An Inventory Model deteriorating itemswith Demand Dependent Production Rate under Permissible Delay Payment

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Abstract

This paper considers a single supplier offering trade credit to demand a dependent production rate with ramp type demand, which has not been reported in the literature. Previously, two inventory models have been developed under the above conditions.Secondly, order quantity and replenishment cycle time algorithms are designed to be optimized. Thefindings of the above study show that lowering the production rate, decreasing the order insertion frequency and decreasing the production rate leads to cost reduction, with no effect on the optimal solution as well as demand-dependent production rates.The retailer will cut the replenishment cycle when the growth time is higher than the business loan; when it is small, within the maturity period of the goods, the optimal order cycle and the quantity of the optimal quantity will have no effect on the trade credit. Finally, numerical examples are discussed to demonstrate sensitivity analysis of optimal solutions.

Key words: EPQ Inventory Model, deterioration, Ramp Type Demand Rate, Inflation and Trade Credit.

Introduction

In the conventional inventory economic order quantity model, it is supposed that the supplier must get paid by the retailer for the products while receiving them. But with changing trends and growing competition in the market, take credit has become a popular payment mode in transactions to endorse sales and reduce inventory. According to this, the supplier gives a fixed time period to the retailer to settle down the payments without claiming any interest. During this uncharged period the retailer starts to collect the revenue on the sales and the level of the inventory decreases. If he is unable to pay the amount in the fixed period to the retailer, the supplier can charge a heavy interest on the balance remaining.The inventory replenishment policies under the impact of trade credit financing policies have been studied continuously. For the first time, Goyal(1985), studied the EOQ model with the trade credit financing which is later extended by Chu et al(1998)by taking into consideration the deteriorating products.Aggarwal and Jaggi(1995) discussed the models with the exponential deterioration rate of the items. A model on the shortage under delayed payment and deteriorating conditions was developed by Jamel et al. (1997) and Chang and Dye(2001). The unit selling price is different from the unit product costs in a model designed by Teng(2002). Chung and Huang(2003) introduced an EPQ model considering the manufacturer offering the retailer the delayed payment policy. An analysis on the supplychain of a single vendor and a single buyer for s single product, taking into consideration the effect of deterioration and credit period incentives with a constant demand and the infinite replenishment ratewas developed by Uthaya Kumar and Parvathi(2011). Du et al. (2013)presented the coordination of two-echelon supply chains using wholesale price discount and credit option. Wu et al.(2014)developed the optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing.The papers in published literature have broadly studied inventory problem by assumingdemand rate to be ramp type. Using this assumption, Hwang and Shinn(1997)prepared a paper on the optimal replenishment policy with ramp type time dependent demand for perishable seasonal products in a season. An inventory model with ramp type demand rate, partial backlogging, and weibull deterioration rate was discussed in 2008 by Panda et al. Later Skouri et al. (2009) have done some modifications and presented namely, an economic production quantity models for deteriorating items with ramp type demand. Manna and Chaudhuri(2006) discussed an EOQ model for weibell distributed deteriorating items under ramp type demand and shortages. After that Mandal(2010) analysed that the supply chain model with stochastic lead time under imprecise partially backlogging and fuzzy ramp type demand for expiring items. S. R. Singh and C Singh(2010) purposed an inventory model with shortages, for ramp type demand, time dependent deterioration items with salvage value. Mishra and Singh (2011) discussed two-warehouse inventory model having ramp-type as demand rate and partially backlogged shortages. Again Roy and Chaudhari(2011) purposed a computation approach to an inventory model with ramp type demand and linear deterioration. Ahmed et al.(2013) purposed an inventory model having general deterioration rate, ramp type demand rate, the partial backlogging and. Saha(2014) discussed a paper on the optimal order quantity of the retailer with quadratic ramp type demand under supplier trade credit financing Dependent production rate for trade credit, researchers attempting to solve problems mostly assume that the production rate is infinite or constant value. However, the infinite and constant replenishment rate of the inventory model is inconsistent with actual industrial practices. In the traditional EOQ or EPQ model without trade credit financing, Darzanou and Skouri (2011) introduced a production inventory system for commodities deteriorating with demand rate, with a linearly ramp type function of time and production rate of demand rate.Two models with and without deficiencies were discussed. Both models were studied assuming that the point at which demand stabilizes before the production stops.Recently, Skouri et al (2011) developed a more general integrated supplier retailer inventory model with demand rate sensitive to retail price and demand dependent production rate with two- level of trade credit financing, which does not accept the ramp type demand rate.Therefore, based on the literature above we find that none of the models above explain the retailer's optimal replenishment policy under trade credit financing with ramp-type demand and demand-dependent production rates. Buzacott (1975) was the first author to incorporate the concept of inflation in inventory modelling. They showed that the cost increases the effect of the result of inflation.Mishra (1979) proposed the time value of currency for inflation as well as external as well as intra-inflation rate, and proved the effect of interest rate and inflation rate on the replenishment strategy. Vart and Padmanabhan (1990) introduced an EOQ model for stock level dependent demand under inflation levels.Liao et al (2000) developed an inventory model for deteriorating items, which affects the rate of inflation and the value of time when the supplier allows, delay in payment under constant demand.Chung and Lui (2001) designed a discounted cash flow approach inventory model for deteriorating goods with time value of money over a fixed time horizon.Hou (2006) developed an EOQ model, which considers shortages and inflation due to inventory-induced demand reduction. Sarkar et al (2014) introduced an EMQ model with price and time induced demand under the influence of reliability and inflation.Moon and Lee (2000) developed an EOQ model for the value of inflation items and time value of money with normal distributed items. Teng (2006) introduced an EOQ model to find optimal ordering policies by considering the discounted cash flow (DCF) approach.Hsieh and Dye (2010) contributed an EOQ lot size model for commodities deteriorating under inflation using a discounted cash flow (DCF) approach over a finite planning horizon. Tripathi and Mishra (2010) introduced EOQ policies of non-deteriorating commodities with a time dependent demand rate in the presence of trade credit using a discounted cash flow (DCF) approach.A partial backlog inventory model for non-instantaneous deteriorating commodities with a stock-dependent consumption rate under inflation was designed by Horng Jin Chang (2010). Naserabadi et al (2014) developed an inventory model for the fading of commodities with linear demand functions under fuzzy inflation conditions and the time value of money with permitted shortages that are completely backlogged.Ting and Chung (2014) presented an EOQ model for optimal solutions with delays in offering payment and price discounts in supply chain systems. Chen and Teng (2015) developed an EOQmodel not only for the retailer's optimal credit duration and cycle time, but also unique ones. Jaggi et al (2016) presented an inventory model for non-instantaneous deteriorating commodities under inflationary conditions with partially backlog shortages.Kumar and Kumar (2016) further explain a model of inventory of deteriorating items for which GA is used for permissible postponed in funds. Kumar at.el. (2016) discuss the model of deteriorating items of stock dependent demand rate underinflationary circumstances. Kumar and Kumar’s (2017) also present an inventorymodel fordeteriorating items with linear demand for the effective and satisfactory process. Sarkar et al (2018) also present a multi-retailer supply chain model withbackorder and variable production cost. Saha and Sen (2018) developed an inventory model for deteriorating items with time and price dependent demand and shortages under the effect of inflation. Sarkar et al (2019) introduced an application of timedependentholding costs and system reliability in a multi-item sustainable economic energyefficient reliable manufacturing system.

In this paper, we have developed an inventory model for single supplierconsidering trade credit with ramp type demand, order quantity and replenishment cycle time algorithms are designed to be optimized. The retailer will cut the replenishment cycle when the growth time is higher than the permissible delay in payment. The optimal order cycle and the quantity of the optimal quantity will have no effect on the trade credit. Finally, numerical example and sensitivity analysis is carried out to study the various parameters on decision variable and objective function by using Matlab for the feasibility and applicability of our model.

Notations and Assumptions:-

O	Cost for ordering product
Hc	Cost for holding product
Pc	Cost for purchasing product
t_1	Maximum level of inventory system for the case $\mu \le t_1$.
t_1^*	Maximum level of inventory system for the case $t_1 \le \mu$
L	Maximum level of inventory system at every scheduling cycle
U	Sales price per unit
k	Production rate represented as k = rf(t)
Z	Market demand represented as Z = f(t)
μ	Break point of the demand function
Do	Demand Rate
M	Credit period offered by the supplier
EI	Interest earned in the inventory
CI	Interest charged in the inventory

Assumptions

The model is considering Supplier and retailer for single item. And assuming planning Horizon is infinite.Shortages are not allowed. Demand rate is used ramp type. The market rate function is defined with Heaviside's function as follows

$f(t) = D_o(t - (t - \mu) H (t - \mu))$

where,

$$H(t-\mu) = \begin{cases} 1, \; if\; t \ge \mu \\ 0, \; if\; t < \mu \end{cases} \quad (1)$$

Production rate is considering to be k = rf(t), as production is very closely related to demand where r is always greater than 1and is a constant and 'k' is replenishment rate.

Permissible delay is possible as up to t = M he could settle the account without paying any interest but after that a huge amount of interest will be paid by the retailer to the supplier.

