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Research Article

Inverse isolate domination on four–regular graphs withgirth 3 and girth 4

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Abstract

Let G be non-trivial graph. A subset S of the vertex set V(G) of a graph G is called a isolate dominating set of G if every vertex in V – S is adjacent to a vertex in S such that $\delta(<S>) = 0$. The minimum cardinality of an isolate dominating set is called the isolate domination number and is denoted by $\gamma_0(G)$. If V – S contains a dominating set S' of G, then S' is called an inverse isolate dominating set with respect to S. The minimum cardinality of an inverse isolate dominating set is called an inverse isolate dominating number and is denoted by ${\gamma_0}^{-1}(G)$. In this paper we investigate the inverse isolate dominating number of 4–regular graph on n vertices with girth 3and girth 4.

Keywords : Domination, Isolate domination, Inverse domination, 4–regular graph, girth.

Subject Classification Number: 05C15, 05C69.

1. Introduction

By a graph, we mean a finite, undirected graph with neither loops nor multiple edges. For graph theoretic terminology, we refer to the book by Chartrand and Lesniak [2]. All graphs in this paper are assumed to be non-trivial.

In a graph G = (V, E), the degree of a vertex v is defined to be the number of edges incident with v and is denoted by deg v. The minimum of {deg v : v ∈V(G)} is denoted by δ(G) and the maximum of {deg v : v ∈ V(G)} is denoted by Δ(G). The subgraph induced by a set S of vertices of a graph G is denoted by <S> with V(<S>) = S and E(<S>) = {uv ∈ E(G) / u, v ∈ S}. The study of domination and related subset problems is one of the fastest growing area in graph theory. For a detailed survey of domination one can see [5, 7] and [9]. For a set σ ⊆ V, the switching of G by σis the graph $G^{\sigma}(V, E')$, which is obtained from G by removing all edges between σ and its complement V – σ and adding as edges all non–edges between σ andV –

σ[12]. A graph G is r–regular if each vertex in G has degree r. The Corona product of two graphs G and H is defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H and joining the i-th vertex of G to every vertex in the i-th copy of H[3].The concept of rooted product graph was introduced in 1978by Godsil and McKay [4]. Given the graph of order n(G) and a graph H withthe rooted product, G ° v H is defined as the graph obtained from G and H by taking one copy of G and n(G) copies of H and identifying the i^{th} vertex of G with the root vertex v in the i^{th} copy of H for every i ∈ {1, 2, …, n(G)}.

In this sequence, the notion of isolate domination was introduced in [11] as a new basis domination parameter. An isolate dominating set of a graph G is a dominating set S of G such that $\delta(< S >) = 0$ and the isolate domination number denoted by $\gamma_0(G)$, is the minimum cardinality of an isolate dominating set of G. The purpose of this paper is to discuss about some concept of Inverse Isolate dominating number of 4–regular graphs with girth 3 and 4.

Definition 1. 1.[8]Let S be a minimum isolate dominating set of a graph G. If V − S contains a dominating set S' such that $\delta(\langle S' \rangle)$= 0, then S' is called an inverse isolate dominating set with respect to S. If $\delta(\langle S' \rangle)$> 0, then we call S' as weak inverse isolate a dominating set of G. The minimum cardinality of an inverse isolate dominating set is called an inverse isolate domination number and is denoted by γ_0^{-1}(G) and the minimum cardinality of a weak inverse isolate dominating set is called a weak inverse isolate domination number and is denoted by γ_{w0}^{-1}(G).

Definition 1. 2.[10]If v_1 is adjacent tov_{n-1}, v_n, v_2, v_3; v_2 is adjacent tov_n, v_1, v_3, v_4; v_i is adjacent tov_{i-2}, v_{i-1}, v_{i+1}, v_{i+2} where i = 3 to n – 2, v_{n-1} is adjacent tov_{n-3}, v_{n-2}, v_n, v_1 and v_n is adjacent tov_{n-2}, v_{n-1}, v_1, v_2 such that $v_1 v_2 \dots v_n$ forms a cycle, then clearly each vertex is of degree 4 and n ≥ 6. Thus from the construction, we have a 4–regular graph with girth 3 on n vertices and 2n edges.We denote G(n, 4, 3) for4–regular graph on n vertices with girth 3.

Example 1. 3. Consider the graph G(8, 4, 3) given in figure 1.1. ClearlyS = {1, 5} is a minimum isolate dominating set. Then minimum inverse dominating sets with respect to S are {2, 6}, {2, 7}, {3, 6}, {3, 7}, {3, 8}, {4, 7} and {4, 8}.

