Two New Generalized Parametric Intuitionistic Fuzzy Information Measures

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Abstract:

In today's life human thinking depends on uncertainty and fuzziness.Predominantly in the faltering atmosphere, the precise value of some factors is difficult to measure. Though, it can be easily approximated by intuitionistic fuzzy linguistic term in the real-life world problem. To deal with such uncertainty and fuzziness in present communication fuzzy information ordered measure defined on intuitionistic fuzzy set. Baiget. al.[18] and O. Prakash[19] defined a fuzzy entropy measure on fuzzy set and we generalize this entropy on intuitionistic fuzzy set. To verify the validity of proposed measure some properties, tables and numerical example is given.

Keywords: Fuzzy logic, entropy, information theory, fuzzy sets, intuitionistic fuzzy set and image processing.

Introduction:

Quantification, storage and communication of information are main tool in our daily life. For better solution of all these factors the concept was given by C. E. Shannon [2] in 1948. But in decision making there is always uncertainty and hesitation. The prolegomena of fuzzy set was firstly given by Zadeh [3] and for bargain with uncertainty and hesitation Zadeh [4] defined fuzzy entropy measure. In other words we can say that entropy, degree of uncertainty is defined by Zadeh [4]. In modern research of information theory, fuzzy set is back bone and main concept is membership and non-membership value between 0 and 1. With the help of all these factors, four axioms were given by Deluca and Termini [5] for entropy of fuzzy set and under these conditions various authors defined many types of entropy on fuzzy set. Bhandari and Pal [6], Pal and Pal [7, 8], Hooda [9], and Tomar and Ohlan [1] introduced the measure on fuzzy set. These entropies solve lot of problem of uncertainty.A generalization of theory of fuzzy set is given by Atanassov [10]. Intuitionistic fuzzy set introduced by Atanassov [10] parallel to fuzzy set. In crisp theory to deal with uncertainties and impreciseness intuitionistic fuzzy theory has an important role. Axioms defined by Deluca and Termini [4] are extended by Szmidt and Kacprzyk [11] to intuitionistic fuzzy set. Thereafter Hung and Yang [12] defined a entropy measure on intuitionistic fuzzy set. Measure defined by De Luca and Termini [5] is generalized by Zhang and Jiang [13] on intuitionistic fuzzy set. Ye [14] proposed two entropy measures on intuitionistic fuzzy set to extend the entropy measure given by Prakash et al. [15].

Keeping in mind all the above studies of entropy, the present article proposed an ordered entropy measure on intuitionistic fuzzy set which is called intuitionistic entropy measure. The proposed measure is generalization of entropy defined by Baig et. al. [18] and O. Prakash[19]on fuzzy set.

Preliminaries:

In this portion we define some definition and notation about fuzzy set and intuitionistic fuzzy set. The detail of intuitionistic fuzzy set are given which are important for our next discussed

Definition: Entropy is the information required to specify the state of system. Entropy defined by Shannon is given as:

$$H(P) = -\sum_{i=1}^{n} p(y_i)\log\ p(y_i)$$

Definition: Let $X = (y_1, y_2, ... y_n)$ be a finite universe of discourse and a fuzzy set A' on X is given as:

$$A' = \{< y, \mu_{A'}(y) > / y \in X\}$$

where, $\mu_{A'}(y) : X \to [0,1]$ is the membership function of A'.

Pal and Pal [8] introduced the fuzzy exponential entropy for fuzzy set A' given by

$$e^{H(A')} = \frac{1}{n(\sqrt{e}-1)}\sum\left[(\mu_{A'}(x_i))e^{(1-\mu_{A'}(x_i))} + (1-\mu_{A'}(x_i))e^{(\mu_{A'}(x_i))} - 1\right]$$

