REGULAR NUMBER OF SUBDIVISION OF LINE GRAPH OF A GRAPH

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Abstract: The regular number of subdivision of line graph is the minimum number of subsets in to which the set of edges of $L[S(G)]$ are partitioned such that subgraph induced by all the subset is regular and is denoted as $r_{LS}(G)$. Here we introduced few results on $r_{LS}(G)$ expressed in terms of components of G.

Keywords: : Line Graph, Regular Graph, Subdivision Graphs, Regular number of Subdivision of Line Graphs.

Mathematics Subject Classification: 05C76.

1. Introduction

Here we take into account a simple, finite, non-trivial graphs. Usually p and q gives the count of vertices and edges of a G and the maximum degree of a vertex in G is written as $\Delta(G)$.For any real number x, $\lceil x \rceil$ denotes the smaller integer not less than x and $\lfloor x \rfloor$ denotes the greatest integer not greater than x .A vertex v is called cutvertex if removing it from G increases the number of components 'A graph G is called trivial if it has no edges. The maximum distance between any two vertices in G is called the diameter, denoted by $diam(G)$. Moreover a connected acyclic graph is said to be tree. A leaf of an unrooted tree is a node of vertex with degree 1.A vertex of a graph is said to be pendant if its neighborhood contains exactly one vertex. The parameters which are not defined in this paper possibly found in [3]. The concept of Regular Number of a Graph was studied Ashwini Ganesan and Radha R Iyer in [1]. The concept of Line graphs of Subdivision Graphs was studied by Sunilkumar M Hosmani in [11].

2. MAIN RESULTS:

Theorem 1: For any Wheel W_p for $p \geq 4$

$r_{LS}(W_p)$ =2 for $p = 4$

$r_{LS}(W_p)$ =3 for $p > 4$

Proof: For any wheel $W_p, p \geq 4$, we discuss with below two cases for $p = 4$ and $p > 4$

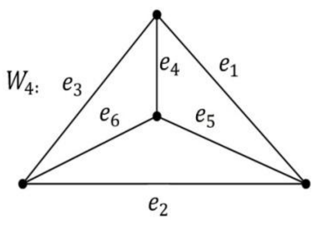
Case1: For a wheel W_p, if $p = 4$, Then we have the edge set of subdivision of W_p as

S$[W_4]$ = $E_1 \cup E_2$ where $E_1 = \{e_1,e_2,e_3, \dots\dots\dots\dots, e_6\}$ the set of edges lies in the interior on embedding in any plane (see fig 1(a)) In $L[S(W_4)]$, $E_1 \cup E_2$ = V$[L(S(w_4))]$.

Now we have minimum edge partition of

$L[S(w_4)]$, is $F_1 = \{(e_1 e_1', e_1' e_6, e_6 e_1), (e_2' e_3', e_3' e_6', e_6', e_2'), (e_2 e_3, e_3 e_4', e_4' e_2), (e_4 e_5, e_5 e_5', e_5' e_4)\}$ and $F_2 = \{(e_1 e_2), (e_3 e_4), (e_4 e_5), (e_5, e_6), (e_1' e_2'), (e_3' e_4'), (e_5' e_6')\}$,

where $\langle F_1 \rangle$ has a 2-regular and $\langle F_1 \rangle$ has 1-regular. Clearly $r_{LS}(w_4) = |F_1, F_2| = 2$ (see fig 1(b))



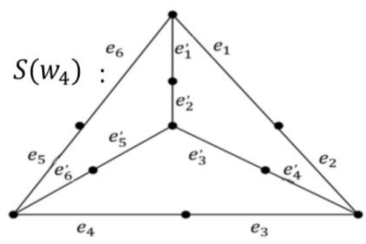


Figure 1(a)

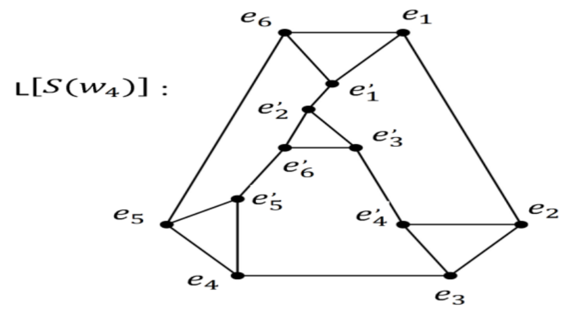


Figure 1 (b)

