New families of Unlike degree Wiener Index (UDWI)for Graph Structures

Dr. S.S. Sandhya[1] ,Sarasree.S[2],
Author1 :Assistant Professor, Department of Mathematics
Sree Ayyappa College for Women, Chunkankadai.
Author 2 :Research Scholar
Regno : 20113182092002
Sree Ayyappa College for Women, Chunkankadai.
[Affiliated to Manonmaniam Sundararajan University,
Abishekapatti – Tirunelveli - 627012, Tamilnadu, India]
Email : shilpavishnu76@gmail.com[1]
sssandhya2009@gmail.com[2]

Abstract :
In this work we introduced a novel concept concept Unlike degree Wiener index (UDWI) for some graph structures .We study the unlike degree Wiener Index for few graph structures such as Caterpillar,$P_n^2, B_{n,n}, K_{1,n,n}$,$B(n,n), Hd_n, S_n$.

Introduction :
The Wiener index is the sole topological index that has been employed in drug discovery Research out of a large number of them .In 1947 ,Chemist Harold Wiener created the Wiener index of a graph (V, E) denoted by $W(G)$.The half-sum of the shortes distances between every pair of vertices in a graph G is the Wiener index

$W(G) = \frac{1}{2}\sum_{i=1}^{n} d(v_i v_j)$where$d(v_i v_j)$ is the smallest distance between the vertices v_i and v_j in Graph G

J.BaskarBabujee and A.Subhashini established the Like Wiener Index (LDWI) in 2006 and it is described as $W_{ud} = \frac{1}{2}\sum_k \sum_{i,j=1}^{n} d({v_i}^k\ {v_j}^k)$.$d({v_i}^k {v_j}^k)$stands for the distance between the two of vertices of same degree . We investigate theUnlike Degree Wiener Index (UDWI) for a set of graph strutures derived from graph operators in this work .

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Key words and Phrases :Wiener Index

Definition 1.1

Unlike Wiener index is ascertain as$W_{ud} = \sum_k \sum_{i,j=1}^{n} d({v_i}^k\ {v_j}^k)$,where $d({V_i}^k, {V_j}^k)$is the distance between the two vertices of different degrees .

2. Main Results

Theorem 1

The Unlike Wiener Index of Caterpillar graph is $W_{ud} = n^2 + n$

Proof:
Let $G = (V, E)$ be a Caterpillar.

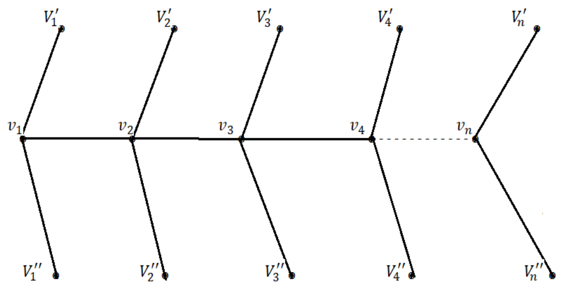


Figure 7.4 Caterpillar

$$V = (v_1, v_2 \dots \dots v_n, v_1{}', v_2{}', \dots \dots v_n{}', v_1'', , v_2'', \dots \dots , v_n'')$$

$$E = \{v_i v_{i+1}, v_i v_i{}', v_i v_i'' \; 1 \le i \le n\}$$

The vertices $v_1{}', v_2{}', \dots \dots v_n{}'$ and $v_1'', , v_2'', \dots \dots , v_n''$ has one and the vertices $v_2, v_3, \dots \dots v_{n-1}$ has degree four except v_1 and v_n having degree three.

$$\deg(v_i') = 1 \quad , \forall 1 \le i \le n$$

$$\deg(v_i'') = 1 \quad , \forall 1 \le i \le n$$

$$\deg(v_i) = 4 \quad , \forall 2 \le i \le n-1$$

$$\deg(v_i) = 3 \quad , for\ i = 1\ and\ i = n$$

W_{ud} (G)

$$= d(v_1, v_2) + d(v_1, v_3) + \cdots \dots \dots + d(v_1, v_n)$$
$$+ \cdots \dots . + d(v_n, v_{n-1}) \;\; + d((v_n, v_{n-2}) + \cdots . . + d((v_n, v_2)$$
$$+ d(v_1 v_1^{'}) + d(v_2, v_2^{'}) + \cdots \dots \; d(v_n, v_n^{'}) + d(v_1, v_1^{''}) + d(v_2, v_2^{''})$$
$$+ d(v_n, v_n^{''})$$
$$= [1 + 2 + \cdots . \dots (n-1)] + [1 + 2 + 3 + \cdots . . + (n-1)]$$
$$+ [1 + 1 + \cdots . . + 1](n \; times) + [1 + 1 + \cdots . \dots .1](n) times$$

$$= 2\left[\frac{(n-1)(n-1+1)}{2}\right] + (n+n)$$
$$= n(n-1) + n + n$$
$$= n^2 - n + n + n$$
$$= n^2 + n$$
$$= n(n+1)$$