Tomar and Ohlan [1] defined exponential ordered fuzzy entropy of order α on fuzzy set which is given as:

$$E_R^{\alpha}(A) = \frac{R}{R-\alpha}\sum_{i=1}^{n}\left[1 - \left\{\mu_A^{\frac{R}{\alpha}}(y_i) + (1-\mu_A(y_i))^{\frac{R}{\alpha}}\right\}^{\frac{\alpha}{R}}\right], \quad 0 < \alpha \le 1,\ R(>0) \ne 1.$$

Baig et. al. [18]defined fuzzy entropy measure as

$$M_R^{\alpha}(A) = \frac{1}{\alpha-1}\sum_{i=1}^{n}\left[\left\{\mu_A^{2-\alpha}(y_i) + (1-\mu_A(y_i))^{2-\alpha}\right\} - 1\right], \quad \alpha > 0,\ \alpha \ne 1$$

O. Prakash[19] defined fuzzy entropy measure as

$$M_R^{\alpha}(A) = \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\left\{\mu_A^{\alpha}(y_i) + (1-\mu_A(y_i))^{\alpha}\right\}^{\beta} - 1\right], \quad \alpha, \beta > 0,\ \alpha \ne 1$$

Definition: Let $X = (y_1, y_2, ... y_n)$ be a finite universe of discourse and Atanassov [10] introduced an intuitionistic fuzzy set A in X as

$A = \{< y, \mu_A(y), \nu_A(y) > / y \in X\}$ where degree of membership and non-membership value of y w.r.t A is denoted by $\mu_A(y)$ *and* $\nu_A(y)$ respectively and $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ with the condition $0 \le \mu_A(y) + \nu_A(y) \le 1$ for all $y \in X$. Let K be a real function defined on intuitionistic fuzzy set such that $K : IFS(X) \to R^+$ and if K satisfies the following properties then it is called entropy measure on intuitionistic fuzzy set

(K_1) $K(A) = 0$ if A is a crisp set

(K_2) $K(A)$ assumes a unique maximum if $\mu_A(y_i) = \nu_A(y_i)$

(K_3) $K(A)$ decreases as set get sharpened

(K_4) $K(A) = K(A^C)$

Definition: Let $X = (y_1, y_2, \ldots.. y_n)$ be a finite universe of discourse and A be a intuitonistic fuzzy set in X . For each A if

$\pi_A(y) = 1 - \mu_A(y) - \nu_A(y)$, then $\pi_A(x)$ is known as hesitation degree of element $x \in X$.

Main Result:

Li, Lu and Cai [16] describe a method for convert fuzzy set into intuitionistic set by putting hesitation degree equal to membership and non-membership value. Various author defined number of entropies on intuitionostin fuzzy set with the help of method given by Li, Lu and Cai [16].Dass and Tomar[17] definedentropy measure for intuitionstics fuzzy set. We proposed a fuzzy entropy measure on intuitionistic fuzzy set which is generalization of entropy given by Baig et. al. [18] and O. Prakash[19]on fuzzy set. The proposed entropies are

$$\Psi_1(A) = \frac{1}{\alpha - 2}\sum_{i=1}^{n}\left\{\begin{aligned}&\left(\frac{\mu_A(y_i)+1-\nu_A(y_i)}{2}\right)^{2-\alpha}\\&+\left(\left(\frac{\nu_A(y_i)+1-\mu_A(y_i)}{2}\right)^{2-\alpha}\right)-1\end{aligned}\right\} \text{ where } \quad 0 < \alpha < 1$$

$$\Psi_2(A) = \frac{1}{(1-\alpha)\beta}\sum_{i=1}^{n}\left[\left\{\begin{aligned}&\left(\frac{\mu_A(y_i)+1-\nu_A(y_i)}{2}\right)^{\alpha}\\&+\left(\left(\frac{\nu_A(y_i)+1-\mu_A(y_i)}{2}\right)^{\alpha}\right)-1\end{aligned}\right\}^{\beta}\right] \text{ where } \quad 0 < \alpha < 1,\ \beta > 0 .$$

Theorem:1 Show that measure $\Psi_1(A)$ is entropy measure of order α over intuitionistic fuzzy set A.