Case 2: For a wheel W_p , $p \geq 4$, we have the subdivision of w_p as $S(W_p)$ has the edge set such as E$[S(W_p)] = E_1 \cup E_2 \cup E_3$.
Now the edge set $E_1 = \{e_1, e_2, e_4, e_5, e_6, \dots\dots\dots\dots\dots, e_{m-1,} e_m\}$;
$E_2 = \{e'_1, e'_2, e'_3, e'_4, e'_5, e'_6{}' \dots\dots\dots\dots\dots, e'_{m-1}, e'_m\}$ and
$E_3 = \{e''_1, e''_2, e''_3, e''_4, e''_5, e''_6, \dots\dots, e''_{m-1}, e''_m\}$

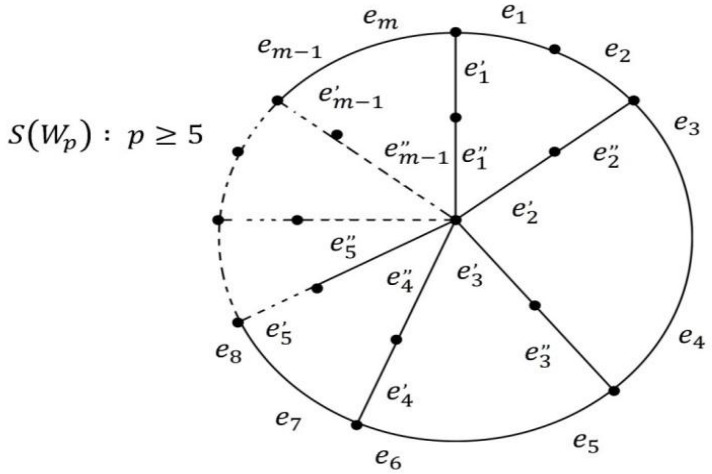


Figure 2 (a)

See fig 2(a). In $[S(w_p)]$, $V[L(s_{wp}))] = E_1 \cup E_{2,} \cup E_3$.
Now $F_1 = \{e_1e_2, e_3e_4, e_5e_6, e_7e_8, \dots\dots\dots, e_{m-1}e_m, e'_1e''_1, e'_2e''_2, e'_3e''_3, \dots\dots\dots, e'_{m-1}e''_{m-1}\}$
$F_2 = \{e_1e'_1e_m, e_2e'_2e_3, e_4e'_3e_5, \dots\dots\dots, e_{m-1}e'_{m-1}e_{m-2}\}$, $F_3 = \{e''_1, e''_2, e''_3, \dots\dots\dots, e''_{m-1}\}$ (see fig 2(b))
where each component of F_1 is K_2 and is 1-regular .For the set F_2, each component is K_3 and is 2-regular For the set F_3, the $\langle F_3\rangle$ is a complete graph K_p and is $p-1$ regular.
Clearly $r_{Ls}[w_p], p > 4 = |F_1, F_2, F_3|$=3.

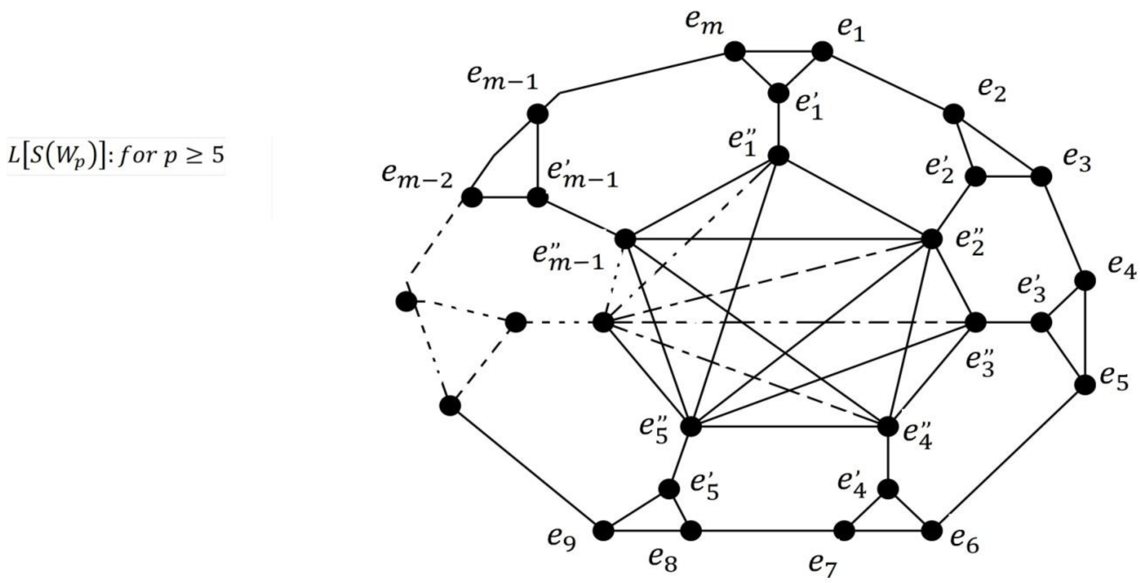


Figure 2 (b)