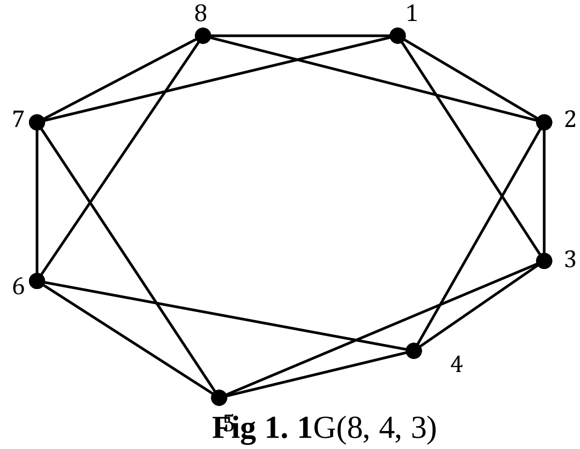


Fig 1. 1G(8, 4, 3)

Definition 1. 4.[10] If v_1 is adjacent tov_{n-2}, v_n, v_2, v_4; v_2 is adjacent tov_{n-1}, v_1, v_3, v_5; v_i is adjacent tov_{i-3}, v_{i-1}, v_{i+1}, v_{i+3} where i = 4 to n – 1, v_{n-1} is adjacent tov_{n-4}, v_{n-2}, v_n, v_2 and v_n is adjacent tov_{n-3}, v_{n-1}, v_1, v_3 such that $v_1 v_2 \dots v_n$ forms a cycle, then clearly each vertex is of degree 4 and n ≥ 7. Thus from the construction, we have a 4–regular graph on girth 4 with n vertices and 2n edges.We denote G(n, 4, 4) for 4–regular graph on n vertices with girth 4.

Example 1. 5. Consider the graph G(10, 4, 4) given in figure 1.2.Clearly S = {1, 6} is a minimum isolate dominating set. Then minimum inverse dominating sets with respect to S are {2, 7}, {3, 8}, {4, 9} and {5, 10}.

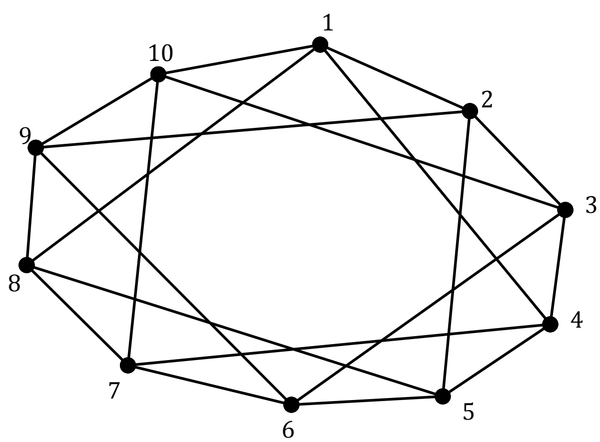


Fig 1. 2G(10, 4, 4)

We use the following results in the subsequent sufficient.

Theorem 1.6. [11] $\gamma_0(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Theorem 1.7. [5] If G is a graph with no isolate vertices, then the complement V – S of every minimal dominating set S is a dominating set.

2. Inverse isolate domination on four – regular graph with girth 3

Notation: We use the notation G(n) to denoteG(n, 3, 4) in this section.

Theorem 2. 1. For the graph G(n),γ_0^{-1}(G(n)) = $\begin{cases} 1 \text{ if } n = 5 \\ \left\lceil \frac{n}{5} \right\rceil \text{ if } n \geq 6 \end{cases}$.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of G(n) such that $v_1 v_2 \dots v_n v_1$ forms a cycle. By Definition 1. 2, n ≥ 6.Let S and S'be anisolate dominating set and inverse isolate dominating set with respect to S respectively. By the definition of four regular graph, n ≥ 5. If n = 5, then G(5) ≅ K_5 and hence$|S| = |S'| = 1$. This implies that γ_0^{-1}(G(5)) = 1. Now, we consider n ≥ 6. If n ≡ 0 (mod 5), then S = $\{v_i, v_{i+5}, v_{i+10}, \dots, v_{i+(n-5)}\}$ is a minimum isolate dominating set and if n ≢ 0 (mod 5), then S = $\{v_i, v_{i+5}, v_{i+10}, \dots, v_{i+(n-4)}\}$ is a minimum isolate dominating set. Therefore $\gamma_0(G(n)) = \left\lceil \frac{n}{5} \right\rceil$.If n ≡ 0 (mod 5), then$S' = \{v_j, v_{j+5}, v_{j+10}, \dots, v_{j+(n-5)} / i \neq j\}$ is a minimum inverse dominating set with respect to S and if n ≢ 0 (mod 5), then$S' = \{v_j, v_{j+5}, v_{j+10}, \dots,$

$v_{j+(n-4)}/ i \neq j\}$is a minimum inverse dominating set with respect to S, all thesuffixes modulo n. In both cases S' is a minimum inverse dominating setand $\delta(\langle S' \rangle)$= 0, which implies that S' is an inverse isolate dominating set of G(n). Clearly $|S| = |S'| = \left\lceil \frac{n}{5} \right\rceil$.Therefore, $\gamma_0^{-1}(G(n)) = \left\lceil \frac{n}{5} \right\rceil$ for n ≥ 6. Hence the theorem.