Proof. To prove entropy measure we have to satisfy the four properties K_1 *to* K_4 .

K_1 : We have to show that $\Psi_1(A)$ is minimum iff A is crisp set i.e $\mu_A(y) = 0$ and $\nu_A(y) = 1$ or $\mu_A(y) = 1$ and $\nu_A(y) = 0$.

We take $\frac{\mu_A(y_i) + 1 - \nu_A(y_i)}{2} = \xi_A(y_i)$ and $\Psi_1(A)$ will be of the form

$$\Psi_1(A) = \frac{1}{\alpha - 2}\left[\sum_{i=1}^{n}\left\{(\xi_A(y_i))^{2-\alpha} + \left((1-(\xi_A(y_i)))^{2-\alpha}\right) - 1\right\}\right]$$

Now $\Psi_1(A) = 0$

iff $\left[\sum_{i=1}^{n}\left\{(\xi_A(y_i))^{2-\alpha} + \left((1-(\xi_A(y_i)))^{2-\alpha}\right) - 1\right\}\right] = 0$

iff $\xi_A(y_i) = 0 \text{ or } 1 \quad \forall \quad y_i \in X$

Therefore $\mu_A(y_i) - \nu_A(y_i) = 1,\ \nu_A(y_i) - \mu_A(y_i) = 1$ *and* $\mu_A(y_i) + \nu_A(y_i) \le 1$

Solving these inequality $\mu_A(y_i) = 0$, $\nu_A(y_i) = 1$. or $\mu_A(y_i) = 1$, $\nu_A(y_i) = 0$.

Therefore, $\Psi_1(A) = 0$ iff either $\mu_A(y_i) = 0$ and $\nu_A(y_i) = 1$ or $\mu_A(y_i) = 1$ and $\nu_A(y_i) = 0\ \ \forall\ i$.

K_2 : We have to show that $\Psi_1(A)$ is maximum iff $\mu_A(y_i) = \nu_A(y_i)\ \forall\ y_i \in X$.

First we suppose that $\mu_A(y_i) = \nu_A(y_i)\ \forall\ y_i \in X$ then clearly $\Psi_1(A) = 1$

Converse: Suppose that $\Psi_1(A) = 1$ .We assume that $\Psi_1(A) = \sum_{i=1}^{n} g(\xi_A(y_i))$, where

$$g(\xi_A(y_i)) = \frac{1}{\alpha - 2}\left[\left\{(\xi_A(y_i))^{2-\alpha} + \left((1 - (\xi_A(y_i)))^{2-\alpha}\right) - 1\right\}\right] \tag{1}$$

Thus, if $\Psi_1(A) = 1$ than $g(\xi_A(y_i)) = \frac{1}{n}\ \ \forall\ i$.

Differentiating (1) both sides w. r. t. $\xi_A(y_i)$, we get

$$\frac{\partial g(\xi_A(y_i))}{\partial \xi_A(y_i)} = \frac{1}{\alpha - 2}\left[(2-\alpha)\xi_A(y_i)^{1-\alpha} - (2-\alpha)(1 - \xi_A(y_i))^{1-\alpha}\right] = 0 \tag{2}$$

Equation (2) held if $\xi_A(y_i) = \frac{1}{2}$ i. e $g\left(\frac{1}{2}\right) = 0$.

We know that $\frac{\partial^2 g(\xi_A(y_i))}{\partial \xi_A^2(y_i)} \le 0$ at $\xi_A(y_i) = 0.5\ \forall\ i$ and $0 < \alpha \le 2$ as shown in fig. 1.

This implies $g(\xi_A(y_i))$ is a concave function obtaining its maximum value at $\xi_A(y_i) = 0.5$.

Therefore $g(\xi_A(y_i))$ is maximum at $\xi_A(y_i) = 0.5$ *i.e.* $\mu_A(y_i) = \nu_A(y_i)$.