Theorem 2: For complete graph k_p with $p \geq 3$ vertices, $r_{SL}(k_p)$ =2,

Proof: For $p = 2$, $S[K_2] = K_{1,2}$ and $L[S(K_2)] = K_2$ Hence $r_{LS}(K_2) \neq 2$. For $p = 3$, $S(K_3) = C_6$ and $r_{LS}(C_6) = 2$. For K_p, $p \geq 5$ we have $\frac{p(p-1)}{2}$ edges. Let $V[K_p] = \{v_1, v_2, v_{3,} \ldots\ldots, v_p\}$ and $V[S(k_p)] = V[K_p] \cup \left\{\frac{p(p-1)}{2}\right\}$, where degree of each vertex in $\left\{\frac{p(p-1)}{2}\right\} \in V[S(k_p)]$ is two (see figure 1(a)). Since the number of edges incident to each vertex of $v_i \in V(K_p) \subseteq V[S(K_p)]$ is $p - 1$ Now in $L[S(w_p)]$ these edges form an induced sub graph k_{p-1} The number of complete induced subgraphs K_{p-1} are p suppose $H = E\left[L\left(S(w_p)\right)\right] - E[p. K_{p-1}], p \geq 5$, then each component of H is K_2 which are edge disjoint subgraphs. Hence $F_1 = \{p. k_{p-1}\}$ and $F_2 = \{p(p-1)\}$ where each component of F_1 is K_{p-1} also each component of F_2 is K_2 (see fig 1 (b)).
Hence $r_{Ls}(K_p) = |F_{1,} F_2| = 2$.

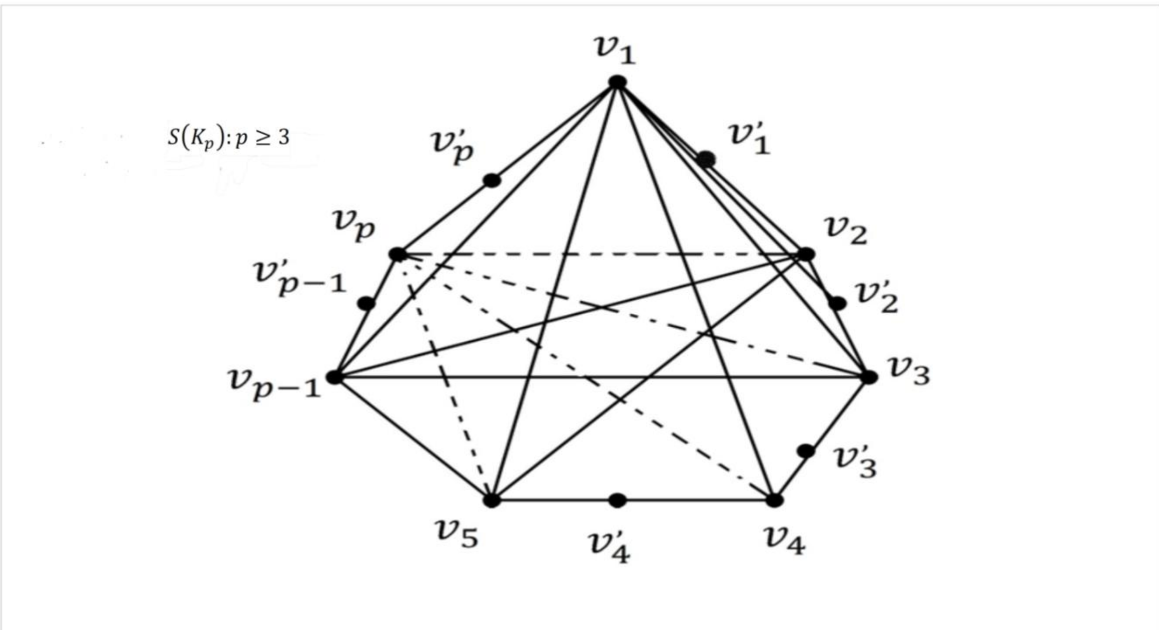


Figure 1 (a)