Theorem 2. 2. For the graph G(n), $\gamma_0^{-1}(\overline{G(n)}) = \begin{cases} 0 \text{ if } n = 5 \\ 3 \text{ if } n \geq 6 \end{cases}$.

Proof. Let G(n) be a 4–regular graph and hence $\overline{G(n)}$is a (n – 5)-regular graph. The vertex v_i is non adjacent to $v_{i\pm1}$ and$v_{i\pm2}$in $\overline{G(n)}$. The vertex v_{i+1} is adjacent to v_{i-2} and $v_{i+2}$is adjacent to v_{i-1} in $\overline{G(n)}$.If n = 5, then$\overline{G(5)} \cong \overline{K}_5$. This implies that S = $\{v_1, v_2, v_3, v_4, v_5\}$ is the unique minimum isolate dominating set of $\overline{G(5)}$and hence the inverse isolate dominating set$S' = \varphi$. Therefore$\gamma_0^{-1}\overline{G(5)} = 0$. Let n $\geq$ 6.For $1 \leq i \leq n$, S = $\{v_i, v_{i+1}, v_{i+2}\}$is a minimum isolate dominating set of$\overline{G(n)}$ where the suffixes modulo n. Clearly $S' = \{v_j, v_{j+1}, v_{j+2}\}$ where $1 \leq j \leq n$, the suffixes modulo n and $|i - j| \geq 3$is a minimum inverse dominating set of $\overline{G(n)}$with respect to S. Therefore$\gamma_0^{-1}(\overline{G(n)})$= 3 for n ≥ 6.

Theorem2. 3. Let v be any vertex of G(n). Then$\gamma_0^{-1}(G(n)^v) = \begin{cases} 0 \text{ if } n = 5 \\ 2 \text{ if } 7 \leq n \leq 10 \\ 3 \text{ if } n = 6, 11 \leq n \leq 16 \\ \left\lceil \frac{n-1}{5} \right\rceil \text{ if } n \geq 17 \end{cases}$.

Proof. Let $G(n)^v$be the graph obtained from G(n) by switching the vertex v. Letv = v_i, $1 \leq i \leq n$.Let S and S' be a minimumisolate dominating set and minimum inverse isolate dominating set with respect to S of $G(n)^v$, respectively. Clearly n $\geq$ 6. We now consider the following cases.

Case 1. n = 5

Then G(5) = K_5 and hence $G(5)^v \cong K_1 \cup K_4$, whichhas theisolate vertex v. By Theorem 1. 7, there does not exist an inverse dominating set and hence $\gamma_0^{-1}(G(5)^v)$= 0.

Case 2. n = 6

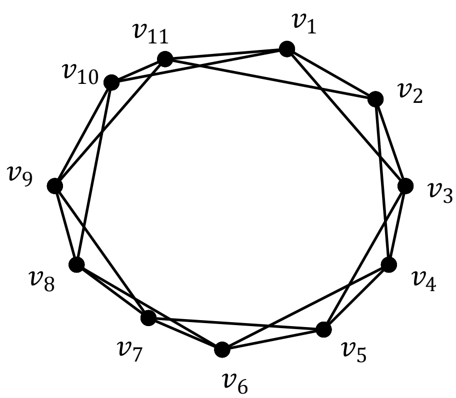
Clearly S =$\{v_{i+3}\}$is the unique minimum isolate dominating set of $G(n)^{v_i}$ and the corresponding minimum inverse dominating set S' is either $\{v_i, v_{i+1}, v_{i+2}\}$ or $\{v_i, v_{i+1}, v_{i-2}\}$ or $\{v_i, v_{i+1}, v_{i-1}\}$ or $\{v_i, v_{i+2}, v_{i-2}\}$ or $\{v_i, v_{i+2}, v_{i-1}\}$ or $\{v_i, v_{i-2}, v_{i-1}\}$, where the suffixes modulo n and $1 \leq i \leq n$ and $\delta(\langle S' \rangle)$= 0.This implies that S' is an inverse isolate dominating set of $G(6)^v$. Hence$\gamma_0^{-1}(G(6)^v)$= 3.

