Research Article

Edge Equitable Connected Domination of Subdivision Graph of a Graphs

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Abstract: Let $G = (V, E)$ be a graph, for any edge $f \in E[S(G)]$, the edge of $f \in uv$ in $S(G)$ is defined $\deg(f) = \deg(u) + \deg(v) - 2$. A set $F^{e'} \subseteq E[S(G)]$is equitable edgedominating set of $S(G)$ if every edge f not in $F^{e'}$is adjacent to at least one edge $f' \in F^{e'}$such that $|\deg(f) - \deg(f')| \leq 1$. The minimum cardinality of such dominating set is called edge equitable domination number of $S(G)$ denoted by $\gamma_{ec}^{'}(S)$ .The set $F^{e'}$ is said to be a edge equitable connected dominating set of $S(G)$, if the induced subgraph $\langle F^{e'} \rangle$ is connected and is denoted by $\gamma_{ecs}^{'}(G)$. In this paper we introduce many bounds for $\gamma_{ecs}^{'}(G)$ and its exact values for some standard graphs are produced.

Keywords: Edge equitable and connected edge equitable dominating set of $S(G)$, Equitable dominating set of subdivision graphs, Edge equitable independent dominating set.

1. Introduction

For definitions and notations which specifically not defined here, can refer to [3]. For more information regarding domination number and for its related concepts, we refer to [4], [10] [12].For the theory of Equitable domination and domination we can refer [1].Commonly we use$\langle X \rangle$as thesub graph induced by the set of vertices X . A vertex set D in a graph G is a dominating set if allvertex in $V - D$is adjacent to few vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. A subset $F^{e'} of E[S(G)]$ is said to be an equitable edge independent set if for any $f \in F^{e'}. f \in N_e(F')$ for all $f' \in F^{e'} - \{f\}$. If anedge $f \in E[S(G)]$ be such that $|\deg(f) - \deg(f')| \geq 2, \forall f' \in N(F)$ then F is an edge equitable dominating set and those edges are called equitable isolates.

Here $f, f' \in E[S(G)$]are equitable adjacent if f and f' are adjoining and $|deg(f) - \deg(f')| \leq 1$, where $\deg(f), deg(f')$ is the degree of the edges $f$and f' respectively. The minimum equitable degree of a edge in G is written as $\delta_e'(G)$, that is $\delta_e'(G) = min_{F \in E(G)} |N_e(f)|$,where $N_e(f)$ is an edge equitable neighbourhood of f.

A edge dominating set X is said as equitable independent edge dominating set .Ifnotwo edges in X are equitably adjacent and its minimum cardinality taken over all equitableindependent edge dominating set of G that gives the domination number as $\gamma_{ei}'(G)$.

2. Main Results:

Theorem 1: An edge equitable connected dominating set $F^{e'}$ of $S(G)$ is minimalif and only iff or all the edge $f \in F^{e'}$one of (a) or (b) holds.

a. Either $N\{f\} \cap F^{e'} = \emptyset$ or $|\deg(g) - \deg(f)| \geq 2$ for all $N(f) \cap F^{e'}$.
b. There exists a edge $g \in E[S(G)] - F^{e'}$ such that $N\{g\} \cap F^{e'} = \{f\}$ and $|\deg(g) - \deg(f)| \leq 1$.

Proof: Let us suppose that $F^{e'}$is a minimal edge equitable connected dominating set of $S(G)$. If (a) and (b) does not hold then for some $f \in F^{e'}$ there exists a edge $g \in N\{f\} \cap F^{e'}$such that $|\deg(g) - \deg(f)| \leq 1$ and for every$g \in E[S(G)] - F^{e'}$, either $N\{g\} \cap F^{e'} \neq \{f\}$ or $|\deg(g) - \deg(f)| \geq 2$ or both. Therefore $F^{e'} - \langle f \rangle$ is an edge equitable γ_c- set of$S(G)$ that contradicts to the minimality of $F^{e'}$thus (a) and (b) holds.

On the contrary let us suppose for all $f \in F^{e'}$one of the statement (a) or (b) holds and assume that $F^{e'}$ is maximal,then there exists $f \in F^{e'}$such that $F^{e'} - \{f\}$ is an edge equitablyconnected γ-set of $S(G)$.Hence edge $g \in F^{e'} - \{f\}$ such that g equitable dominates f.

That is $g \in N\{f\}$ and $|\deg(g) - \deg(f)| \leq 1$. Consequently $f$does not satisfy (a) but must satisfy (b) for whichtheir exists a edge $g \in E[S(G)] - F^{e'}$ such that $N\{g\} \cap F^{e'} = \{f\}$ and $|\deg(g) - \deg(f)| \leq 1$. Here if $F^{e'} - \{f\}$ is an edge equitable dominating set, thenthere exists $j \in F^{e'} - \{f\}$ andj is adjoining with g and j is connected with f and g respectively.Thus $j \in N\{g\} \cap F^{e'}$, $|\deg(j) - \deg(g)| \leq 1$and $j \neq f$, a contradiction to (b). Thus $F^{e'}$ should be minimal edge equitable connected dominating set of $S(G)$.