Inventory level

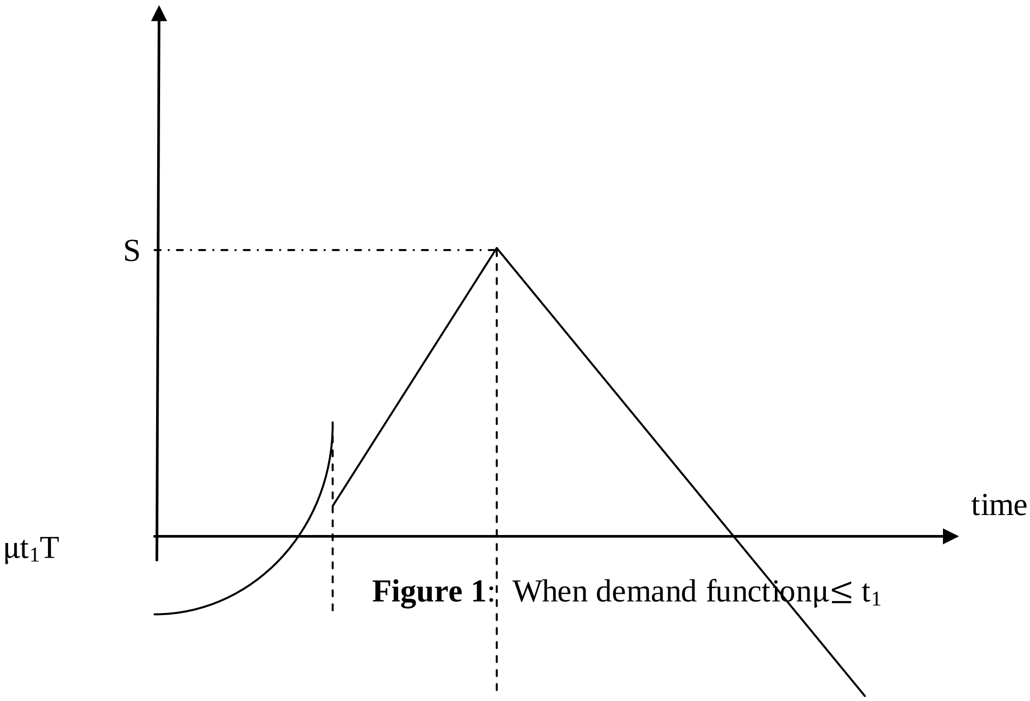


Figure 1: When demand function$\mu \leq t_1$

Inventory level for$\mu \leq t_1$

Firstly, the production starts at t = 0 then inventory level is zero and stops at t= t_1. Then the inventory level starts declining until the inventory will be zero at t = T. The inventory level I(t)with respect to time are described below:

During the early stage (0,μ) , the demand Rate is $D_o t$ and replenishment rate is $rD_o t$.

$$\frac{dI_1(t)}{dt} = (r-1)D_o t\,, \qquad 0 \leq t \leq \mu \qquad (2)$$

With the boundary condition $I_1(0) = 0$.

$$I_1(t) = \frac{(r-1)D_o t^2}{2}, \quad 0 \leq t \leq \mu \qquad (3)$$

During the intermediate stage (μ, t_1) the demand rate is $D_o\mu$ and replenishment rate is $rD_o\mu$.

$$\frac{dI_2(t)}{dt} = (r-1)D_o\mu \quad \mu \leq t \leq t_1 \qquad (4)$$

With the boundary condition $I_2(t_1) = L$.

$$I_2(t) = (r-1)D_o\mu\left(t - \frac{\mu}{2}\right) \mu \leq t \leq t_1 \qquad (5)$$

During the final stage (t_1,T) the demand rate is $D_o\mu$ and production stops.

$$\frac{dI_3(t)}{dt} = -D_o\mu \quad t_1 \leq t \leq T \qquad (6)$$

With the boundary condition $I_3(t_1) = L$ and $I_3(T) = 0$.

$$I_3(t) = D_o\mu(T-t) \qquad t_1 \leq t \leq T \qquad (7)$$

Inventory level

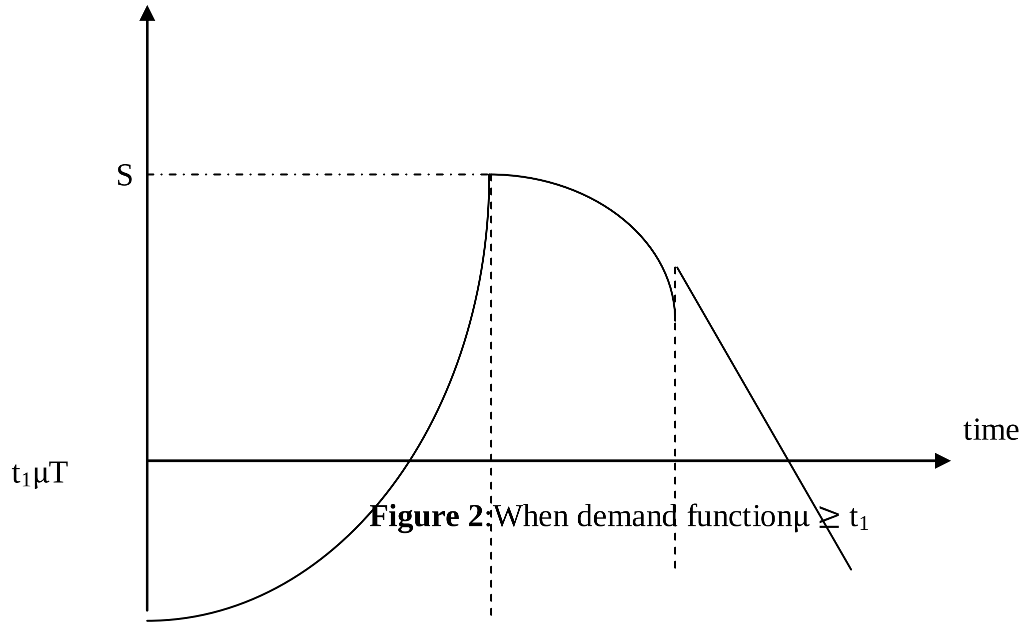


Figure 2: When demand function$\mu \geq t_1$

Inventory level for $\mu \geq t_1$

The inventory level for $\mu \geq t_1$performs same as for $t_1 \geq \mu$. Figure shows the inventory system in three steps.

The inventory level I(t) with respect to time are described below:

During the early stage $(0, t_1)$, the demand Rate is $D_o t$ and replenishment rate is $rD_o t$.

$$\frac{dI_1(t)}{dt} = (r-1)D_o t\,, \qquad 0 \leq t \leq t_1 \quad (8)$$

With the boundary condition $I_1(0) = 0$ and $I_1(t_1) = L$.

$$I_1(t) = \frac{(r-1)D_o t^2}{2}, \quad 0 \leq t \leq \mu \qquad (9)$$

During the stage (t_1, μ) the demand rate is $D_o\mu$ and replenishment rate does not exists.

$$\frac{dI_2(t)}{dt} = -D_o t \quad \mu \leq t \leq t_1 \qquad (10)$$

With the boundary condition $I_2(t_1) = L$.

$$I_2(t) = \frac{1}{2}D_o(-t^2 + \mu^2 + 2\mu(T-\mu)) \quad \mu \leq t \leq t_1 \qquad (11)$$

During the final stage (μ, T) the demand rate is $D_o\mu$ and production stops.

$$\frac{dI_3(t)}{dt} = -D_o\mu \quad \mu \leq t \leq T \qquad (12)$$

With the boundary condition$I_3(T) = 0$.

$$I_3(t) = D_o\mu(T-t) \quad \mu \leq t \leq T \qquad (13)$$

Mathematical models and solutions

1. Cost for ordering product = O.

2. Cost for holding product (excluding interest charges)

(a) When $\mu \le t_1$

$$= H_c\left(\int_0^{\mu} I_1(t)e^{-Rt}dt + \int_{\mu}^{t_1} I_2(t)e^{-Rt}dt + \int_{t_1}^{T} I_3(t)e^{-Rt}dt\right)$$

$$= H_c D_o \mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3}-\frac{R\mu}{4}\right)-\frac{2R}{3}(t_1^3-\mu^3)+(t_1-\mu)\left(\mu+(t_1+\mu)\left(1-\frac{R}{2}\right)\right)\right\}\right.$$

$$\left.+(T-t_1)\left\{T+\frac{(T-t_1)}{2}(\mu RT+1)+\frac{R}{3}(T^2+t_1(t_1+T))\right\}\right] \quad (14)$$

(b) When $t_1 \le \mu$

$$= H_c\left(\int_0^{t_1^*} I_1(t)e^{-Rt}dt + \int_{t_1^*}^{\mu} I_2(t)e^{-Rt}dt + \int_{\mu}^{T} I_3(t)e^{-Rt}dt\right)$$

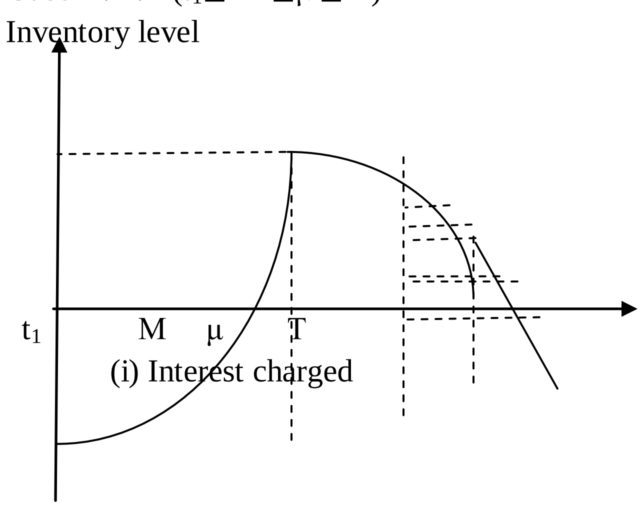
$$= H_c\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3}-\frac{Rt_1^*}{4}\right)+D_o\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]\right.$$

$$\left.+D_o(\mu-t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2+t_1^*(t_1^*+\mu)}{3}-\frac{R(t_1^*+\mu)}{2}\right\}+\left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2+\mu(T-\mu)\right\}\right]\right] \quad (15)$$

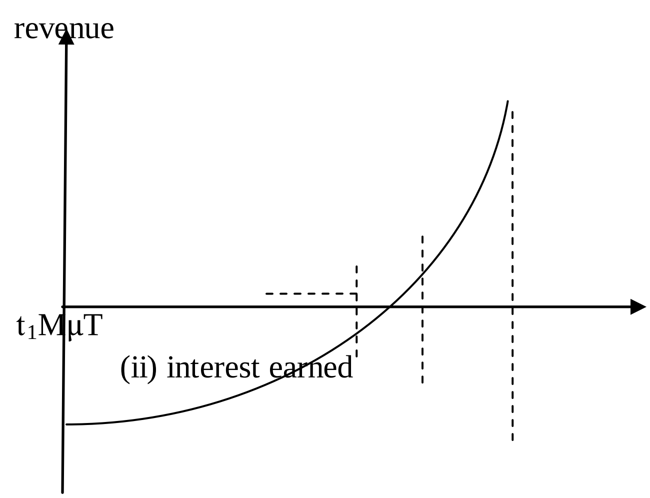
3. There exist total eight cases for interest charged and earned for the items kept in inventory. There are two possibilities for M i.e. $M \le \mu$ and $M \ge \mu$. Following are the discussion of eight cases with these possibilities.