Case 3. $7 \leq n \leq 10$

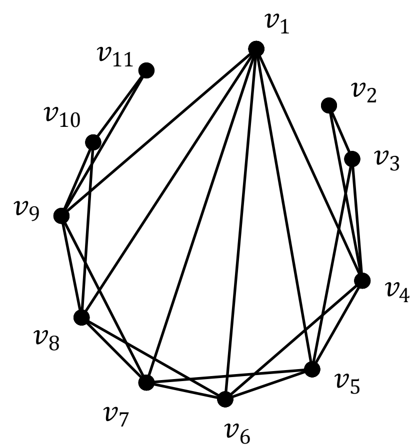
If n = 7, then S = $\{v_{i+1}, v_{i+4}\}$ is a minimum isolate dominating set and the corresponding minimum inverse dominating $S'$is $\{v_{i-1}, v_{i+3}\}$where the suffixes modulo n, $1 \leq i \leq n$ and $\delta(\langle S' \rangle)$= 0. If $8 \leq n \leq 10$, then S = $\{v_{i-2}, v_{i+3}\}$ is a minimum isolate dominating set and the corresponding minimum inverse dominating S' is either$\{v_{i-1}, v_{i+4}\}$or$\{v_{i-3}, v_{i+2}\}$where the suffixes modulo n, $1 \leq i \leq n$. In both cases $\delta(\langle S' \rangle)$= 0, which implies that S' is an inverse isolate dominating set of$G(n)^v$. Hence $\gamma_0^{-1}(G(n)^v)$ = 2

Case 4. n = 11

Here S = $\{v_{i+3}, v_{i-3}\}$ is the unique minimum isolate dominating set of $G^v(n)$ and the corresponding inverse dominating set S' is either $\{v_i, v_{i+1}, v_{i-2}\}$ or $\{v_i, v_{i+1}, v_{i-1}\}$ or $\{v_i, v_{i+2}, v_{i-2}\}$ or $\{v_i, v_{i+2}, v_{i-1}\}$ or $\{v_{i+1}, v_{i+2}, v_{i-4}\}$ or $\{v_{i+1}, v_{i+4}, v_{i-4}\}$ or $\{v_{i+1}, v_{i+4}, v_{i-2}\}$ or $\{v_{i+1}, v_{i+5}, v_{i-4}\}$ or $\{v_{i+1}, v_{i+5}, v_{i-2}\}$ or $\{v_{i+1}, v_{i+5}, v_{i-1}\}$ or $\{v_{i+1}, v_{i-5}, v_{i-4}\}$ or $\{v_{i+1}, v_{i-5}, v_{i-2}\}$ or $\{v_{i+2}, v_{i+5}, v_{i-2}\}$or $\{v_{i+2}, v_{i+5}, v_{i-1}\}$ or $\{v_{i+2}, v_{i-5}, v_{i-2}\}$ or $\{v_{i+2}, v_{i-4}, v_{i-2}\}$ or$\{v_{i+2}, v_{i-4}, v_{i-1}\}$ or $\{v_{i+4}, v_{i+5}, v_{i-1}\}$ or $\{v_{i+4}, v_{i-5}, v_{i-1}\}$ or $\{v_{i+4}, v_{i-4}, v_{i-1}\}$ or $\{v_{i+4}, v_{i+5}, v_{i-1}\}$, where the suffixes modulo n. In all possible cases $\delta(\langle S' \rangle)$= 0, which implies that S' is an inverse isolate dominating set of $G(11)^v$. Hence $\gamma_0^{-1}(G(11)^v)$ = 3.



G(11)



$G(11)^{v_1}$

Case 5. 12≤ n ≤ 16

In this case, S = $\{v_{i-1}, v_i, v_{i+1}\}$ is a minimum isolate dominating set and the corresponding minimum inverse dominating set $S'$is $\{v_{i-3}, v_{i+3}, v_{i+8}\}$, where the suffixes modulo n and $\delta(\langle S' \rangle)$= 0, which implies that S' is an inverse isolate dominating set of $G(n)^v$. Hence$\gamma_0^{-1}(G(n)^v)$= 3.

Case 6. n ≥ 17

In this case, a minimum isolate dominating set S = $\{v_i, v_{i+1}, v_{i-2}\}$and the corresponding inverse dominating set $S'$is given by, $S' = \{v_{i+3}, v_{i+8}, v_{i+13}, \ldots, v_{i-3}\}$for n ≡ 0, 1 (mod 5) and$S'$ = $\{v_{i+3}, v_{i+8}, v_{i+13}, \ldots, v_{i-2}\}$forn ≡ 2, 3, 4 (mod 5), where the suffixes modulo n and also $\delta(\langle S' \rangle)$= 0. This implies that S' is an inverse isolate dominating set of $G(n)^v$. Hence $\gamma_0^{-1}(G(n)^v)$= $\left\lceil \frac{n-1}{5} \right\rceil$.The theorem follows from all the six cases.