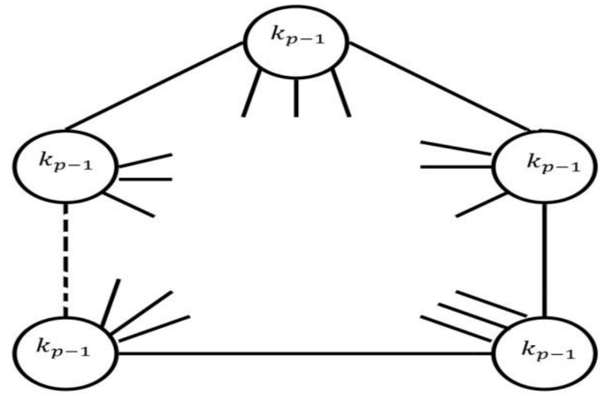


Figure 1 (b) copies of k_{p-1}

Theorem 3: For any star $K_{1,p}$ with p ≥ 2 vertices $r_{LS}[K_{1,p}]$ = 2.
Proof: Let $G = K_{1,p}, p \geq 2$ and $v = \Delta(G)$ in $K_{1,p}$ .Now the edge set of $K_{1,p}$ be $vv_1, vv_2, vv_3, \dots\dots, vv_m$. In $S[K_{1,p}]$ edge set be$\{vv_1', v_1'v\}, \{v_2v_2', v_2'v_2\}, \dots\dots\dots, \{vv_m', v_m'v_m\}$
such that in each set the edges are consecutive edges. Now the $E[S(K_{1,p})] = \{vv_1' = e_1', v_1'v_1 = e_1, vv_2' =$ *e2,, v2,v=e3,.....,vvm,=em,,vm,vm=em.*
In forming $L[S(K_{1,p})]$, $V\left[L\left(S(K_{1,p})\right)\right] = \{e_1'e_1, e_2', e_2, e_3', e_3, \dots\dots, e_m', e_m\}$.
Now in $L[S(K_{1,p})$, the set $S = \{e_1', e_2', e_3', \dots\dots, e_m'\}$
gives $\langle S\rangle$ as K_m which is regular. Hence $F_1 = \{e_1', e_2', e_3', \dots, e_m'\}$ and $F_2 = \{e_1'e_1, e_2'e_2, e_3'e_3, \dots\dots, e_m'e_m\}$,
Where $e_1'e_1 = e_2'e_2 = e_3'e_3 = K_2$ Hence $r_{LS}(K_{1,n}) = |F_1, F_2| = 2$.

Theorem 4: For any wheel w_p, with $p \geq 4$ vertices, $r_{LS}(W_p) \leq r_L(W_p)$.
Proof: Let $v_1, v_2, v_3, \dots\dots\dots\dots\dots, v_p$ be the vertex set of W_p in which $\Delta(W_p) = v_p$ and $degv_i = 3$, $1 \leq i \leq p-1$ suppose $e_1, e_2, e_3, \dots, e_{p-1}, e_1', e_2', \dots\dots, e_{p-1}'$ be the edges of W_p, Such that $e_i = v_iv_{i+1}$, $1 \leq i \leq p-2$, $e_{p-1} = v_1v_{p-1}$ and $e_i' = v_jv_p$ for $1 \leq i \leq p-1$ in$L[W_p]$, $\{e_1, e_2, e_3, \dots\dots\dots, e_{p-1}, e_1', e_2', e_3', \dots\dots\dots\dots, e_{p-1}'\} = V[L(W_p)]$ which corresponds to the E[W_p]. Let $F_1 = \{e_1, e_2, e_3, \dots\dots\dots\dots, e_{p-1}, e_{p-2}, e_1', e_2', e_3', \dots\dots\dots, e_{p-2}', e_{p-1}'\}$ in whichdeg$(e_i) = 4 = $ deg(e_j'), $\forall\, e_ie_j \epsilon F_1$ and $\langle F_1\rangle$ is 4-regular.Similarly for
$F_2 = \{e_1', e_3', e_5', \dots\dots\dots, e_{p-1}', e_2', e_4', e_6', \dots\dots\dots, e_{p-6}', e_{p-3}', e_1'\}$; $\forall e_k' \in F_2$, deg$(e'k) = p-4$;
$1 \leq k \leq p-3$ then $\langle F_2\rangle$ is $p-4$ regular. Hence $r_L(W_p) = |F_1, F_2| = 2$.
For the graph W_p and by Theorem 1
$r_{LS}(W_p) \leq r_L(W_p)$.