(A) The model for $M \le \mu$

Case A.1 :- ($t_1 \le M \le \mu \le T$)



(i) Interest charged



(ii) interest earned

The interest charge will be paid for the unsold inventory after the time period M. For the time period (M,μ), the inventory level is $I_2(t)$ and for the time period (μ, T), the inventory level is $I_3(t)$. Hence the interest charged for unsolved inventory is

$$P_C C_I \left\{ \int_M^{\mu} I_2(t) e^{-Rt} dt + \int_{\mu}^{T} I_3(t) e^{-Rt} dt \right\}$$

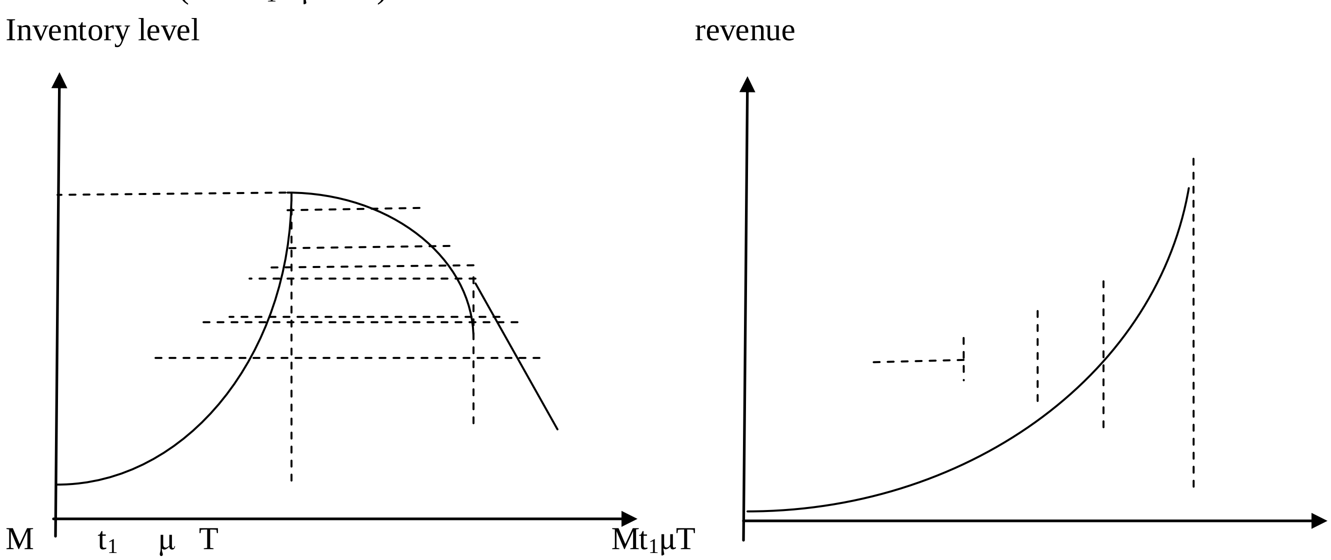
$$= P_C C_I D_o \left[(\mu - M) \left[\frac{-1}{2} \left\{ \frac{\mu^2 + M(M+\mu)}{3} - \frac{R(M+\mu)}{2} \right\} + \left\{ 1 - \frac{R}{2}(\mu + M) \right\} \left\{ \frac{1}{2}\mu^2 + \mu(T-\mu) \right\} \right] \right.$$

$$\left. + \mu(T-\mu) \left[T - \frac{(T+\mu)(RT+1)}{2} + \frac{R}{3}(T^2 + \mu(\mu+T)) \right] \right] \quad (16)$$

Interest earned during the time interval (0,M) is given by

$$E_I L \left\{ \int_0^M (M-t) e^{-Rt} D_o t \, dt \right\} = \frac{E_I L D_o M^3}{6} \left(5 - \frac{MR}{2} \right) (17)$$

Case A.2 :- $(M \le t_1 \le \mu \le T)$



(i) Interest charged (ii) interest earned

The interest charged will be paid for the unsold inventoryafter the time period M, For the time period (M, t_1^*), the inventory level is $I_1(t)$, for the time period (t_1^*, μ), the inventory level is $I_2(t)$ and for the time period (μ, T), the inventory level is $I_3(t)$. Hence the interest charged for unsolved inventory is

$$P_C C_I \left\{ \int_M^{t_1^*} I_1(t) e^{-Rt} dt + \int_{t_1^*}^{\mu} I_2(t) e^{-Rt} dt + \int_{\mu}^{T} I_3(t) e^{-Rt} dt \right\}$$

$$= P_C C_I \left[\frac{(r-1)D_o}{2} \left(\frac{t_1^{*3} - M^3}{3} - \frac{R(t_1^{*4} - M^4)}{4} \right) \right.$$

$$+ D_o(\mu - t_1^*) \left[\frac{-1}{2} \left\{ \frac{\mu^2 + t_1^*(t_1^* + \mu)}{3} - \frac{R(t_1^* + \mu)}{2} \right\} + \left\{ 1 - \frac{R}{2}(\mu + t_1^*) \right\} \left\{ \frac{1}{2}\mu^2 + \mu(T-\mu) \right\} \right]$$

$$+D_o\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]\Bigg](18)$$

Interest earned during the time interval (0, M) is given by

$$E_I L\left\{\int_0^M (M-t)e^{-Rt}D_o t\,dt\right\} = \frac{E_I L D_o M^3}{6}\left(5-\frac{MR}{2}\right)(19)$$

Case A.3:- (M ≤ t_1≤μ ≤ T)

The interest charged will be paid for the unsold inventory after the time period M. For the time period (M,μ), the inventory level is $I_1(t)$, for the time period (μ, t_1), the inventory level is $I_2(t)$ and for the time period (t_1, T), the inventory level is $I_3(t)$. Hence the interest charged for unsolved inventory is

$$P_C C_I\left\{\int_M^{\mu} I_1(t)e^{-Rt}dt+\int_{\mu}^{t_1} I_2(t)e^{-Rt}dt+\int_{t_1}^{T} I_3(t)e^{-Rt}dt\right\}$$

$$=P_C C_I\left[\frac{(r-1)D_o}{2}\left(\frac{\mu^3-M^3}{3}-\frac{R(\mu^4-M^4)}{4}\right)+D_o\mu(r+1)(t_1-\mu)\left\{\frac{t_1+\mu}{2}+\frac{R(t^2+\mu(\mu+t))}{2}\right\}\right.$$

$$\left.+D_o\mu(T-\mu)\left[T+(T+t_1)\left(\frac{1}{2}-RT\right)+\frac{R}{3}(T^2+t_1(t_1+T))\right]\right](20)$$

Interest earned during the time interval (0, M) is given by

$$E_I L\left\{\int_0^M (M-t)e^{-Rt}D_o t\,dt\right\} = \frac{E_I L D_o M^3}{6}\left(5-\frac{MR}{2}\right)(21)$$

Total cost of replenishment policy is given by following

$$TC_1=\begin{cases}TC_{11}, & t_1\le M\le\mu\le T\\ TC_{12}, & M\le t_1\le\mu\le T\\ TC_{13}, & M\le\mu\le t_1\le T\end{cases}\quad(22)$$

Where,

$$\text{TC}_{11}=A+H_c\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3}-\frac{Rt_1^*}{4}\right)+D_o\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]\right.$$

$$\left.+D_o(\mu-t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2+t_1^*(t_1^*+\mu)}{3}-\frac{R(t_1^*+\mu)}{2}\right\}+\left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2+\mu(T-\mu)\right\}\right]\right]$$

$$+P_C C_I D_o\left[(\mu-M)\left[\frac{-1}{2}\left\{\frac{\mu^2+M(M+\mu)}{3}-\frac{R(M+\mu)}{2}\right\}+\left\{1-\frac{R}{2}(\mu+M)\right\}\left\{\frac{1}{2}\mu^2+\mu(T-\mu)\right\}\right]\right.$$

$$\left.+\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]\right]-\frac{E_I L D_o M^3}{6}\left(5-\frac{MR}{2}\right)$$

$$\mathrm{TC}_{12} = A + H_C\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3}-\frac{Rt_1^*}{4}\right) + D_o\mu(T-\mu)\left[T - \frac{(T+\mu)(RT+1)}{2} + \frac{R}{3}(T^2+\mu(\mu+T))\right]\right.$$

$$+ D_o(\mu - t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2 + t_1^*(t_1^*+\mu)}{3} - \frac{R(t_1^*+\mu)}{2}\right\} + \left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2 + \mu(T-\mu)\right\}\right]\Bigg]$$

$$+ D_o\mu(T-\mu)\left[T - \frac{(T+\mu)(RT+1)}{2} + \frac{R}{3}(T^2+\mu(\mu+T))\right] - \frac{E_I L D_o M^3}{6}\left(5 - \frac{MR}{2}\right)$$

$$+P_C C_I\left[D_o(\mu - t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2 + t_1^*(t_1^*+\mu)}{3} - \frac{R(t_1^*+\mu)}{2}\right\} + \left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2 + \mu(T-\mu)\right\}\right]\right.$$

$$\left. + \frac{(r-1)D_o}{2}\left(\frac{t_1^{*3} - M^3}{3} - \frac{R(t_1^{*4} - M^4)}{4}\right)\right]$$

$$TC_{13} = A + H_c D_o \mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3}-\frac{R\mu}{4}\right) - \frac{2R}{3}(t_1^3 - \mu^3) + (t_1-\mu)\left(\mu + (t_1+\mu)\left(1-\frac{R}{2}\right)\right)\right\}\right.$$

$$\left. + (T-t_1)\left\{T + \frac{(T-t_1)}{2}(\mu RT+1) + \frac{R}{3}(T^2 + t_1(t_1+T))\right\}\right]$$

$$+P_C C_I\left[\frac{(r-1)D_o}{2}\left(\frac{\mu^3 - M^3}{3} - \frac{R(\mu^4 - M^4)}{4}\right) + D_o\mu(r+1)(t_1-\mu)\left\{\frac{t_1+\mu}{2} + \frac{R(t^2+\mu(\mu+t))}{2}\right\}\right.$$

$$\left. + D_o\mu(T-\mu)\left[T + (T+t_1)\left(\frac{1}{2} - RT\right) + \frac{R}{3}(T^2 + t_1(t_1+T))\right]\right] - \frac{E_I L D_o M^3}{6}\left(5 - \frac{MR}{2}\right)$$

