MATHEMATICAL MODEL FOR THE RESPONSE OF HPA AXIS APPLYING GOMPERTZ - MAKEHAM DISTRIBUTION IN ASSOCIATION WITH ANALYTIC UNIVALENT FUNCTIONS

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Abstract: The study here is to find the activity of Hypothalamic Pituitary Adrenal axis - **HPA** which is a major part of human system which controls reaction to stress by finding Cortisol level and the objective is to observe the response of the system over time due to stress by applying the Gompertz - Makeham distribution. Here we develop two functions $f_1(z)$and $f_2(z)$ by using the class of analytic univalent functions in the open unit disc whose coefficients are considered as Probability density function of the above mentioned distribution, for which the Subordination property, Convex and Star likeness hold.

Current study leads us to a real life application by considering the effects on the response of HPA axis to acute stress. The concluded results coincide with the medical findings.

Key Words

Hypothalamic Pituitary Adrenal (HPA), Cortisol, Gompertz-Makeham Distribution, Analytic functions, Univalent functions and Subordination, Convex and Starlike functions.

AMS Classification: 60E, 62E, 30C45, 30C50, 30C80

1.Introduction

Gompertz Makeham distribution is widely used for Biological systems **[1,6,10].** For comparing Random Variables **[9]** Stochastic dominance is widely used. The most common application of Stochastic dominance is based on the comparison of the Cumulative Distribution Function.

In our model, the assumptions are improved in this direction, and we utilize the approach of **[3]** and special cases are derived for our model.

1.1 Random Variables and Analytic Functions

Let be the class of functions of the form $f(z) = z + \sum_{t=2}^{\infty} a_t$ which are analytic in the open unit disc $\mathcal{U} = \{ z: z\epsilon \, \mathbb{C} \; and \; |z| <$ and represents the class of all functions in which are univalent in **[2]** .

Before proceeding to the main result, we make use of the following concepts.

- **Stochastic Dominance:[7,9]**
- **Subordination**: **[4]**

Here we focus on the Random Variable and using the concept of subordination for the considered analytic functions $f_1($ and $f_2($ by assuming $X \sim$with density having different parameters.

Before going for the main result, we make use of the following Lemma and Definition.

Lemma [5] : Let $f(z) = \sum_{k=0}^{\infty} a_k$ be analytic in and $g(z) = \sum_{k=0}^{\infty} b_k$ be analytic and convex in . If $f(z) \prec g($, then $|a_k| \leq |b$, for = 1,2,…..

Definition [7] A random variable X, is said to have a convex distribution if, for any and and any $\lambda \; \epsilon \; [0$, the following relation is satisfied for the density f :

$$f\{ \lambda x_1 + (1-\lambda)x_2\} \leq \lambda f(x_1) + \; (1- \; \lambda) f(x_2)$$

A function $f($ defined in the form $f(z) = z + \sum_{t=2}^{\infty} a_t$ which are analytic in the open unit disc $\mathcal{U} = \{ z: z\epsilon \, \complement \; and \; |z| <$ is said to be convex of order $(0 \leq \alpha <$ satisfies the condition $\mathbb{R}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \, , z\epsilon$

2. Development of Mathematical Model

Several studies **[8]** highlights the action of the **Hypothalamic Pituitary Adrenal** axis that controls reaction to stress and the response of the hormones. Here we consider the response of Cortisol to stress.

2.1 Assumptions of the Model

1. Participants are exposed with number of stresses.
2. Stress effect is the source for increase in Cortisol levels.

2.2 Classification

Subjects - Participated in two experiments.
Case 1. HPA axis response in Evening.
Case 2. HPA axis response in Morning

2.3.1 Application

Participants response to HPA axis measured in morning and evening.
Here we work with two analytic functions by applying Gompertz Makeham density

$$f = \left(\alpha + \beta e^{\lambda t}\right) \exp\left\{-\alpha t - \frac{\beta}{\lambda}\left(e^{\lambda t} - 1\right)\right\} \alpha, \beta, \lambda >$$

Now consider the class of functions of the form

$f_1(z) = z + \sum_{t=2}^{\infty} a_t$ and $f_2(z) = z + \sum_{t=2}^{\infty} b_t$

which are analytic in the open unit disc whose coefficients and are considered as Probability density function of life time distribution.

We provide an application by using the class of analytic univalent functions in the open unit disc.

On substituting $\alpha, \; \beta, \; anc$ and vary we have obtained the coefficient for power series and is given in the following table (2.3.1).

t	a_t	b_t
1	= .25	=.003
2	=.01	b_{20}

Table 2.3.1. Coefficient of Power series

$$f_1(z) = z + \sum_{t=2}^{\infty} a_t z^t \;, |z| < 1$$

$$= z + a_2 z^2 + \text{(neglecting higher order)}$$

$$= z + a_2$$

$$f_1(z) = z + .01 z$$

and

$$f_2(z) = z + \sum_{t=2}^{\infty} b_t z^t$$

$= z + b_2 z^2 + b_3$ (neglecting higher order)
$= z$
Here we focus our attention on Subordination Principle
Let $f_1(z) = z + .01z$ and $f_2(z) =$ be analytic in .
There exist $w(z) = si$, analytic in with $w(0) =$ and $|w(z)| < 1(z \epsilon$, such that $f_2(z) = f_1(w(z))\ (z \epsilon$. Further $f_2($ is univalent in , then $f_2(z) \prec f_1(z)\ (z\epsilon$)
$\Leftrightarrow f_2(0) = f_1($ and $f_2(\mathcal{U}) \subset f_1($.
Hence we say that is subordinate to and write $f_2(z) \prec f_1($
Thus the definition and the lemma holds good for the considered functions $f_1($ and $f_2($.