Theorem 5: For any non-trivial tree T, $r_{Ls}(T) \leq \Delta(T)$.
Proof: Suppose T be a non trivial tree$T$, then T has maximum degree vertex v such that deg$(v) = \Delta(T)$. Let $A = \{v_1, v_2, v_3, \dots\dots\dots\dots\dots, v_n\}$ be the intermediate vertices and $v \notin A$.Hence deg$(v_1) <$ deg(v) $\forall\, v_i \in A$, $1 \leq i \leq n$ and $V(T) = A \cup \{v\}$. In $S[T], E[S(T)] = 2q(T) = V[L(S(T))]$.Now in $L[S(T)]$, each block is a complete induced sub graph of $L[S(T)]$. Since deg$(v) = \Delta(T) = \Delta[S(T)]$.
Now we discuss the following cases:
Case1: Suppose the degree of each vertex in A have similar degree such that deg$(v_i) <$ deg(v) , $\forall\, v_i \epsilon A$ and $v \in \Delta[S(T)]$, then in $L[S(T)]$ each block is complete .Let $\{B_1, B_2, \dots\dots, B_n\}$ be the block set which are complete r-regular,$1 \leq r \leq n$ and denote it as $F_1 = \{B_1, B_2, \dots\dots\dots\dots, B_n\}$. Now the edges which are incident to v forms a complete block B_{n+1} with $r+1$ regular.Hence $F_2 = \{B_{n+1}\}$ .Since T is a tree, in $L\{S(T)\}$, the set $C = \{B_1', B_2', B_3', \dots\dots\dots\dots, B_m'\}$ where each block is complete and is K_2. Thus the minimum regular partition of $L[S(T)] = |F_1, F_2, F_3| = 3$.
Clearly $r_{Ls}(T) < \Delta(T)$.
Case2: Suppose the degree of each vertex in A$\cup \{v\}$ is alike then in $S(T)$ they preserve the same degree and $N(v_j) = v_k$, $\forall v_j \in A \cup \{v\}$; $v_k \in V[S(T)]$ such that deg$(v_k) = 2$. In $L[S(T)]$; $F_1 = \{B_1, B_2, \dots\dots\dots\dots, B_n, B_v\}$ be the block set and each is r-regular complete disjoint sub graphs of $L[S(T)]$. Let $F_2 = \{B_1', B_2', B_3' \dots\dots\dots, B_K'\}$ be

the set of blocks that forms bridges of $L[S(T)]$ or end blocks which are K_2 of $L[S(T)]$. Hence the minimal edge partition of $L[S(T)]$ are F_1 and F_2 thus $r_{Ls}(T) = |F_1, F_2| = 2$ and gives $r_{Ls}(T) = \Delta(T)$.

Case3: Suppose $degv_i \leq degv_2 \leq, \dots \dots \dots, \leq degv_n$ for the set $A = \{v_1, v_2, \dots \dots \dots, v_n\}$and $degv = \Delta(T)$ then in $S(T)$, $\forall\ v_i \epsilon A \cup \{v\}, N(v_i) = v_j\ , 1 \leq j \leq A \cup \{v\}$, Thus $\deg(v_j) = 2$.

In $L[S(T)]$ each block is complete sub graph of $L[S(T)]$ with proper that the set of blocks in $L[S(T)]$ are $k_2, k_3, k_4, k_5, \dots \dots \dots, k_p$.

The minimal edge partition of $L[S(T)]$ is denoted as $F_1 = \{K_2\},\ F_2 = \{K_3\}, \dots \dots \dots \dots, F_p = \{k_p\}$ Hence $r_{Ls}(T) = |F_1, F_2, \dots \dots \dots \dots, F_p| = \Delta(T) - 1$ Thus $r_{Ls}(T) < \Delta(T)$. By considering all the three cases, we have $r_{Ls}(T) \leq \Delta(T)$.

Theorem 6: For any graph G, for $p \geq 4\ r_{Ls}(G) \leq p - 2$.