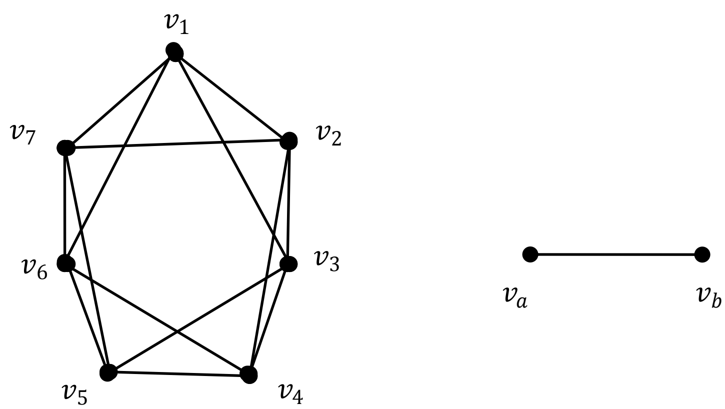
Theorem2. 4. Let G be the Cartesian product of G(n) and K_2.Then γ_0^{-1}(G)

$$=\begin{cases} \left\lceil \frac{n}{3} \right\rceil & \text{if } n \equiv 1 (\text{mod} 6) \\ 2\left\lceil \frac{n}{6} \right\rceil & \text{if } n \not\equiv 1 (\text{mod} 6) \end{cases}.$$

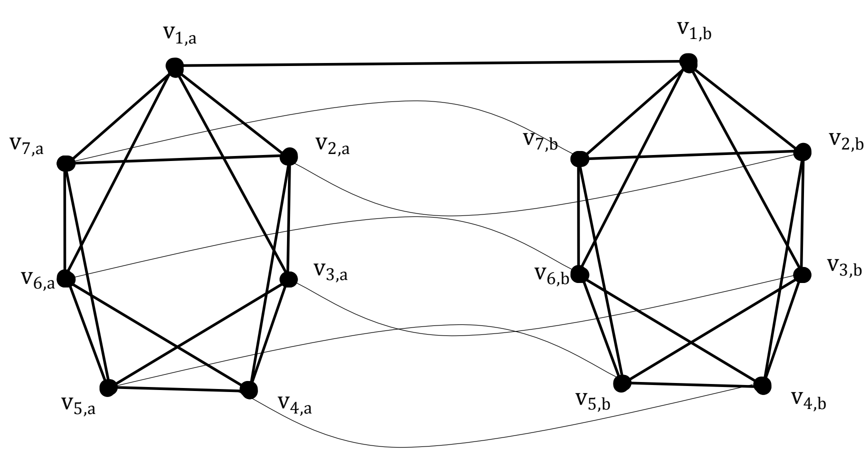
Proof. Let G = G(n) $\times K_2$.Let V(G(n)) = $\{v_1, v_2, \ldots, v_n\}$ andE(G(n)) = $\{v_i v_{i+1}, v_i v_{i+2}, v_i v_{i-1}, v_i v_{i-2} / 1 \leq i \leq n\}$ be the vertex set and edge set of G(n) respectively where the suffixes modulo n. Let V(K_2) = $\{v_a, v_b\}$ and E(K_2) = $\{v_a v_b\}$. By the definition of Cartesian product V(G) = $\{(v_i, v_a), (v_i, v_b) / 1 \leq i \leq n\}$ and E(G) = $\{(v_i, v_a)(v_j, v_b) / i = j$ and$v_a v_b \in E(K_2)$ or a = b and$v_i v_j \in E(G(n))\}$.Denote (v_i, v_a) by $v_{i,a}$and (v_i, v_b) by $v_{i,b}$, $1 \leq i \leq n$. We now consider the following two cases.

Case 1. n $\equiv 1(\mathrm{mod}6)$

In this case a minimumisolate dominating set S = $\{v_{i,a}, v_{i+3,b}, v_{i+6,a}, \ldots, v_{i+(n-4),b}, v_{i+(n-1),a}\}$, where the suffixes modulo nand the corresponding inverse dominating setS' = $\{v_{j,a}, v_{j+3,b}, v_{j+6,a}, \ldots, v_{j+(n-4),b}, v_{j+(n-3),a}\}$,where the suffixes modulo n andi $\neq$ j. Clearly $\delta(\langle S' \rangle)$= 0, which implies that S' is an inverse isolate dominating set of G. Hence $\gamma_0^{-1}(G) = \left\lceil \frac{n}{3} \right\rceil$.