Theorem 2: A graph $S(G)$, has a single minimal edge equitable dominating set if and only if all edge equitable isolate set forms an edge equitable dominating set and is also minimally connected.

Proof: Let $S(G)$ has a single edge equitable dominating set$F^{e'}$. Also let us consider another edge set $F' \in N(F^{e'})$ where $\langle F' \cup F^{e'} \rangle$forms a minimal edge equitable connectedγ-set where as $F' \subseteq F^{e'}$. Now let $F = \{f \in E[S(G)]/f\}$is an edge equitable isolate.Then $F \subseteq (F^{e'} \cup F')$.Let us prove that $F = F^{e'} \cup F$, Suppose $(F^{e'} \cup F' - F = \emptyset$.Let $g \in (Fe' - F') - F$,where g is not a edge equitable isolate $ESG - \{g\} \in N(Fe' - F')$ is an edge equitable γ_c- set which is also connected. Thus $\langle F' \cup F^{e'} \rangle \subseteq E[S(G)] - \{g\}$ and$< F^{e'} \cup F' > \neq (F^{e'} \cup F') - F$, which contradictsthat $S(G)$ has single minimal edge equitable γ_c-set.

Theorem 3: An edge equitable dominating set $F^{e'}$ of $S(G)$is minimally connected if for each edge $f' \in F^{e'}$ for any of the following conditions:

a. $N_e\{f'\} \cup F^{e'} \neq \emptyset$.

b. There exists an edge $g' \in E[S(G)] - F^{e'}$ such that $N_e(g') \cap F^{e'} = \{f'\}$.

Proof: If we assume that $F^{e'}$ is minimally connected edge equitable dominating set of $S(G)$. Then for some $f' \in F^{e'}$ there exists an edge $g' \in N_e(f') \cup F^{e'}$ and for each edge$k' \in E[S(G)] - F^{e'}$, $N_e\{k'\} \cap F^{e'} \neq \{f'\}$. Therefore $F^{e'} - \{f'\}$is an edge equitable γ_c-set of $S(G)$ for which (a) and (b) holds. Now if suppose $F^{e'}$ is not minimal for $S(G)$, then there exists an edge $g' \in F - \{f\} \subset E[S(G)]$ such that $g \in N_e(f')$. Hence f'does not hold good for (a). Then f'must satisfy (b) for which an edge $g' \in E[S(G)] - F^{e'}$ is existed,such that $N_e\{g'\} \cap F = \{f\}$. Moreover $(F^{e'} - \{f'\}) \cup N_e\{f'\}$ is an edge equitable connected γ-set for which an edge $f'' \in F^{e'} - \{f\}$ is existed where as f'' is adjacent to g'equitably. Consequently $f'' \in N_e\{g'\} \cup F^{e'}$ which proves for $F^{e'}$ is minimally edge equitable connected dominating set of $S(G)$.

Theorem 4: For any edge $f \in E[S(G)] - F^{e'}$,then there is only one edge $f' \in F^{e'}$ Such that $N_e(f') \cap F^{e'} = \{f'\}$ then for any γ'_{ec}-set of $S(G)$, $|E[S(G)] - F^{e'}| \leq \sum_{f \in F^{e'}} deg_e(f')$.

Proof: Since each edge $E[S(G)] - F^{e'}$is adjacent equitably to more than one edge of$F^{e'}$.

Thus all the edges in $E[S(G)] - F^{e'}$contributes atleast one to the sum of edge equitability$F^{e'}$in $S(G)$.

Hence $|E[S(G)] - F^{e'}| \leq \sum_{f \in F^{e'}} deg_e(f')\ If \left|E[S(G)] - F^{e'}\right| = \sum_{f \in F^{e'}} deg_e(f')$

Clearly each edgein$E[S(G)] - F^{e'}$ is counted in the summation of$\sum_{f \in F^{e'}} deg_e(f')$

Hence iff_1' and$f_2'$are adjacent equitably, where$f_1'$is countedin $deg_e(f_1')\ and\ f_2' in\ deg_e(f_2')$. So that the sum exceeds more than two.

Now suppose if $N_e(f) \cap F^{e'} \geq 2$, for some edge $f \in E[S(G)] - F^{e'}$, where $(f_1', f_2') \in N_e(f') \cap F^{e'}$.

Hence $\sum_{f \in F^{e'}} deg_e(f')$ exceeds by atleast one .Since$f_1'$is counted twice in$deg_e(f_1')$and once in $deg_e(f_2')$. Hence the above statement of the theorem holds good.