Proof: Suppose $p \leq 3$ for $G = K_2, L[S(G)] = G$ .Then $r_{Ls}(G) = 1 > p - 2$, a contradiction for $p = 3$, then G is either $K_{1,2}$ or C_3Assume $G = K_{1,2}$.Then $L[S(G)] = p_4$. clearly $r_{Ls}(G) = 2 > p - 2$. For $= C_3$,$r_{Ls}(C_3) = 2 > p - 2$ again a contradiction. Hence $p \geq 4$ .we consider the following cases:

Case1: Suppose $G = T$ and $B = \{v_1 v_2, v_3 \dots \dots \dots \dots, v_n\}$ be the set of non-end vertices with $\deg(v_1) \geq \deg(v_2) \geq, \dots \dots \dots \dots \dots, \geq \deg(v_n)$. Again we have the following sub cases.

Sub case 1.1: Assume $\deg(v_1) = \deg(v_2) = \dots \dots = \deg(v_n)$ Let $C = \{v_1, v_2, v_3, \dots \dots \dots, v_m\}$ be vertex set with $\deg(v_1) = 2, 1 \leq i \leq m$ in $S(G)$ In $L[S(G)]$ each block is complete with same regularity. Further the edges incident to the vertices of C are exactly two, thus these edges gives K_2 as an induced sub graph with mK_2 copies in $L[S(G)]$.Hence minimal regular edge partition $F_1 = \{B\}$, $F_2 = \{mK_2\}$.

Thus $r_{LS}(G) = |F_1, F_2| \leq p - 2$ gives $r_{LS}(G) \leq p - 2$.

Sub case 1.2: Assume $\deg(v_1) > \deg(v_2) \dots \dots \dots \dots \dots \dots > \deg(v_n)$ for the set B then in $[S(G)]$, the edges that are incident to the corresponding vertices of B gives the different edge disjoint induced regular subgraphs. Hence the minimal edge regular partitions are as

$\{F_1, F_2, F_3, \dots \dots \dots \dots, F_n\}$ with respect to these the set $\{B\}$. Further as in sub case 1.1, the remaining edge disjoint induced subgraphs gives the partition $F_2 = \{mK_2\}$

Thus $r_{LS}(G) = |F_1, F_2| \leq p - 2$ gives

$r_{Ls}(G) \leq p - 2$.

Case2:Suppose $G \neq T$. Then it produces more than one block that does not belongs to any edge .Now for $L[S(G)]$, the same argument will exists as in the above cases. Hence the partition $\{F_1, F_2, F_3, \dots, F_m\}$ be the edge partition of $L[S(G)]$, such that$|\{F_1, F_2, F_3, \dots \dots, F_m\}| \leq p - 2$.

Theorem 7: For every r-regular graph G with $r \geq 2$, $r_{LS}(G) = 1$.

Proof: For any graph G, if G is r-regular, then $L(G)$ is $r + 1$ regular. If G is r-regular and $S(G)$ is not regular, but edge degree of each is r-regular. Now in $L[S(G)]$, $E[S(G)] = V[L(G)]$,since edge degree of each edge in $S(G)$ is r, then $\deg(v_i) = r$; $\forall v_i \in L[S(G)]$.Hence by theorem1 , $r_{LS}(G)$ has exactly one partition of edges .Clearly $r_{Ls}(G) = 1$.

Theorem 8: For any graph $G = P_p$ with $p \geq 3$ vertices,$r_{LS}(P_p) = 2$.

Proof: Suppose $G = P_P$ with $p \geq 3$ then $E[P_p] = p - 1$. In $S(P_p)$, $E[S(P_p)] = 2(p - 1) = V\left[L[S(P_p)]\right]$. Since $E[L(G)] = 2(p - 1) - 1 = 2p - 3$, we have $E[L(G)] = \{e_1, e_2, e_3, e_4, \dots \dots \dots \dots \dots \dots, e_{2p-3}\}$ which gives $F_1 = \{e_1, e_3, e_5, \dots \dots \dots \dots \dots \dots, e_{2p-3}\}$ and $F_{2=}\{e_2, e_4, e_6, \dots \dots \dots \dots \dots \dots \dots, e_{2(p-1)}\}$.

Clearly $r_{Ls}(P_p) = |F_1, F_2| = 2$ with $p \geq 3$.

Conclusion:

The beginning of the regular number of some families of Line subdivision of graphs was carried out, and few results and bounds were discussed. we have defined and verified some results of it. Now we conclude that the above theorems are of some trends in Regular number of line subdivision of a graph are cycle or complete block graphs and it's satisfied for isomorphic.

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