Total cost of replenishment policy per unit time is given by following

$$TPC_1 = \begin{cases} TPC_{11}, & t_1 \le M \le \mu \le T \\ TPC_{12}, & M \le t_1 \le \mu \le T \quad (23) \\ TPC_{13}, & M \le \mu \le t_1 \le T \end{cases}$$

Where,

TPC_{11}

$$= \frac{A}{T} + \frac{1}{T}\left[H_C\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3}-\frac{Rt_1^*}{4}\right) + D_o\mu(T-\mu)\left[T - \frac{(T+\mu)(RT+1)}{2} + \frac{R}{3}(T^2+\mu(\mu+T))\right]\right.\right.$$

$$+ D_o(\mu - t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2 + t_1^*(t_1^*+\mu)}{3} - \frac{R(t_1^*+\mu)}{2}\right\} + \left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2 + \mu(T-\mu)\right\}\right]\Bigg]$$

$$+P_C C_I D_o\left[(\mu-M)\left[\frac{-1}{2}\left\{\frac{\mu^2+M(M+\mu)}{3}-\frac{R(M+\mu)}{2}\right\}+\left\{1-\frac{R}{2}(\mu+M)\right\}\left\{\frac{1}{2}\mu^2+\mu(T-\mu)\right\}\right]\right.$$

$$\left.+\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]\right]-\frac{E_I L D_o M^3}{6}\left(5-\frac{MR}{2}\right)\Bigg]$$

TPC_{12}

$$=\frac{A}{T}+\frac{1}{T}\left[H_c\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3}-\frac{Rt_1^*}{4}\right)+D_o\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]\right.\right.$$

$$\left.+D_o(\mu-t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2+t_1^*(t_1^*+\mu)}{3}-\frac{R(t_1^*+\mu)}{2}\right\}+\left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2+\mu(T-\mu)\right\}\right]\right]$$

$$+D_o\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]-\frac{E_I L D_o M^3}{6}\left(5-\frac{MR}{2}\right)\Bigg]$$

$$+P_C C_I\left[D_o(\mu-t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2+t_1^*(t_1^*+\mu)}{3}-\frac{R(t_1^*+\mu)}{2}\right\}+\left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2+\mu(T-\mu)\right\}\right]\right.$$

$$\left.\left.+\frac{(r-1)D_o}{2}\left(\frac{t_1^{*3}-M^3}{3}-\frac{R(t_1^{*4}-M^4)}{4}\right)\right]\right]$$

$$TPC_{13}=\frac{A}{D}+\frac{1}{T}\left[H_c D_o\mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3}-\frac{R\mu}{4}\right)-\frac{2R}{3}(t_1^3-\mu^3)+(t_1-\mu)\left(\mu+(t_1+\mu)\left(1-\frac{R}{2}\right)\right)\right\}\right.\right.$$

$$\left.+(T-t_1)\left\{T+\frac{(T-t_1)}{2}(\mu RT+1)+\frac{R}{3}(T^2+t_1(t_1+T))\right\}\right]$$

$$+P_C C_I\left[\frac{(r-1)D_o}{2}\left(\frac{\mu^3-M^3}{3}-\frac{R(\mu^4-M^4)}{4}\right)+D_o\mu(r+1)(t_1-\mu)\left\{\frac{t_1+\mu}{2}+\frac{R(t^2+\mu(\mu+t))}{2}\right\}\right.$$

$$\left.\left.+D_o\mu(T-\mu)\left[T+(T+t_1)\left(\frac{1}{2}-RT\right)+\frac{R}{3}(T^2+t_1(t_1+T))\right]\right]-\frac{E_I L D_o M^3}{6}\left(5-\frac{MR}{2}\right)\right]$$

Theorem 1

Case 1 ($x < 0$)

$TPC_{13}(T)$ is a convex function of T $(0,\infty)$.

The optimal cost of T_{13} is attained by min[$TPC_{13}(T_{13})$, $T_{13}=(1/2)(r+1)\mu$]

Case 2 ($x \geq 0$)

$TPC_{13}(T)$ is an increasing function of $T(0,\infty)$.

For T belongs to $((1/2)(r+1)\mu,\infty)$, the optimal value of T_{13} is analyzed when $T_{13}=((1/2)(r+1)\mu)$.

By using theorem 1, the optimal solutions can easily be analyzed. So by following algorithm we can obtain the optimal replenishment cycle and the ordering quantity.

Steps for Algorithm 1:

1: enter the values of all the parameters which satisfy the condition $M \le \mu$.

2: make a comparison between M *and* $(\mu/\sqrt{r})$.

3: if $M \le \mu/\sqrt{r}$ then continue with steps in (1.a).

4:if $M \ge \mu/\sqrt{r}$ then continue with steps in (1.b).

Steps for Algorithm (1.a):

1: Compute $TPC_{11}(T_{11})$, if $\mu \le T_{11} \le (M^2r+\mu^2)/2\mu$ and $f_{11}^{"}(T_{11})>0$.

Find the min[$TPC_{11}(T_{11})$, $TPC_{11}(\mu)$, $TPC_{11}((M^2r+\mu^2)/2\mu)$] and according to it set T_{11}.

Compute $TPC_{11}(T_{11})$ and Q_{11}. Else go to next step.

2: Calculate min[$TPC_{11}(\mu)$, $TPC_{11}((M^2r+\mu^2)/2\mu)$]and according to it set T_{11}.

Compute $TPC_{11}(T_{11})$and Q_{11}.

3:Compute $TPC_{12}(T_{12})$ and Q_{12}, if $(M^2r+\mu^2)/2\mu \le T_{12} \le (r+1)2\mu$ and $f_{12}^{"}(T_{12})>0$.

Find the min[$TPC_{12}(T_{12})$, $TPC_{12}((r+1)\mu/2)$, $TPC_{12}((M^2r+\mu^2)/2\mu)$] and according to it set T_{12}.

Compute $TPC_{12}(T_{12})$ and Q_{12}. Else go to next step.

4: Calculate min[$TPC_{12}((r+1)\mu/2)$, $TPC_{12}((M^2r+\mu^2)/2\mu)$]and according to it set T_{12}.

Compute $TPC_{12}(T_{12})$and Q_{11}.

5:Compute $TPC_{13}(T_{13})$ and Q_{13}, if $(r+1)\mu/2 \le T_{13}$.

6: Select $T_{13}=(r+1)\mu/2$ .compute $TPC_{13}(T_{13})$and Q_{13}.

7: Find the min[$TPC_{11}(T_{11})$, $TPC_{12}(T_{12})$, $TPC_{13}(T_{13})$] and according to it set T_{11}.

Select the optimal value. Stop.

Steps for Algorithm (1.b):

1: Compute $TPC_{12}(T_{12})$, if $\mu \le T_{12} \le (r+1)\mu/2$ and $f_{12}^{"}(T_{12})>0$.

Find the min[$TPC_{12}(T_{12})$, $TPC_{12}((r+1)\mu/2)$, $TPC_{12}(\mu)$] and according to it set T_{12}.

Compute Q_{12} and $TPC_{12}(T_{12})$. Else go to next step.

2: Calculate min[$TPC_{12}(((r+1)\mu/2))$, $TPC_{12}(\mu)$]and according to it set T_{12}.

Compute Q_{12} and $TPC_{12}(T_{12})$.

3:Compute $TPC_{13}(T_{13})$ and Q_{13}, if $(r+1)\mu/2 \le T_{13}$ .Else go next.

4: Select T_{13}= $((r+1)\mu/2)$ and compute Q_{13} and TPC13(T_{13}).

5: Find the min[$TPC_{12}(T_{12})$, $TPC_{13}(T_{13})$] and according select the optimal value.

Stop.

(B) The model for μ ≤ M

Case B.1:- $t_1 \le \mu \le M \le T$

Same as case A, interest charged for unsolved inventory after the time period M is given by

$$P_c C_I \int_M^T I_3(t)e^{-Rt}dt$$

$$= P_c C_I D_o \mu (T-M)\left[T+\frac{(T+M)}{2}(1-RT)+\frac{R}{3}(T^2+M(M+T))\right] (24)$$

And the earned interest is given by

$$LE_I\left\{\int_0^{\mu}(M-T)D_o te^{-Rt}dt+\int_{\mu}^{M}(M-T)D_o \mu e^{-Rt}dt\right\}$$

$$= LE_I D_o \mu^2\left[\frac{M}{2}-\frac{\mu}{3}\left\{(1+MR)+\frac{R\mu^2}{4}\right\}+\frac{(M-\mu)}{\mu}\left\{M+\frac{(M+\mu)}{2}(1-MR)+\frac{R}{3}(M^2+\mu(M+\mu))\right\}\right]$$

(25)

Case B.2:- ($\mu \le t_1 \le M \le T$)

Interest charged for unsold inventory after the time period M is given by

$$P_C C_I \int_M^T I_3(t)e^{-Rt}dt$$

$$= P_C C_I D_o \mu (T-M)\left[T+\frac{(T+M)}{2}(1-RT)+\frac{R}{3}(T^2+M(M+T))\right] (26)$$

And the earned interest is given by

$$LE_I\left\{\int_0^{\mu}(M-T)D_o te^{-Rt}dt+\int_{\mu}^{M}(M-T)D_o \mu e^{-Rt}dt\right\}$$

$$= LE_I D_o \mu^2\left[\frac{M}{2}-\frac{\mu}{3}\left\{(1+MR)+\frac{R\mu^2}{4}\right\}+\frac{(M-\mu)}{\mu}\left\{M+\frac{(M+\mu)}{2}(1-MR)+\frac{R}{3}(M^2+\mu(M+\mu))\right\}\right]$$

(27)

Case B.3 :- ($\mu \le M \le t_1 \le T$)