2.3.2 Application cont.

Convex order

Now $\mathbb{R}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha, z\epsilon$
For case 1
$\mathbb{R}\left\{1 + \frac{z(.02)}{1+(.02)z}\right\} >$, $(0 \leq \alpha <$
For case 2
$= \mathbb{R}\left\{1 + \frac{z(0)}{1}\right\} = 1 > \alpha, (0 \leq \alpha <$

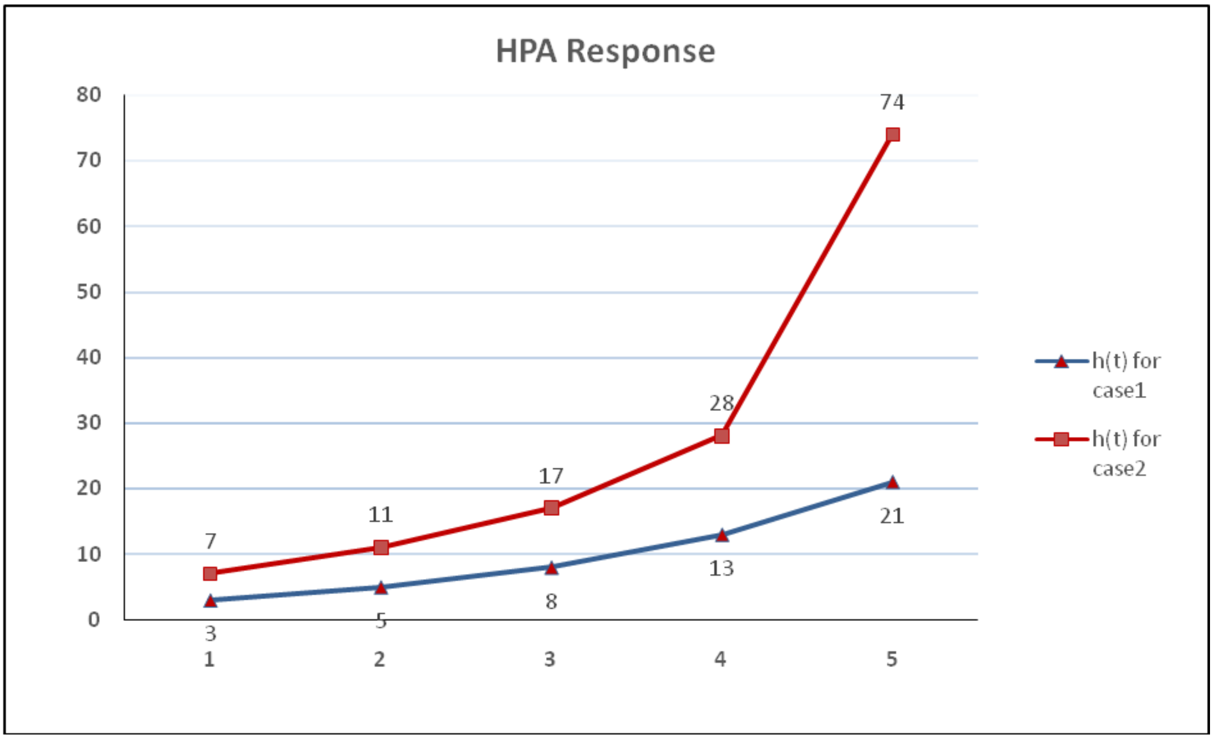
Starlike

$$\mathbb{R}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha, (0 \leq \alpha < 1)\ for\ all\ z\epsilon\,\mathcal{U}$$

For case 1
$$\mathbb{R}\left\{\frac{1 + (.02)z}{1 + (.01)z]}\right\} > \alpha, (0 \leq \alpha < 1)$$
For case 2
$$\mathbb{R}\left\{\frac{zf'(z)}{f(z)}\right\} = 1 > \alpha, (0 \leq \alpha < 1)$$

Results

Here we find the shape of hazard rate for both cases of our study



4.Conclusion

The result shows that HPA response to stress was higher in the morning than in the evening. It is clear from the results obtained that, as time increases hazard also increases during stress and the graph shows that the hazard is more in morning than evening. On taking the Probability density function as coefficients for the class of functions used above, the considered analytic functions satisfy the Subordination principle , convex of order α and starlikeness property. Since the results are consistent it is concluded that HPA axis activity response to stress is more in the morning than in the evening. Related life data fitted with our model and the results coincide with the medical findings.

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