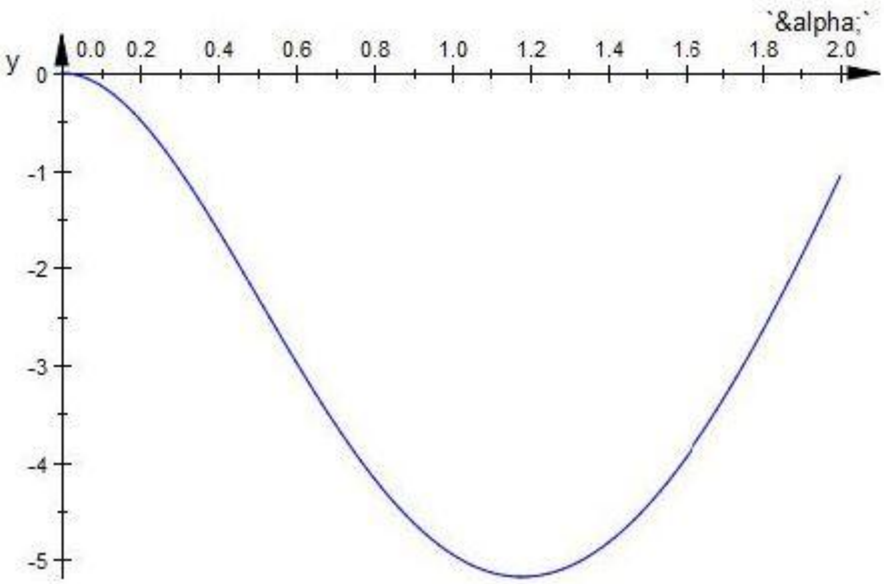

figure.1

K_3 : In order prove that $I_\alpha(A)$ satisfy the K_3, it is sufficient to prove that function

$$g(p,q)=\left[\left\{\left(\frac{p+1-q}{2}\right)^{2-\alpha}+\left(\frac{q+1-p}{2}\right)^{2-\alpha}\right\}-1\right] \text{ where } p,q\in[0,1]$$

is increasing w. r. t. p and decreasing for q.

Now,

$$\frac{\partial g}{\partial p}=2-\alpha\left(\left(\frac{p+1-q}{2}\right)^{1-\alpha}-(2-\alpha)\left(\frac{q+1-p}{2}\right)^{1-\alpha}\right)$$

$$\frac{\partial g}{\partial q}=-(2-\alpha)\left(\left(\frac{p+1-q}{2}\right)^{1-\alpha}+(2-\alpha)\left(\frac{q+1-p}{2}\right)^{1-\alpha}\right)$$

In order to find the critical point of g, put $\frac{\partial g}{\partial p}=0, \frac{\partial g}{\partial q}=0$

$\Rightarrow \frac{\partial g}{\partial p}\geq 0$ if $p\leq q$ and $\frac{\partial g}{\partial q}\geq 0$ if $p\geq q$

Therefore, g is increasing function if $p\leq q$ and g is decreasing function if $p\geq q$.

Thus, by containment property, monotonicity of function $g(p,q)$ and K_2, we get

$\Psi_1(A)\leq\Psi_1(B)$ for $A\subseteq B$.

K_4 : We have $A^C=(<y,\nu_A(y_i),\mu_A(y_i)>/\,y\in X)$ for all $y_i\in X$

$\nu_A(y_i)=\mu_{A^C}(y_i)$ and $\nu_{A^C}(y_i)=\mu_A(y_i)$. Now clearly by description of complement of intuitionistic fuzzy set and by equation (1), we have, $\Psi_1(A)=\Psi_1(A^C)$.

Hence $\Psi_1(A)$ is a valid entropy measure for intuitionistic fuzzy set.