Theorem 5: If $G = (V, E)$ is without equitable isolate edges, then for all minimal edge equitable connected dominating set $F^{e'}, E - F^{e'}$ is also edge equitable dominating set of $S(G)$.

Proof: Let $F^{e'}$ be a minimal set of connected equitable edge dominating set of $S(G)$.Assume that $E - F^{e'}$is not edge equitable γ_c-set of $S(G)$that consists an edge f', where as f' is not equitablyneighbouring edge $E - F^{e'}$ then in $S(G)$,their is non equitable isolated edges, then f' is equitably adjoining at least one edge in $F^{e'} - \{f'\}$,as a result $F^{e'} - \{f'\}$ is edge equitable γ_c- set of $S(G)$that contradicts the minimality of $F^{e'}$in $S(G)$. Hence $E - F^{e'}$is edge equitable connected dominating set.

Theorem 6: An edge equitable connected independent set $F_i^{e'}$of $S(G)$ is maximal independent set, if it is edge equitable independent and edge equitable dominating set of $S(G)$.

Proof: If $F_i^{e'}$is maximal and is also connected,then for every edge $f \in E[S(G)] - F_i^{e'}$, then set $F_i^{e'} \cup \{f\}$ is not edge equitable independent, that is for each edge $f \in E[S(G)] - F_i^{e'}$, there is an edge $f' \in F_i^{e'}$ such that f is edge equitable adjacent to f'.Thus $F_i^{e'}$ is an edge equitable dominating set and if$\langle F_i^{e'} \rangle$ is connected then it gives $\gamma'_{ecs}(G)$. Hence $F_i^{e'}$is edge equitable independent and also edge equitable dominating set of $S(G)$.

On the other hand let us assume $F_i^{e'}$ is edge equitable independent as well as edge equitable γ-set of $S(G)$. Now if $F_i^{e'}$ is not maximal, then there exists an edge $f \in E[S(G)] - F_i^{e'}$ such that $F_i^{e'} \cup \{f\}$ is edge equitable independent , then no edge in$F_i^{e'}$is not aedge equitable dominating set which is contradiction.For thegiven statement.

Theorem 7: For any tree T, with $e = uv$ as maximum edge degreeΔ', then $\gamma'_{ecs}(T) \leq q - \Delta'(T)$.

Proof: Let $S(T)$ be the subdivision of T where $p = q - 1$ vertices and also $\gamma' = q - \Delta'$. Let M be the set of all pendant edges of $S(T)$. Since $E[S(T)] \cup M$ is dominating edge set. Now there exists a edge set $F_1^{e'} = \{e_1, e_2, e_3, \ldots\ldots\ldots e_j\} \subseteq E[S(T)]$ is an edge equitable γ-set of $S(T)$, if every edge $f \in F_1^{e'}$ is adjoining to more than one edge $f' \in F_1^{e'}$ and$|\deg(f) - \deg(f')| \leq 1$. Now let $F' = e_i ; i \leq j$ be an another edge set that belongs to neighbourhood $F_1^{e'}$ i.e $F' \in N\{F_1^{e'}\}$ such that $\langle F_1^{e'} \cup F' \rangle$ is connected$\gamma'_{es}(T)$. So that $|F_1^{e'} \cup F'| = \gamma'_{ecs}(T)$. Since in every subdivision of tree T, there exists atleast one edge with maximum edge degree $e = uv \in \Delta'(T) < q$ and also each non-pendent edges is adjacent to a pendent edge of T.

Thus $\left|F_1^{e'} \cup F'\right| \leq E(T) - max(\deg(e))$.

$$\gamma'_{ecs}(T) \leq q - \Delta'(T).$$

Theorem 8: For any graph G , with $p \geq 3$ vertices $\gamma'_{ecs}(G) \leq p - 3$.

Proof: Let$G$ be graph with $p \geq 3$ vertices. Now suppose $F^{e'} = \{f_1, f_2, f_3, \ldots\ldots\ldots\ldots f_i\}$ is edge equitable connected dominating set of$S(G)$, whereas $F^{e'} \subset E[S(G)]$ in which for every edge $f \notin F^{e'}$ is in the neighbourhood ofatleast one edge $f' \in F^{e'}$ and $|\deg(f') - \deg(f)| \leq 1$ where as induced subgraph $\langle F^{e'} \rangle$ is connected, than $\left|F^{e'}\right| = \gamma'_{ecs}(G)$. Since $F_i = u_i v_i \in \gamma(G)$ in which every vertex is incident to each edge of $\langle F^{e'} \rangle$ which consequently proves the required result.

3.Conclusion

In this paper the concepts of edge equitable connected domination of subdivision of Graphs, edge equitable domination, equitable edge independent set of $S(G)$ was introduced.Some interesting results related with above are proved

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