Interest charged for unsold inventory after the time period M is given by

$$P_C C_I \int_M^{t_1} I_2(t)e^{-Rt}dt+\int_{t_1}^{T} I_3(t)e^{-Rt}dt$$

$$= P_C C_I D_o \mu\left[(r-1)\left\{\frac{(t_1-M)(t_1+M-\mu)}{2}-R(t_1-M)\left(\frac{\mu}{2}+t_1^2+M(M+t_1)\right)\right\}\right.$$

$$\left.+T-\frac{T^2-t_1^2}{2}(1-RT)+\frac{R(T^3-t_1^3)}{3}\right] (28)$$

And the earned interest is given by

$$LE_I\left\{\int_0^{\mu}(M-T)D_o te^{-Rt}dt+\int_{\mu}^{M}(M-T)D_o\mu e^{-Rt}dt\right\}$$

$$=LE_I D_o\mu^2\left[\frac{M}{2}-\frac{\mu}{3}\left\{(1+MR)+\frac{R\mu^2}{4}\right\}+\frac{(M-\mu)}{\mu}\left\{M+\frac{(M+\mu)}{2}(1-MR)+\frac{R}{3}(M^2+\mu(M+\mu))\right\}\right] \quad (29)$$

(C) The model for T <M

Case C.1:-($t_1 \le \mu \le T < M$)

Interest charged for unsold inventory after the time period M is zero and the earned interest is given by

$$LE_I\left\{\int_0^{T}(M-T)D_o te^{-Rt}dt+\int_0^{t_1^*}(M-T)K(t)e^{-Rt}dt\right\}$$

$$=LE_I D_o\left[T^2\left[\frac{M}{2}-\frac{T}{3}(1+MR)+\frac{RT^2}{4}\right]+r(M-T)\left\{\frac{t_1^{*2}}{2}-\frac{Rt_1^{*3}}{3}\right\}\right] \quad (30)$$

Case C.2 :-($\mu \le t_1\mu \le T < M$)

Interest charged for unsold inventory after the time period M is zero and the earned interest is given by

$$LE_I\left\{\int_0^{T}(M-T)D_o te^{-Rt}dt+\int_0^{t_1}(M-T)K(t)e^{-Rt}dt\right\}$$

$$=LE_I D_o\left[T^2\left[\frac{M}{2}-\frac{T}{3}\left\{(1+MR)+\frac{RT^2}{4}\right\}\right]+r(M-T)\left\{\frac{t_1^{*2}}{2}-\frac{Rt_1^{*3}}{3}\right\}\right] \quad (31)$$

Total cost of replenishment policy is given by following

$$TC_2=\begin{cases}TC_{21}, & t_1\le\mu\le M\le T\\ TC_{22}, & \mu\le t_1\le M\le T\\ TC_{23}, & \mu\le M\le t_1\le T\\ TC_{31}, & t_1\le\mu\le T\le M\\ TC_{32}, & \mu\le t_1\le T\le M\end{cases} \quad (32)$$

Where,

$$TC_{21} = A + H_C\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3} - \frac{Rt_1^*}{4}\right) + D_o\mu(T-\mu)\left[T - \frac{(T+\mu)(RT+1)}{2} + \frac{R}{3}(T^2+\mu(\mu+T))\right]\right.$$

$$\left. + D_o(\mu - t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2 + t_1^*(t_1^*+\mu)}{3} - \frac{R(t_1^*+\mu)}{2}\right\} + \left\{1 - \frac{R}{2}(\mu + t_1^*)\right\}\left\{\frac{1}{2}\mu^2 + \mu(T-\mu)\right\}\right]\right]$$

$$+ P_c C_I D_o \mu(T-M)\left[T + \frac{(T+M)}{2}(1-RT) + \frac{R}{3}(T^2 + M(M+T))\right]$$

$$-LE_I D_o \mu^2\left[\frac{M}{2} - \frac{\mu}{3}\left\{(1+MR) + \frac{R\mu^2}{4}\right\} + \frac{(M-\mu)}{\mu}\left\{M + \frac{(M+\mu)}{2}(1-MR) + \frac{R}{3}(M^2 + \mu(M+\mu))\right\}\right]$$

$$TC_{22} = A + H_c D_o \mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3} - \frac{R\mu}{4}\right) - \frac{2R}{3}(t_1^3 - \mu^3) + (t_1-\mu)\left(\mu + (t_1+\mu)\left(1 - \frac{R}{2}\right)\right)\right\}\right.$$

$$\left. +(T-t_1)\left\{T + \frac{(T-t_1)}{2}(\mu RT+1) + \frac{R}{3}(T^2 + t_1(t_1+T))\right\}\right]$$

$$+P_C C_I D_o \mu(T-M)\left[T + \frac{(T+M)}{2}(1-RT) + \frac{R}{3}(T^2 + M(M+T))\right]$$

$$-LE_I D_o \mu^2\left[\frac{M}{2} - \frac{\mu}{3}\left\{(1+MR) + \frac{R\mu^2}{4}\right\} + \frac{(M-\mu)}{\mu}\left\{M + \frac{(M+\mu)}{2}(1-MR) + \frac{R}{3}(M^2 + \mu(M+\mu))\right\}\right]$$

$$TC_{23} = A + H_c D_o \mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3} - \frac{R\mu}{4}\right) - \frac{2R}{3}(t_1^3 - \mu^3) + (t_1-\mu)\left(\mu + (t_1+\mu)\left(1 - \frac{R}{2}\right)\right)\right\}\right.$$

$$\left. +(T-t_1)\left\{T + \frac{(T-t_1)}{2}(\mu RT+1) + \frac{R}{3}(T^2 + t_1(t_1+T))\right\}\right] + P_C C_I D_o \mu\left[(r-1)\left\{\frac{(t_1-M)(t_1+M-\mu)}{2}\right.\right.$$

$$\left.\left. -R(t_1-M)\left(\frac{\mu}{2} + t_1^2 + M(M+t_1)\right)\right\} + T - \frac{T^2 - t_1^2}{2}(1-RT) + \frac{R(T^3 - t_1^3)}{3}\right]$$

$$-LE_I D_o \mu^2\left[\frac{M}{2} - \frac{\mu}{3}\left\{(1+MR) + \frac{R\mu^2}{4}\right\} + \frac{(M-\mu)}{\mu}\left\{M + \frac{(M+\mu)}{2}(1-MR) + \frac{R}{3}(M^2 + \mu(M+\mu))\right\}\right]$$

$$TC_{31} = A + H_C\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3} - \frac{Rt_1^*}{4}\right) + D_o\mu(T-\mu)\left[T - \frac{(T+\mu)(RT+1)}{2} + \frac{R}{3}(T^2+\mu(\mu+T))\right]\right.$$

$$\left. + D_o(\mu - t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2 + t_1^*(t_1^*+\mu)}{3} - \frac{R(t_1^*+\mu)}{2}\right\} + \left\{1 - \frac{R}{2}(\mu + t_1^*)\right\}\left\{\frac{1}{2}\mu^2 + \mu(T-\mu)\right\}\right]\right]$$

$$-LE_I D_o\left[T^2\left[\frac{M}{2} - \frac{T}{3}(1+MR) + \frac{RT^2}{4}\right] + r(M-T)\left\{\frac{t_1^{*2}}{2} - \frac{Rt_1^{*3}}{3}\right\}\right]$$

$$TC_{32} = A + H_c D_o \mu \left[\frac{(r-1)}{2} \left\{ \mu^2 \left(\frac{1}{3} - \frac{R\mu}{4} \right) - \frac{2R}{3}(t_1^3 - \mu^3) + (t_1 - \mu)\left(\mu + (t_1+\mu)\left(1 - \frac{R}{2}\right)\right) \right\} \right.$$
$$\left. +(T-t_1)\left\{ T + \frac{(T-t_1)}{2}(\mu RT + 1) + \frac{R}{3}(T^2 + t_1(t_1+T)) \right\} \right]$$
$$-LE_I D_o \left[T^2 \left[\frac{M}{2} - \frac{T}{3}\left\{ (1+MR) + \frac{RT^2}{4} \right\} \right] + r(M-T)\left\{ \frac{t_1^{*2}}{2} - \frac{Rt_1^{*3}}{3} \right\} \right]$$

Total cost of replenishment policy per unit time is given by following

$$TPC_2 = \begin{cases} TPC_{21}, & t_1 \le \mu \le M \le T \\ TPC_{22}, & \mu \le t_1 \le M \le T \\ TPC_{23}, & \mu \le M \le t_1 \le T \\ TPC_{31}, & t_1 \le \mu \le T \le M \\ TPC_{32}, & \mu \le t_1 \le T \le M \end{cases} \qquad (33)$$

Where,

$$TPC_{21} = \frac{A}{T} + \frac{1}{T}\left[\frac{(r-1)D_o t_1^{*3}}{2}\left(\frac{1}{3} - \frac{Rt_1^*}{4} \right) + D_o \mu (T-\mu)\left[T - \frac{(T+\mu)(RT+1)}{2} + \frac{R}{3}(T^2 + \mu(\mu+T)) \right] \right.$$
$$+ D_o(\mu - t_1^*)\left[\frac{-1}{2}\left\{ \frac{\mu^2 + t_1^*(t_1^*+\mu)}{3} - \frac{R(t_1^*+\mu)}{2} \right\} + \left\{ 1 - \frac{R}{2}(\mu + t_1^*) \right\}\left\{ \frac{1}{2}\mu^2 + \mu(T-\mu) \right\} \right]$$
$$+ P_c C_I D_o \mu (T-M)\left[T + \frac{(T+M)}{2}(1-RT) + \frac{R}{3}(T^2 + M(M+T)) \right]$$
$$\left. -LE_I D_o \mu^2 \left[\frac{M}{2} - \frac{\mu}{3}\left\{ (1+MR) + \frac{R\mu^2}{4} \right\} + \frac{(M-\mu)}{\mu}\left\{ M + \frac{(M+\mu)}{2}(1-MR) + \frac{R}{3}(M^2 + \mu(M+\mu)) \right\} \right] \right]$$