Theorem. 2 Consider two intuitionistic fuzzy set P and Q then

$$\Psi_1(P\cup Q)+\Psi_1(P\cap Q)=\Psi_1(P)+\Psi_1(Q).$$

Proof: Universe of discourse can be divided into two subsets as

$$X_1=\{y\,/\,y\in X,\mu_P(y_i)\leq\mu_Q(y_i)\},$$

$$X_2=\{y\,/\,y\in X,\mu_P(y_i)\geq\mu_Q(y_i)\}$$

In X_1, $\mu_P(y_i)\leq\mu_Q(y_i)\Rightarrow\nu_P(y_i)\geq\nu_Q(y_i)$, then

$$\mu_{P\cup Q}(y)=\max\{\mu_P(y),\mu_Q(y)\}=\mu_Q(y),$$

$$\nu_{P\cup Q}(y)=\min\{\nu_P(y),\nu_Q(y)\}=\nu_Q(y),$$

$$\mu_{P\cap Q}(y)=\min\{\mu_P(y),\mu_Q(y)\}=\mu_P(y),$$

$$\nu_{P\cap Q}(y)=\max\{\nu_P(y),\nu_Q(y)\}=\nu_P(y).$$

Now from equation (1), we have

$$\Psi_1(P \cup Q) = \frac{1}{\alpha - 2}\sum_{i=1}^{n}\left[\left\{\left(\frac{\mu_{P\cup Q}(y_i) + 1 - \nu_{P\cup Q}(y_i)}{2}\right)^{2-\alpha} + \left(1 - \left(\frac{\mu_{P\cup Q}(y_i) + 1 - \nu_{P\cup Q}(y_i)}{2}\right)\right)^{2-\alpha} - 1\right\}\right]$$

In X_1 we have $\Psi_1(P \cup Q) = \frac{1}{\alpha - 2}\sum_{i=1}^{n}\left[\left\{\left(\frac{\mu_{Q}(y_i) + 1 - \nu_{Q}(y_i)}{2}\right)^{2-\alpha} + \left(1 - \left(\frac{\mu_{Q}(y_i) + 1 - \nu_{Q}(y_i)}{2}\right)\right)^{2-\alpha} - 1\right\}\right]$

For $0 < \alpha \neq 2.$, again from equation (1), we have

$$\Psi_1(P \cap Q) = \frac{1}{\alpha - 2}\sum_{i=1}^{n}\left[\left\{\left(\frac{\mu_{P\cap Q}(y_i) + 1 - \nu_{P\cap Q}(y_i)}{2}\right)^{2-\alpha} + \left(1 - \left(\frac{\mu_{P\cap Q}(y_i) + 1 - \nu_{P\cap Q}(y_i)}{2}\right)\right)^{2-\alpha} - 1\right\}\right]$$

For $0 < \alpha \leq 2$ in X_1,

$$\Psi_1(P \cap Q) = \frac{1}{\alpha - 2}\sum_{i=1}^{n}\left[\left\{\left(\frac{\mu_{P}(y_i) + 1 - \nu_{P}(y_i)}{2}\right)^{2-\alpha} + \left(1 - \left(\frac{\mu_{P}(y_i) + 1 - \nu_{P}(y_i)}{2}\right)\right)^{2-\alpha} - 1\right\}\right]$$

For $0 < \alpha \leq 2$, from above equations, we get

$\Psi_1(P \cup Q) + \Psi_1(P \cap Q) = \Psi_1(P) + \Psi_1(Q).$ in X_1. Similarly the result hold for X_2.

In particular: For any $P \in IFS(X), P^C \in IFS(X)$ where P^C the complement of fuzzy set P, it get

$$\Psi_1(P) = \Psi_1(P^C) = \Psi_1(P \cup Q^C) + \Psi_1(P \cap P^C).$$

Theorem: 3 Show that measure $\Psi_2(A)$ is valid entropy measure of order α over intuitionistic fuzzy set A and satisfy the property $\Psi_2(P \cup Q) + \Psi_2(P \cap Q) = \Psi_2(P) + \Psi_2(Q).$

Proof. We can prove the validity of entropy measure proposed in $\Psi_2(A)$ as above entropy and their properties.

