(x_n, p) \to 0\ as\ n \to \infty$.

Example 4.1: Consider (X, d) is a complete b-metric space**.** Assume that$T$ is aKannancontration mapping from X into itself. Then, T is well-posed.Indeed $Fix(T) = \{p\}$and suppose a sequence $\{x_n\}_{n\in N}$ in X such that $d(x_n, Tx_n) \to 0$ as $n \to \infty$.

We know that;

$$d(x_n, p) \le t[d(x_n, Tx_n) + d(Tx_n\ , Tp)]$$
$$\le t\{d(x_n, Tx_n) + s[d(x_n, Tx_n) + d(p, Tp)]\}$$
$$\le td(x_n, Tx_n) + tsd(x_n, Tx_n) + tsd(p, Tp)$$
$$\le t(1+s)d(x_n, Tx_n) \to 0\ \ as\ n \to \infty.$$
$$d(x_n, p) \to 0\ \ as\ \ n \to \infty$$

5. Conclusion

In the present work we investigate theKIFS in complete b-metric space and establish the Hutchinson-Barnsley results for an IFSof kannanmappingbythe new setting of the space. We demonstrated a fewinteresting theory forKIFS in the usualcomplete b −metric space. Some of the recent results established in this paper which are true for KIFS. We also established Collage theorem which can be devoted to obtain anattractor for a KIFS and determined inverse problems of reconstruction the fractal objects. In addition, we proved well-posedness problem in a different setting.

References

1. B. B. Mandelbrot, The fractal geometry of nature, W. H. Freeman and Company, New York, (1983).
2. J. E. Hutchinson, Fractals and self similarity, Indiana University Mathematics Journal, 30 (5) (1981), 713-747.
3. M. F. Baransley and S. Demko, Iterated function systems and global construction of fractals, Proc. Roy. Soc. London. Ser A, 399 (1985),243-275.
4. M.F. Baransley, Fractals Everywhere, New York, Academic press, (1988).
5. D. R. Sahu, A. Chakraborty, K-iterated function system, Fractals 18(1) (2010), 139-144.
6. S. C. Shrivastava, Padmavati, Iterated function system for commuting mapping, International Journal of Pure and Applied Mathematics, 70(7) (2011), pp. 975-982.
7. S. C. Shrivastava, Padmavati, Iterated function system in D-metric space, Applied Mathematical Sciences, 5(35) (2011), pp. 1705-1711.
8. S. C. Shrivastava ,Padmavati, An iterated function system due to Riech, Bulletin Culcutta Mathematical Society, 104(3) (2012), pp. 211-218.
9. I. A. Backhtin, The contraction mapping principle in almost metric spaces, Funct. Anal. UnianowskGos. Ped. Inst., 30 (1989), 26-37.
10. S. Czerwik, Contraction mappings in b-metric spaces, Acta mathematical etInformaticaUnivercitatisOstraviensis 1(1993), 5-11.
11. M. Boriceanu, M. Bota, A. Petrusel, Multivalued fractals in b-metric space, Cent. Eur. J. Math., 8(2) (2010), 367-377.
12. R. Uthaykumar, G. A. Prabakar, An Iterated Function System for Reich contraction in complete b-metric space, World Academy of Science, Engineering and Technology, International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, 7(11) (2013), 1640-1643.
13. S. Phiangsungnoen, P. Kumama, Fuzzy fixed point theorem for multivalued fuzzy contractions in b-metric spaces, J. Nonlinear Sci. Appl. 8(2015), 55-63.
14. T. Nazir, S. Silvestrov, M. Abbas, Fractals of generalized F-Hutchinson operator, Waves, Wavelets Fractals, 2(2016), 29-40.
15. S. Kumari, R. Chungh, J.Cao, C.Hung, Multi fractals of generalized multivalued iterated function system in b-metric spaces with applications, 7(967)(2019),1-17, doi:10.3390/math7100967.
16. S. Czerwik, Nonlinear Set valued contraction mappings in b-metric spaces, Atti Sem. Mat. Univ. Modena, 46(1998), 263-276.
17. S. L. Singh and Bhagwati Prasad, Some coincidence theorems and stability of iterative procedures, Computers and Mathematics with Applications 55(2008) 25122520.
18. M. Boriceanu, Fixed point theory for multivalued generalized contraction on a set with two b-metric, studia, univ Babes, Bolya: Math, Liv (3)(2009), 1-14.
19. N.Kir, H.Kiziltun, On some well known fixed point theorems in b-metric spaces, Turk. J. Anal.Number Theory 2013, 1, 13-16.

20. R. Kannan, Some results on fixed points-II, Am. Math. Mon. 76(1969), 405-408.
21. F. S. De Blasi and J. Myjak, Sur la porosite des contractions sans point fixe, C.R. Acad. Sci. Paris. 308(1989), 51-54.
22. S. Reich and A. J. Zaslavski, Well posedness of fixed point problems, Far East J. Math. Sci. Special Volume Part III (2001), 393-401.