$$TPC_{22} = \frac{A}{T} + \frac{1}{T}\left[H_c D_o \mu \left[\frac{(r-1)}{2}\left\{ \mu^2\left(\frac{1}{3} - \frac{R\mu}{4} \right) - \frac{2R}{3}(t_1^3 - \mu^3) + (t_1-\mu)\left(\mu + (t_1+\mu)\left(1 - \frac{R}{2} \right) \right) \right\} \right. \right.$$
$$\left. +(T-t_1)\left\{ T + \frac{(T-t_1)}{2}(\mu RT + 1) + \frac{R}{3}(T^2 + t_1(t_1+T)) \right\} \right]$$
$$+ P_C C_I D_o \mu (T-M)\left[T + \frac{(T+M)}{2}(1-RT) + \frac{R}{3}(T^2 + M(M+T)) \right]$$
$$\left. -LE_I D_o \mu^2 \left[\frac{M}{2} - \frac{\mu}{3}\left\{ (1+MR) + \frac{R\mu^2}{4} \right\} + \frac{(M-\mu)}{\mu}\left\{ M + \frac{(M+\mu)}{2}(1-MR) + \frac{R}{3}(M^2 + \mu(M+\mu)) \right\} \right] \right]$$

$$TPC_{23}=\frac{A}{T}+\frac{1}{T}\left[H_cD_o\mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3}-\frac{R\mu}{4}\right)-\frac{2R}{3}(t_1^3-\mu^3)+(t_1-\mu)\left(\mu+(t_1+\mu)\left(1-\frac{R}{2}\right)\right)\right\}\right.\right.$$

$$\left.\left.+(T-t_1)\left\{T+\frac{(T-t_1)}{2}(\mu RT+1)+\frac{R}{3}(T^2+t_1(t_1+T))\right\}\right]+P_CC_ID_o\mu\left[(r-1)\left\{\frac{(t_1-M)(t_1+M-\mu)}{2}\right.\right.$$

$$\left.\left.-R(t_1-M)\left(\frac{\mu}{2}+t_1^2+M(M+t_1)\right)\right\}+T-\frac{T^2-t_1^2}{2}(1-RT)+\frac{R(T^3-t_1^3)}{3}\right]$$

$$\left.-LE_ID_o\mu^2\left[\frac{M}{2}-\frac{\mu}{3}\left\{(1+MR)+\frac{R\mu^2}{4}\right\}+\frac{(M-\mu)}{\mu}\left\{M+\frac{(M+\mu)}{2}(1-MR)+\frac{R}{3}(M^2+\mu(M+\mu))\right\}\right]\right]$$

$$TPC_{31}=\frac{A}{T}+\frac{1}{T}\left[\frac{(r-1)D_ot_1^{*3}}{2}\left(\frac{1}{3}-\frac{Rt_1^*}{4}\right)+D_o\mu(T-\mu)\left[T-\frac{(T+\mu)(RT+1)}{2}+\frac{R}{3}(T^2+\mu(\mu+T))\right]\right.$$

$$+D_o(\mu-t_1^*)\left[\frac{-1}{2}\left\{\frac{\mu^2+t_1^*(t_1^*+\mu)}{3}-\frac{R(t_1^*+\mu)}{2}\right\}+\left\{1-\frac{R}{2}(\mu+t_1^*)\right\}\left\{\frac{1}{2}\mu^2+\mu(T-\mu)\right\}\right]$$

$$\left.-LE_ID_o\left[T^2\left[\frac{M}{2}-\frac{T}{3}\left\{(1+MR)+\frac{RT^2}{4}\right\}\right]+r(M-T)\left\{\frac{t_1^{*2}}{2}-\frac{Rt_1^{*3}}{3}\right\}\right]\right]$$

$$TPC_{32}=\frac{A}{T}+\frac{1}{T}\left[H_cD_o\mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3}-\frac{R\mu}{4}\right)-\frac{2R}{3}(t_1^3-\mu^3)+(t_1-\mu)\left(\mu+(t_1+\mu)\left(1-\frac{R}{2}\right)\right)\right\}\right.\right.$$

$$\left.+(T-t_1)\left\{T+\frac{(T-t_1)}{2}(\mu RT+1)+\frac{R}{3}(T^2+t_1(t_1+T))\right\}\right]$$

$$\left.-LE_ID_o\left[T^2\left[\frac{M}{2}-\frac{T}{3}\left\{(1+MR)+\frac{RT^2}{4}\right\}\right]+r(M-T)\left\{\frac{t_1^{*2}}{2}-\frac{Rt_1^{*3}}{3}\right\}\right]\right]$$

Theorem 2

Case 1 ($y < 0$)

$TPC_{23}(T)$ is a convex function of $T(0,\infty)$.

The optimal value of T_{23} is obtained by min[$TPC_{23}(T_{23})$, T_{23}= M r - (1/2) (r - 1) μ]

Case 2 ($y \geq 0$)

$TPC_{23}(T)$ is an increasing function of T $(0,\infty)$. Where T belongs to (M r - (1/2) (r - 1) μ,∞), the optimal value of T_{23} is analyzed when T_{23} =(M r - (1/2) (r - 1) μ).

By using theorem 1, the optimal solutions can easily be analyzed. So by following algorithm we can obtain the optimal replenishment cycle and the ordering quantity with $M \geq \mu$.

Steps for Algorithm 2:

1: Enter the values of all the parameters which satisfy the condition $M \geq \mu$.

2: Make a comparison between M *and* $(r+1)\mu/2$.

3: if $(r+1)\mu/2 \geq M \geq \mu$ then continue with steps in algorithm (2.a).

4:if $M \geq (r+1)\mu/2$ then continue with steps in (2.b).

Steps for Algorithm (2.a):

1: Find the minimum value of $TPC_{31}(T)$ as follows.

2: Compute $TPC_{31}(T_{31})$, if $\mu \le T_{31} \le M$ and $f_{31}^{"}(T_{31}) > 0$.

Find the min[$TPC_{31}(T_{31})$, $TPC_{31}(\mu)$, $TPC_{31}(M)$] and according to it set T_{31}.

Compute Q_{31} and $TPC_{31}(T_{31})$. Else go to next step.

3: Calculate min[$TPC_{31}(\mu)$, $TPC_{31}(M)$]and according to it set T_{31}.

Compute Q_{31} and $TPC_{31}(T_{31})$.

4: Find the minimum value of $TPC_{21}(T)$ as follows

5:Compute $TPC_{21}(T_{21})$, if $M \le T_{21} \le (r+1)2\mu$ and $f_{21}^{"}(T_{21}) > 0$.

Find the min[$TPC_{21}(T_{21})$, $TPC_{21}(M)$, $TPC_{21}((r+1)\mu/2)$] and according to it set T_{21}.

Compute Q_{21} and $TPC_{21}(T_{21})$. Else go to next step.

6:Calculate min [$TPC_{21}(M)$, $TPC_{12}((r+1)\mu/2)$]and according to it set T_{21}.

Compute Q_{21} and $TPC_{12}(T_{12})$.

7:Find the minimum value of $TPC_{22}(T)$ as follows

8: Compute $TPC_{22}(T_{22})$, if $(r+1)\mu/2 \le T_{22} \le Mr-(r+1)\mu/2$ and $f_{22}^{"}(T_{22}) > 0$

Find the min [$TPC_{22}(T_{22})$, $TPC_{22}((r+1)\mu/2)$, $TPC_{31}(Mr-(r+1)\mu/2)$] and according to it set T_{22}.

Compute Q_{22} and $TPC_{22}(T_{22})$. Else go to next step.

9: Calculate min [$TPC_{22}((r+1)\mu/2)$, $TPC_{31}(Mr-(r+1)\mu/2)$]and according to it set T_{22}.

Compute $Q_{22}$and $TPC_{22}(T_{22})$.

10: Find the minimum value of $TPC_{23}(T)$ as follows

11: Compute $TPC_{23}(T_{23})$, if $(r+1)\mu/2 \le T_{23}$.

Compute Q_{23} and $TPC_{23}(T_{23})$. Else go to next step.

12: Select T_{23}= $(r+1)\mu/2$.

Compute Q_{23} and $TPC_{23}(T_{23})$.

13: Find the min [$TPC_{31}(T_{31})$, $TPC_{21}(T_{21})$, $TPC_{22}(T_{22})$, $TPC_{23}(T_{23})$,
and according select the optimal value. Stop.

Steps for Algorithm (2.b):

1: Find the minimum value of $TPC_{31}(T)$ as follows.

2: Compute $TPC_{31}(T_{31})$, if $\mu \le T_{31} \le (r+1)\mu/2$ and $f_{31}^{"}(T_{31}) > 0$.

Find the min [$TPC_{31}(T_{31})$, $TPC_{31}(\mu)$, $TPC_{31}((r+1)\mu/2)$] and according to it set T_{31}.

Compute Q_{31} and $TPC_{31}(T_{31})$. Else go to next step.

3: Calculate min [$TPC_{31}(\mu)$, $TPC_{31}((r+1)\mu/2)$]and according to it set T_{31}.

Compute Q_{31} and $TPC_{31}(T_{31})$.

4: Find the minimum value of $TPC_{32}(T)$ as follows

5:Compute $TPC_{32}(T_{32})$, if $(r+1)2\mu \le T_{32} \le M$ and $f_{32}^{"}(T_{32}) > 0$.

Find the min [$TPC_{32}(T_{32})$, $TPC_{32}((r+1)\mu/2)$, $TPC_{32}(M)$] and according to it set T_{32}.

Compute Q_{32} and $TPC_{32}(T_{32})$. Else go to next step.

6:Calculate min [$TPC_{32}((r+1)\mu/2)$, $TPC_{32}(M)$]and according to it set T_{32}.

Compute Q_{32} and $TPC_{32}(T_{32})$.

7:Find the minimum value of $TPC_{22}(T)$ as follows

8: Compute $TPC_{22}(T_{22})$, if $M \le T_{22} \le Mr-(r+1)\mu/2$ and $f_{22}^{''}(T_{22})>0$

Find the min [$TPC_{22}(T_{22})$, $TPC_{22}(M)$, $TPC_{22}(Mr-(r+1)\mu/2)$] and according to it set $T_{22.}$

Compute Q_{22} and $TPC_{22}(T_{22})$. Else go to next step.

9: Calculate min [$TPC_{22}(M)$, $TPC_{22}(Mr-(r+1)\mu/2)$]and according to it set T_{22}.

Compute Q_{22} and $TPC_{22}(T_{22})$.