Numerical Example:

Let $\Phi = (x_1, x_2, ..., \ x_n)$ be a finite universe of discourse and $B = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle / x_i \in \Phi\}$ be an intuitionistic fuzzy set. We assume that intuitionistic fuzzy set on universal set Ψ which follows as:

$$B^n = \{\langle x_i, [\mu_A(x_i)]^n, 1 - [1 - \nu_A(x_i)]^n \rangle / x_i \in \Phi\} \tag{4}$$

We consider an intuitionistic fuzzy set Q on Φ which is defined as

$$B = \{(x_{1,}\ 0.1, 0.8), (x_{2,}\ 0.5, 0.4), (x_{3,}\ 0.3, 0.5), (x_{4,}\ 0.9, 0.0), (x_{5,}\ 1.0, 0.0)\}$$

Now with the help of operations defined in equation (4) the following intuitionstic fuzzy set are created:

$$B^{\frac{1}{2}}, B,\ B^2, B^3, B^4$$

which are defined as follows:

$B^{\frac{1}{2}}$ may be assumed as "More or less LARGE"

B may be assumed as "LARGE"

B^2 may be assumed as "very LARGE"

B^3 may be assumed as "quite very LARGE"

B^4 may be assumed as "very very LARGE"

and the corresponding set of above notation are given as

$$B^{\frac{1}{2}} = \begin{Bmatrix} (y_1, 0.3162\ , 0.5528\ ), (y_2, 0.7746\ , 0.1056\ ), \\ (y_3, 0.5477\ , 0.2929\ ), (y_4, 0.9487\ , 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B = \begin{Bmatrix} (y_1, 0.1, 0.8), (y_2, 0.5, 0.4), (y_3, 0.3, 0.5), \\ (y_4, 0.9, 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B^2 = \begin{Bmatrix} (y_1, 0.010\ , 0.9600\ ), (y_2, 0.2500\ , 0.6400\ ), \\ (y_3, 0.0900\ , 0.7500\ ), (y_4, 0.8100\ , 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B^3 = \begin{Bmatrix} (y_1, 0.001\ , 0.9920\ ), (y_2, 0.0125\ , 0.7840\ ), \\ (y_3, 0.0270\ , 0.8750\ ), (y_4, 0.7290\ , 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B^4 = \begin{Bmatrix} (y_1, 0.0001\ , 0.9984\ ), (y_2, 0.6250\ , 0.8704\ ), \\ (y_3, 0.0081\ , 0.9375\ ), (y_4, 0.6591\ , 0), (y_5, 1.0, 0) \end{Bmatrix}$$

According to the fuzzy mathematical operation the proposed entropy on different set should be in following order

$$\Psi_1\left(B^{\frac{1}{2}}\right) > \Psi_1(B) > \Psi_1(B^2) > \Psi_1(B^3) > \Psi_1(B^4)$$

Table is constructed for batter comparison between entropies on intuitionistic fuzzy set

Different values of $\Psi_1(A)$ for different values of $0 < \alpha < 1$

Table. 1

α	$\Psi_1\left(B^{\frac{1}{2}}\right)$	$\Psi_1(B)$	$\Psi_1(B^2)$	$\Psi_1(B^3)$	$\Psi_1(B^4)$
0.1	0.6685	0.6490	0.4459	0.3284	0.2711
0.2	0.6488	0.6305	0.4347	0.3213	0.2654
0.5	0.54204	0.529079	0.370839	0.278488	0.230741
0.7	0.405737	0.397882	0.283467	0.216184	0.179734
0.9	0.173924	0.17162	0.124959	0.097199	0.08122

From the above tables it is clear that required order is maintained by proposed entropy.

Conclusions:

As we know that entropy measure has an important role in uncertainty. So our work introduces a new parametric entropy measure for intuitionistic fuzzy set. Some properties of this measure have been also studied. This measure generalizes parametric fuzzy entropy given by Baig et. al. [18] and O. Prakash[19].The theory of parametric entropy on intuitionistic fuzzy set provides new flexibility and wider application in different situations.

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