Theorem 2

The Unlike degree Wiener Index of (P_n^2) is $W_{ud} = P_n^2 = 4(n-2)$

Proof :
Let$P_n^2 = (V, E)$

$$V = (v_1, v_2, \dots . v_n)$$

$E = (v_i v_{i+1}, v_j v_{j+1}; 1 \leq i \leq n - 1.1 \leq j \leq n - 2)$

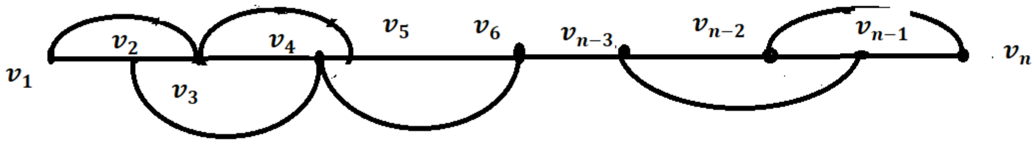


Figure 7.5P_n^2

The vertices v_1 and v_n have degree two ,the vertices v_2 and v_{n-1} have degree three and all other vertices $v_3, v_4, \dots \dots v_{n-2}$ have degree four.

$$\deg(v_1) = \deg(v_n) = 2$$

$$\deg(v_2) = \deg(v_{n-1}) = 3$$

$$\deg(v_k) = 4\,; 3 \le k \le n-2$$

$$W_{ud}\,(P_n^2)$$

$$= d(v_1, v_2) + d(v_1, v_3) + \cdots \dots \dots + d(v_1, v_{n-1})$$

$$+ \cdots \dots . + d(v_n, v_{n-1}) \ \ + d((v_n, v_{n-2}) + \cdots .. + d((v_n, v_2) +)$$

$$+ d(v_2, v_3) + d(v_2, v_4) + \cdots \dots \ d(v_2, v_{n-2}) + d(v_{n-1}, v_{n-2})$$

$$+ d(v_{n-1}, v_{n-3}) + \cdots + d(v_{n-1}, v_3)$$

$$= (1 + 1 + \cdots . \dots + 1)(n-1)\ times$$

$$+ [(1 + 1 + 1 + \cdots .. + 1)(n - 1\ times\ )] + [1 + 1 + \cdots .. + 1](n$$

$$- 3\ times\ ) + [1 + 1 + \cdots . \dots .1](n-3) times$$

$$= (n-1) + (n-1) + (n-3) + (n-3)$$

$$=2(n-1) + 2(n-3)$$

$$= 2n - 2 + 2n - 6$$

$$= 4n - 8$$

$$= 4(n-2)$$

Theorem 3

The Unlike degree Wiener Index of Double Graph $K_{1,n,n}$ is $W_{ud}\left(K_{1,n,n}\right) = 4n$

Proof:

$V = (v_0, v_1, v_2 \dots \dots v_n, v_1{}', v_2{}', \dots\dots v_n{}')$

$E = \{v_0 v_i, v_i v_i{}'; 1 \le i \le n\}$

The vertices $v_1, v_2 \dots \dots v_n$ has degree two and the vertices $v_1{}', v_2{}', \dots\dots v_n{}'$ has degree one except $v_0{}'$

$\deg(v_0) = n$

$\deg(v_i) = 2$;$\forall 1 \le i \le n$ and

$\deg(v_i{}') = 1; \forall\ 1 \le i \le n$

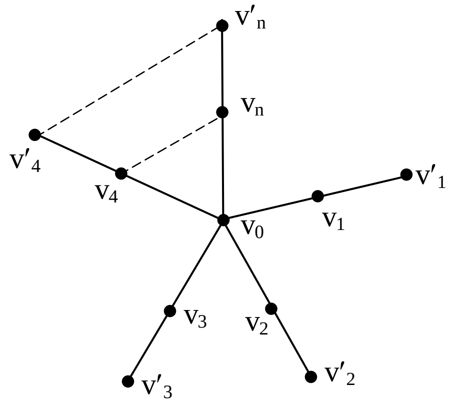


Figure 7.7 $\boldsymbol{K_{1,n,n}}$

$$W_{ud}(K_{1,n,n}) = d(v_0, v_1) + d(v_0, v_2) + \cdots \dots \dots + d(v_0, v_n) + d((v_0, v_n{}') + d((v_1, v_1{}') + d((v_2, v_2{}') + \cdots \dots \dots + d((v_n, v_n{}')$$