G(7) K_2

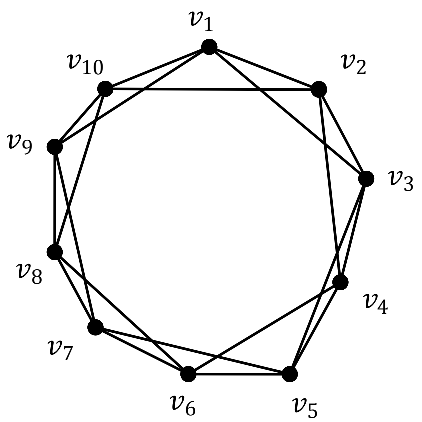


G(7) $\times K_2$

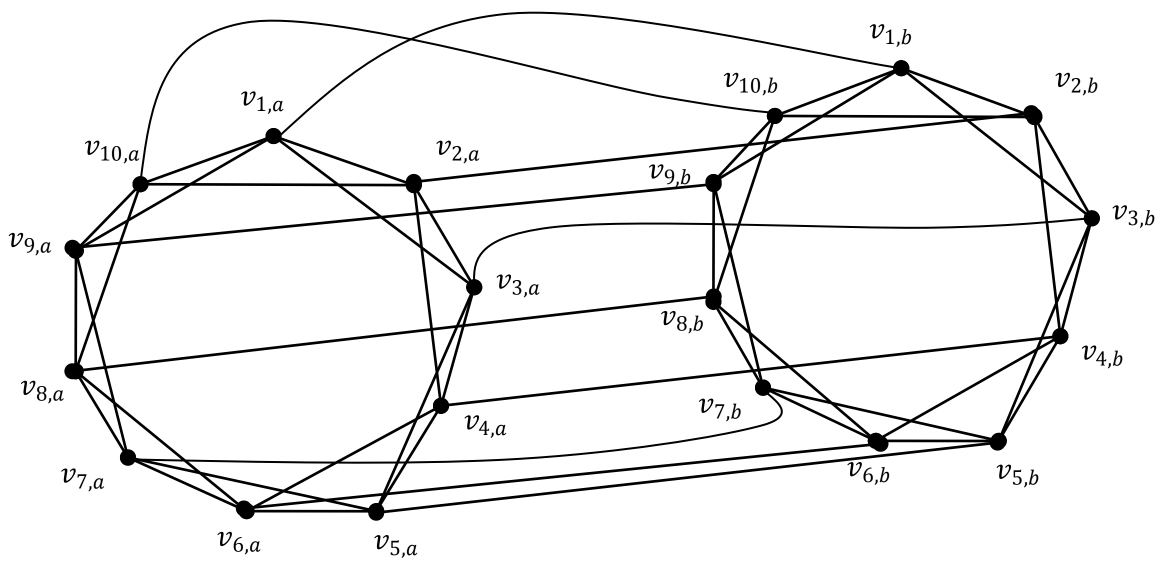
Case 2. n $\not\equiv 1(\mathrm{mod}6)$

In this case the isolate dominating sets S of G is $\{v_{i,a}, v_{i+3,b}, v_{i+6,a}, \ldots, v_{i+(n-6),a}, v_{i+(n-3),b}\}$for n $\equiv 0(\mathrm{mod}6)$, $\{v_{i,a}, v_{i+3,b}, v_{i+6,a}, \ldots, v_{i+(n-3),a}, v_{i+(n-2),b}\}$ for n $\equiv 2, 3(\mathrm{mod}6)$, $\{v_{i,a}, v_{i+3,b}, v_{i+6,a}, \ldots, v_{i+(n-4),a}, v_{i+(n-2),b}\}$ for n $\equiv 4(\mathrm{mod}6)$ and

$\{v_{i,a}, v_{i+3,b}, v_{i+6,a}, \ldots, v_{i+(n-5),a}, v_{i+(n-2),b}\}$ for $n \equiv 5(mod 6)$and the corresponding inverse dominating sets are $\{v_{j,a}, v_{j+3,b}, v_{j+6,a}, \ldots, v_{j+(n-6),a}, v_{j+(n-3),b}\}$,$\{v_{j,a}, v_{j+3,b}, v_{j+6,a}, \ldots, v_{j+(n-3),a}, v_{j+(n-2),b}\}$, $\{v_{j,a}, v_{j+3,b}, v_{j+6,a}, \ldots, v_{j+(n-4),a}, v_{j+(n-2),b}\}$and $\{v_{j,a}, v_{j+3,b}, v_{j+6,a}, \ldots, v_{j+(n-5),a}, v_{j+(n-2),b}\}$, where the suffixes modulo n and$i \neq j$.In all possible cases $\delta(\langle S' \rangle)$= 0, which implies that S' is an inverse isolate dominating set of G. Hence$\gamma_0{}^{-1}(G) = 2\left\lceil \frac{n}{6} \right\rceil$.