10: Find the minimum value of $TPC_{23}(T)$ as follows

11: Compute $TPC_{23}(T_{23})$, if $(r+1)\mu/2 \le T_{23}$.

Compute Q_{23} and $TPC_{23}(T_{23})$. Else go to next step.

12: Select T_{23}= $(r+1)\mu/2$.

Compute Q_{23} and $TPC_{23}(T_{23})$.

13: Find the min [$TPC_{31}(T_{31})$, $TPC_{21}(T_{21})$, $TPC_{32}(T_{32})$, $TPC_{22}(T_{22})$, and according select the optimal value. Stop.

Numerical Example:-

(1) Input parameters

O = Rs 50 per order, D_0 =500 units, r = 2, L = Rs 100 per unit, H_C = Rs 5 per unit per year, P_C = Rs 40 per unit, R = 0.13, E_I = 0.15, C_I = 0.08, M = 0.3 years and $\mu = 0.04$ years.

Values obtained are $TPC_{11}(T_{11})$ = 291.3335 for T_{11} = 0.4000, $TPC_{12}(T_{12})$ = 375.25025for T_{12} = 0.4250, $TPC_{13}(T_{13})$ =101.56675for T_{13} = 0.6000. Therefore the optimal order cycle for the retailer is obtained to be T_{13} = 0.6000 and the total minimum cost is 101.56.

(2) Input parameters

O = Rs 50 per order, D_0 =500 units, r = 2, L = Rs 100 per unit, H_C = Rs 5 per unit per year, P_C = Rs 40 per unit, R = 0.13, E_I = 0.15, C_I= 0.08, M = 0.2 years and $\mu = 0.04$ years.

Values obtained are $TPC_{12}(T_{12})$ = 386.01607 for T_{12} = 0.40, $TP0C_{13}$ (T_{13}) =174.20633 for T_{13} = 0.6000. Therefore the optimal order cycle for the retailer is obtained to be T_{13} = 0.6000 and the total minimum cost is 174.21.

(3) Input parameters

O = Rs 50 per order, D_0 =500 units, r = 2, L = Rs 40 per unit, H_C = Rs 5 per unit per year, P_C = Rs 20 per unit, R = 0.13, E_I = 0.15, C_I= 0.08, M = 0.6 years and $\mu = 0.04$ years.

Values obtained are $TPC_{31}(T_{31})$ = 81.1925for T_{31} = 0.40, $TPC_{21}(T_{21})$ =348.0103for T_{21} = 0.6000, $TPC_{22}(T_{22})$ = 89.03 for T_{22} = 0.8000, $TPC_{23}(T_{23})$ = 447.1427 for T_{23} = 1.4. Therefore the optimal order cycle for the retailer is obtained to be T_{31} = 0.40 and the total minimum cost is 81.1925.

(4) Input parameters

O = Rs 50 per order, D_0 =500 units, r = 2, L = Rs 40 per unit, H_C = Rs 5 per unit per year, P_C = Rs 20 per unit, R = 0.13, E_I = 0.15, C_I= 0.08, M = 0.9 years and $\mu = 0.04$ years.

Values obtained are $TPC_{31}(T_{31})$ = 523.6765 for T_{31} = 0.6367, $TPC_{32}(T_{32})$ =224.0127 for T_{32} = 0.8000, $TPC_{22}(T_{22})$ = 381.6820 for T_{22} = 0.9000, $TPC_{23}(T_{23})$ = 1019.3333 for T_{23} = 2.3. Therefore the optimal order cycle for the retailer is obtained to be T_{31} = 0.40 and the total minimum cost is 224.0127.

Sensitivity analysis

In our considered model we have taken various numbers of parameters and the results obtained are influenced by them. Some parameters are already studied by a number of researchers and different results are obtained according to the different parameters such as ordering cost. Now in this present paper we will study the impact of three parameters namely, a constant used in production rate, offered credit period by the supplier and point where demand function breaks i.e. r, M, μ .

(a)Impact due to changes in the parameter r

For case $M \le \mu$ we consider the values of the parameters as, O = Rs 50 per order, D_0 =500 units, L = Rs 100 per unit, H_C = Rs 5 per unit per year, P_C = Rs 40 per unit, R = 0.13, E_I = 0.15, C_I= 0.08, M = 0.3 years and $\mu = 0.06$ years. The alterations occur due to the changes of the parameter r with respect to the total cost and retailer's choice is shown in the graphs (1) below.

For case $M \ge \mu$ we consider the values of the parameters as, O = Rs 250 per order, D_0 =500 units, L = Rs 40 per unit, H_C = Rs 5 per unit per year, P_C = Rs 40 per unit, R = 0.13, E_I = 0.15, C_I= 0.08, M = 0.7 years and $\mu = 0.03$ years. The alterations occur due to the changes of the parameter r with respect to the total cost and retailer's choice is shown in the graphs (2) below.

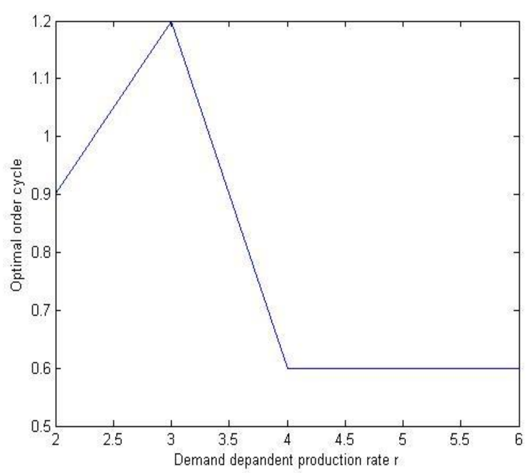
Influence on the retailer's decision

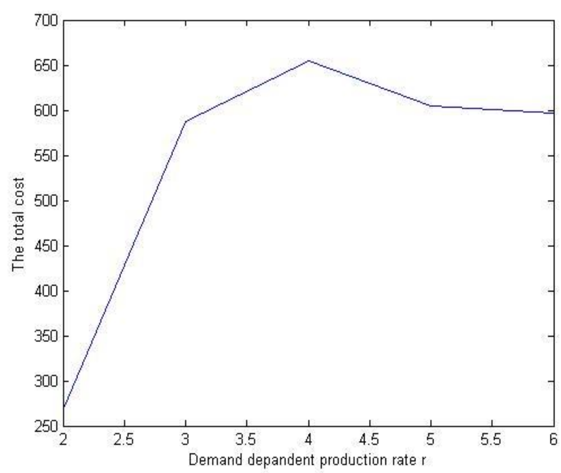
From thegraphs (1) and (2) we observed that if we increase the value of the parameter r, the optimal order quantity and optimal order cycle increases but the production rate dependent to demand rate decreases. So we can say that if rate of production is small, the retailer will reduce the frequency of placing orders to cut order cost. Higher the production rate dependent to demand rate and optimal order cycle and as it reaches to infinity it will lost its influence in the decision.

Influence on the total cost

With the increase in the value of parameter r, the value of total cost increases firstly and then starts to decrease. Low production rate results in low cost. As the value of the production rate increases, the profit of the retailer decreases.

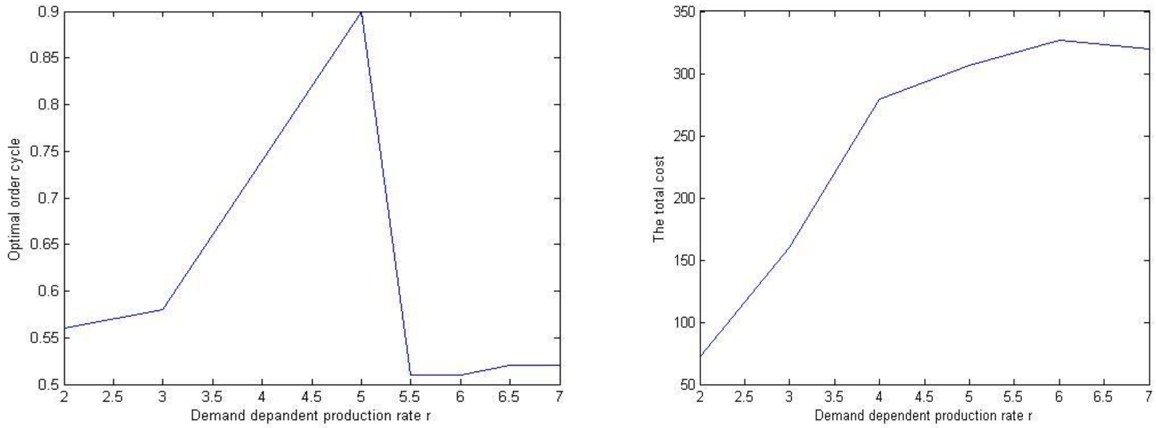
Graphs (1)





The impact due to the changes of the parameter r when $M \le \mu$

Graphs (2)



The impact due to the changes of the parameter r when $M \geq \mu$

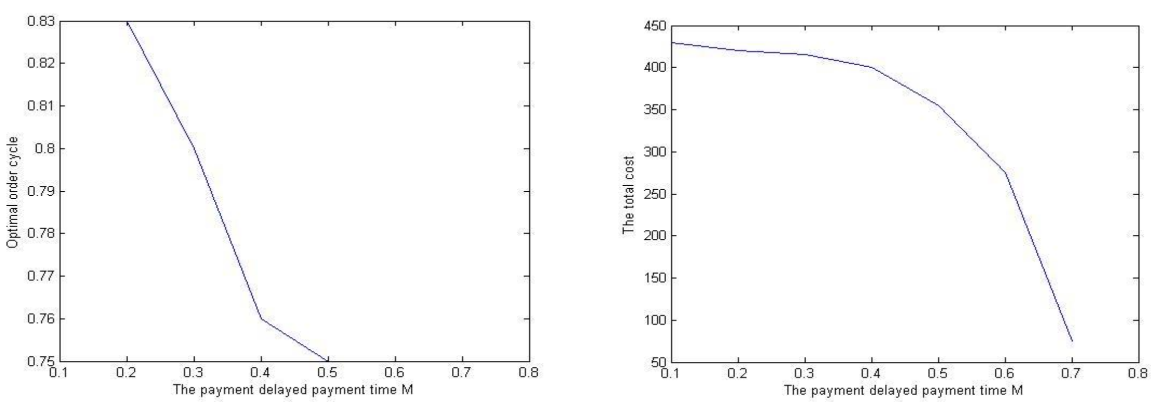
(b)Impact due to changes in the parameter M

We consider the values of the parameters as, O = Rs 250 per order, D_0 =500 units, L = Rs 40 per unit, H_C = Rs 5 per unit per year, P_C = Rs 20 per unit, R = 0.13, E_I = 0.15, C_I= 0.08, r = 2 and $\mu = 0.05$ years. The alterations occur due to the changes of the parameter M are shown in the graphs (3).