$$= (1 + 1 + 1 + \cdots \ldots + 1)(n\ times\ ) + (2 + 2 + 2 + \cdots \ldots + 2)(n\ times\ ) + (1 + 1 + 1 + \cdots . +1)(n\ times)$$

$$= n + 2n + n$$

$$= 4n$$

Theorem 4

The Unlike degree Wiener Index of Bistar Hd_n is $(W_{ud}(Hd_n) = n^2 - 2$

Proof:

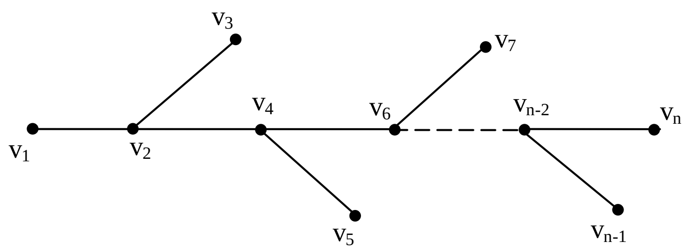
Let $(\mathrm{Hd_n}) = (V, E)$

$V = \{v_1, v_2, v_3, \ldots \ldots v_n\}$ have n vertices and $n - 1$ edges

$E = \{v_i v_{i+1}, v_i v_{i+2}; 1 \leq i \leq n\}$

$deg(v_1) = deg(v_3) = deg(v_5) = deg(v_7) \ldots \ldots . = deg(v_n) = 1$

$deg(v_2) = deg(v_4) = \cdots .. deg(v_{n-2}) = 3$



$$W_{ud}(\mathrm{Hdn}) = d(v_1, v_2) + d(v_1, v_4) + \cdots \ldots \ldots + d(v_1, v_{n-2}) + d((v_n, v_{n-2}) + \cdots .. + d((v_n, v_2) + d((v_n, v_1) + d((v_2, v_3) + d(v_4, v_5) + d(v_{n-1}, v_{n-2})$$

$$= \begin{bmatrix} 1 + 2 + 3 + \cdots \ldots + 1 \\ (n-1) \end{bmatrix} + [1 + 2 + 3 + \cdots .. + (n-1)] + [1 + 1 + \cdots .. + n](n-2) times$$

$$= 2\left[\frac{(n-1)(n-1+1)}{2}\right] + (n-2) \times 1$$

$$= n(n-1) + (n-2)$$

$$= n^2 - n + n - 2$$

$$= n^2 - 2$$

Theorem :5

The Unlike degree Wiener index of star graph, S_n is $W_{ud}(S_n) = n$

Proof:

Let $S_n = (V, E)$

$V = \{v_0, v_1, v_2, \ldots \ldots v_n\}$ have $n + 1$ vertices and n edges.

$$E = \{v_0 v_i ; 1 \leq i \leq n\}$$

All the vertices $v_1, v_2, \ldots \ldots v_n$ has degree one except v_0

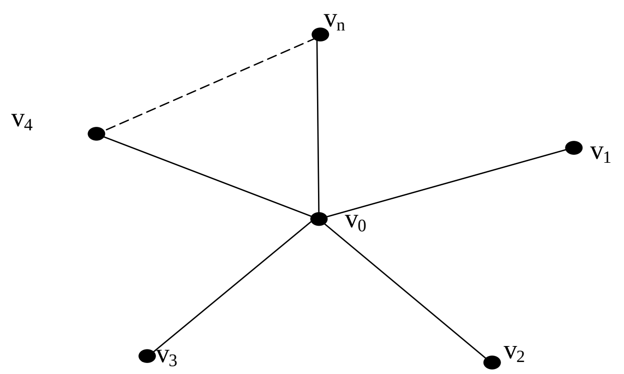


Figure 7.6

$deg\ (v_0) = n$ and $deg(v_i) = 1 \forall\ 1 \leq i \leq n$

$$W_{ud} = d(v_0, v_1) + dv_0, v_2) + \cdots + d(v_0, v_n)$$
$$= 1 + 1 + 1 + \cdots 1(n\ times\ )$$
$$= n$$

Conclusion :

We have studied Unlike degree Wiener Index for various graph structures derived using graph operators. In future a comparison study will be carried out with Wiener index and Unlike degree Wiener Index for various compounds.

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