G(10) K_2



$G(10) \times K_2$

3. Inverse isolate domination on four–regular graph with girth 4

Note: We denote the graph G(n, 4, 4) by G*(n) in this section.

Theorem 3. 1. For the graph G*(n), $\gamma_0{}^{-1}(G^*(n)) = \begin{cases} 4 \text{ if } n = 8 \\ \left\lceil \frac{n}{5} \right\rceil + 1 \text{ if } n \equiv 4 \ (mod\ 5) \\ \left\lceil \frac{n}{5} \right\rceil \text{ if } n \not\equiv 4 \ (mod\ 5) \end{cases}$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of G(n) and such that $v_1 v_2 \ldots v_n v_1$ form a cycle. By the definition of 4-regular graph with girth 4, $n \geq 7$. Let S and S' be the isolate dominating set and inverse isolate dominating set respectively. We now consider the following three cases.

Case 1. $n = 8$

In this case a minimum isolate dominating set of $G^*(n)$ is, S = $\{v_1, v_3, v_5, v_7\}$ and the corresponding minimum inverse isolate dominating set S' = $\{v_2, v_4, v_6, v_8\}$. Clearly $\delta(\langle S' \rangle)$= 0, which implies that S' is an inverse isolate dominating set of $G^*(8)$. This implies that $\gamma_0^{-1}(G^*(8)) = 4$.

Case 2. $n \equiv 4 \pmod 5$

In this case a minimum isolate dominating set set of $G^*(n)$ is, S = $\{v_i, v_{i+5}, v_{i+10}, \ldots, v_{i+(n-2)} / 1 \leq i \leq n\}$ and the corresponding minimuminverse dominating set S' = $\{v_j, v_{j+5}, v_{j+10}, \ldots, v_{j+(n-2)} / j = i+1\}$, where the suffixes modulo n and $\delta(\langle S' \rangle)$= 0. Hence S' is an inverse isolate dominating set of $G^*(n)$. This implies that $\gamma_0^{-1}(G^*(n)) = \left\lceil \frac{n}{5} \right\rceil + 1$.

Case 3. $n \not\equiv 4 \pmod 5$

In this case the minimum isolate dominating set S is$\{v_i, v_{i+5}, v_{i+10}, \ldots, v_{i+(n-5)}\}$forn $\equiv 0 \pmod 5$, $\{v_i, v_{i+5}, v_{i+10}, \ldots, v_{i+(n-1)}\}$ for $n \equiv 1,3 \pmod 5$ and $\{v_i, v_{i+5}, v_{i+10}, \ldots, v_{i+(n-2)}\}$for $n \equiv 2 \pmod 5$. Then the corresponding minimuminverse dominating sets $S'$is $\{v_j, v_{j+5}, v_{j+10}, \ldots, v_{j+(n-5)}\}$forn $\equiv 0 \pmod 5$, $\{v_j, v_{j+5}, v_{j+10}, \ldots, v_{j+(n-3)}\}$for $n \equiv 1,3 \pmod 5$ and$\{v_j, v_{j+5}, v_{j+10}, \ldots, v_{j+(n-2)}\}$ for $n \equiv 2 \pmod 5$, where the suffixes modulo n and j = i + 1 and $\delta(\langle S' \rangle)$= 0. Hence S' is an inverse isolate dominating set of $G^*(n)$. This implies that $\gamma_0^{-1}(G^*(n)) = \left\lceil \frac{n}{5} \right\rceil$.

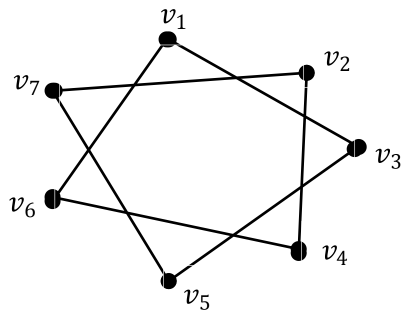
Thus the theorem follows from cases 1, 2 and 3.

Theorem3. 2. For the graph $(\overline{G^*(n)}), \gamma_0^{-1}(\overline{G^*(n)}) = \begin{cases} 3 \text{ if } n = 7 \\ 2 \text{ if } n \geq 8 \end{cases}$.

Proof. Let $(\overline{G^*(n)})$be the complement graph of G(n).Let $V(\overline{G^*(n)}) = \{v_1, v_2, \ldots, v_n\}$. Fori = 1 to n, v_i is adjacent to v_j in $(\overline{G^*(n)})$, $1 \leq j \leq n$ $j \neq i \pm 1, i \pm 3$. Let S be aminimum isolate dominating set and S' be aminimuminverse isolate dominating set with respect to S in $(\overline{G^*(n)})$. We now consider the following two cases.