For case $M = \mu$, as the payment delayed time starts to increase and reaches up to the condition$M = \mu$, the optimal order quantity and the optimal order cycle does not increase which means that if than the growth stage tome of the new product is greater than the delayed payment time, then the retailer will make the replenishment cycle short so that he can take advantage of the trade credit.

For case $M > \mu$, it means the breakpoint of the demand is less than the delayed payment time. The delayed payment time has no effect on the optimal order quantity during the maturity stage and during this stage, the replenishment policy is not influenced by the offered delayed time.

Graphs (3) for M



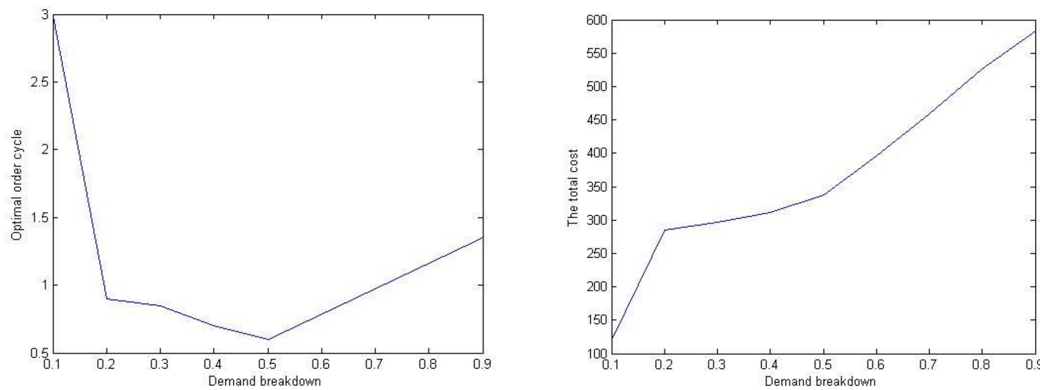
The impact due to changes in the parameter M

(c)Impact due to changes in theparameter μ

We consider the values of the parameters as, O = Rs 250 per order, D_0 = 500 units, L = Rs 40 per unit, H_C = Rs 5 per unit per year, P_C = Rs 20 per unit, R = 0.13, E_I = 0.15, C_I= 0.08, r = 2 and $M = 0.05$ years. The alterations occur due to the changes of the parameter M are shown in the graphs below (4).

With the help of the graph, we can say that as the breakpoint of demand is increasing the optimal order quality and quantity is decreasing in the first short interval and then increases. It means that as the breakpoint of demand rate is big the maturity period moves at slow rate and the various changes in demand are slow. So, with the increase in the growth rate of the life of the product the cost of the retailer will increase. Therefore, some measures must be taken by the retailer to move the product demand.

Graph (4) for μ



The impact due to changes in theparameter μ

Conclusions

If the demand dependent rate is increased, then the demand dependent production rate has no effect on the optimal order cycle. If the production rate decreases then the retailer will reduce the frequency of placing orders to cut order costs. For the limit case, when the production rate is infinite, as in most papers presented, the production rate has no effect on members' decisions.

When the delayed payment time is shorter than the time of the development phase of the introduction of new products, the retailer will shorten the replenishment cycle to take advantage of trade credits more often to accumulate interest. On the other hand, the short order cycle may allow the retailer to adjust its ordering decisions more quickly to meet the variable demand within the development phase of the products. When the delayed payment time is longer than the breakpoint of demand, then within the maturity stage of the products, the delayed payment time has no effect on the optimal ordering cycle. The total cost for the retailer is decreasing with delayed payment times. Therefore, the retailer hopes that the supplier can offer them as long a delay in payment as possible.As the demand breakpoint is increasing, firstly the optimal order cycle is decreasing and then increasing. The total cost demand for the retailer is increasing with the breakpoint. Therefore, for products, if the development phase of the product life cycle is long, the retailer will have a higher cost. Therefore, the retailer must take some measures, such as advertising, to drive demand for the products to the maturity stage at the earliest.

Appendices

1: Proof of Theorem 1

The first derivative for a minimum of TPC_{13} is

$$\frac{\partial TPC_{13}}{\partial T} = -\frac{1}{T^2} f_{13}(T) + \frac{1}{T} f_{13}^{'}(T)$$

Where,

$$f_{13}(T) = A + H_c D_o \mu \left[\frac{(r-1)}{2} \left\{ \mu^2 \left(\frac{1}{3} - \frac{R\mu}{4} \right) - \frac{2R}{3}(t_1^3 - \mu^3) + (t_1 - \mu)\left(\mu + (t_1 + \mu)\left(1 - \frac{R}{2}\right)\right)\right\} \right.$$

$$\left. + (T - t_1)\left\{ T + \frac{(T-t_1)}{2}(\mu RT + 1) + \frac{R}{3}(T^2 + t_1(t_1 + T)) \right\} \right]$$

$$+ \frac{(r-1)D_o}{2}\left(\frac{\mu^3 - M^3}{3} - \frac{R(\mu^4 - M^4)}{4} \right) + D_o \mu (r+1)(t_1 - \mu)\left\{ \frac{t_1 + \mu}{2} + \frac{R(t^2 + \mu(\mu + t))}{2} \right\}$$

$$+ D_o \mu (T - \mu)\left[T + (T + t_1)\left(\frac{1}{2} - RT \right) + \frac{R}{3}(T^2 + t_1(t_1 + T)) \right] - \frac{E_I L D_o M^3}{6}\left(5 - \frac{MR}{2} \right)$$

$$f_{13}^{'}(T) = H_c D_o \mu \left[(1 - t_1)\left\{ T + \frac{(T - t_1)}{2}(\mu RT + 1) + \frac{R}{3}(T^2 + t_1(t_1 + T)) \right\} \right.$$

$$\left. + (T - t_1)\left\{ 1 + \frac{(T - t_1)(\mu R)}{2} + \frac{(\mu RT + 1)}{2} + \frac{R}{3}(2T + t_1) \right\} \right]$$

$$+ D_o \mu (1 - \mu)\left[T + (T + t_1)\left(\frac{1}{2} - RT \right) + \frac{R}{3}(T^2 + t_1(t_1 + T)) \right]$$

$$+ D_o \mu (T - \mu)\left[1 + (1 + t_1)\left(\frac{1}{2} - RT \right) - R(T + t_1) + \frac{R}{3}(2T + t_1) \right]$$

The second derivative for a minimum of TPC_{13} is

$$\frac{\partial^2 TPC_{13}}{\partial T^2} = \frac{2}{T^3} f_{13}(T) - \frac{2}{T^2} f_{13}^{'}(T) + \frac{1}{T} f_{13}^{''}(T)$$

Where,

$$f_{13}^{"}(T)=H_cD_o\mu\begin{bmatrix}2(1-t_1)\left\{1+\frac{(T-t_1)(\mu R)}{2}+\frac{(\mu RT+1)}{2}+\frac{R}{3}(2T+t_1)\right\}\\+(T-t_1)\left\{\frac{\mu R(2-t_1)}{2}+\frac{2R}{3}\right\}\end{bmatrix}$$
$$+2D_o\mu(1-\mu)\left[1+(1+t_1)\left(\frac{1}{2}-RT\right)-R(T+t_1)+\frac{R}{3}(2T+t_1)\right]$$
$$+D_o\mu(T-\mu)\left[(1+t_1)\left(\frac{1}{2}-2R\right)+\frac{2R}{3}\right]$$

1: Proof of Theorem 2

The first derivative for a minimum of $TPC_{23}$is

$$\frac{\partial TPC_{13}}{\partial T}=-\frac{1}{T^2}f_{13}(T)+\frac{1}{T}f_{13}^{'}(T)$$

$$f_{23}(T)=A+H_cD_o\mu\left[\frac{(r-1)}{2}\left\{\mu^2\left(\frac{1}{3}-\frac{R\mu}{4}\right)-\frac{2R}{3}(t_1^3-\mu^3)+(t_1-\mu)\left(\mu+(t_1+\mu)\left(1-\frac{R}{2}\right)\right)\right\}\right.$$
$$\left.+(T-t_1)\left\{T+\frac{(T-t_1)}{2}(\mu RT+1)+\frac{R}{3}(T^2+t_1(t_1+T))\right\}\right]+P_CC_ID_o\mu\left[(r-1)\left\{\frac{(t_1-M)(t_1+M-\mu)}{2}\right.\right.$$
$$\left.\left.-R(t_1-M)\left(\frac{\mu}{2}+t_1^2+M(M+t_1)\right)\right\}+T-\frac{T^2-t_1^2}{2}(1-RT)+\frac{R(T^3-t_1^3)}{3}\right]$$
$$-LE_ID_o\mu^2\left[\frac{M}{2}-\frac{\mu}{3}\left\{(1+MR)+\frac{R\mu^2}{4}\right\}+\frac{(M-\mu)}{\mu}\left\{M+\frac{(M+\mu)}{2}(1-MR)+\frac{R}{3}(M^2+\mu(M+\mu))\right\}\right]$$

$$f_{23}^{'}(T)=H_cD_o\mu\begin{bmatrix}(1-t_1)\left\{T+\frac{(T-t_1)}{2}(\mu RT+1)+\frac{R}{3}(T^2+t_1(t_1+T))\right\}\\(T-t_1)\left\{1+\frac{(\mu RT+1)}{2}+\frac{\mu R(T-t_1)}{2}+\frac{R}{3}(2T-t_1)\right\}\end{bmatrix}$$
$$+P_CC_ID_o\mu\left[1+\frac{R(T^2-t_1^2)}{2}+2RT^2\right]$$

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