Case 1. n = 7

In this case the resulting graph$(\overline{G^*(7)}) \cong C_7$. By Theorem 2. 2,$\gamma_0^{-1}(\overline{G^*(n)}) = 3$



$(\overline{G^*(8)})$

Case 2. n ≥ 8

Clearly the resulting graph$(\overline{G^*(n)})$ is a (n – 5)-regular graph. A minimumisolate dominating set$S_i = \{v_i, v_{i+1}\}$, where the suffixes modulo n, 1≤ i ≤ n and the corresponding minimum inverse dominating set $S_i^{'}$is $\{v_j, v_{j+1}\}$,where the suffixes modulo n, 1≤ j ≤ n, j ≠ i, i + 1 and also $\delta(\langle S^{'}\rangle)$= 0, which implies that $S^{'}$ is an inverse isolate dominating set of $\overline{G^*(n)}$. Hence $\gamma_0{}^{-1}(\overline{G^*(n)})$ = 2.

Thus the theorem follows from cases 1 and 2.

Theorem 3. 3. For the graph $G^v(n)$,

$$\gamma_0{}^{-1}\left(G^*(n)\right)^v = \begin{cases} 5 \text{ if } n = 8 \\ 2 \text{ if } n = 9 \\ \left\lceil \frac{n}{5} \right\rceil + 1 \text{ if } n \equiv 4 \ (\mathrm{mod}\ 5) \text{and } n \geq 10 \\ \left\lceil \frac{n}{5} \right\rceil \text{ if } n \not\equiv 4 \ (\mathrm{mod}\ 5) \text{ and } n = 7, n \geq 10 \end{cases}.$$

Proof. Let $[G^*(n)]^v$be the graph obtained by switching of the vertex v. Without loss of generality, let v =v_i. Let S and$S^{'}$ be the isolate dominating set and inverse isolate dominating set of $[G^*(n)]^v$, respectively. We now considerthe following four cases.

Case 1. n = 7

The minimum isolate dominating set of $[G^*(7)]^{v_i}$is S = $\{v_{i+2}, v_{i-3}\}$and the corresponding inverse dominating sets are $S^{'} = \{v_{i+3}, v_{i-2}\}$ respectively. Claerly $\delta(\langle S^{'}\rangle)$= 0. Hence $S^{'}$ is an inverse isolate dominating set of $[G^*(7)]^{v_i}$. Hence $\gamma_0{}^{-1}([G^*(7)]^{v_i}) = 2$.

Case 2. n = 8

In this case the isolate dominating set of $[G^*(8)]^{v_i}$is S = $\{v_{i+2}, v_{i+4}, v_{i-2}\}$ and the corresponding inverse dominating set $S^{'} = \{v_i, v_{i+1}, v_{i+3}, v_{i-3}, v_{i-1}\}$. Clearly $\delta(\langle S^{'}\rangle)$= 0. Hence$S^{'}$ is an inverse isolate dominating set of $[G^*(8)]^{v_i}$. Hence $\gamma_0{}^{-1}\left(G^*(5)\right)^v$ = 5.

Case 3. n = 9

In this case the minimum isolate dominating setof $[G^*(9)]^{v_i}$ is S = $\{v_{i+2}, v_{i-2}\}$ and the corresponding inverse dominating sets $S^{'} = \{v_{i+1}, v_{i+4}, v_{i-3}, v_{i-1}\}$.Clearly $\delta(\langle S^{'}\rangle)$= 0, which implies that $S^{'}$ is an inverse isolate dominating set of $[G^*(9)]^{v_i}$. Hence$\gamma_0{}^{-1}([G^*(9)]^{v_i})$ = 2.

Case 4. n ≥ 10 and n ≡ 4 (mod 5)

The minimumisolate dominating set of $[G^*(n)]^{v_i}$ is S = $\{v_i, v_{i+3}, v_{i-2}\}$ and the corresponding inverse dominating set S' = $\{v_{i+2}, v_{i+}, v_{i+12}, \dots, v_{i-3}\}$, where the suffixes modulo n.Clearly $\delta(\langle S' \rangle)$= 0. Therefore $S'$is aninverse isolate dominating set of$[G^*(n)]^{v_i}$. Hence$\gamma_0{}^{-1}([G^*(n)]^{v_i})$ =$\left\lceil \frac{n}{5} \right\rceil + 1$.

Case 5. n = 7,n ≥ 10 and n $\not\equiv 4 \pmod 5$

The minimum isolate dominating set of $[G^*(n)]^{v_i}$is S = $\{v_i, v_{i+3}, v_{i-2}\}$ and the corresponding inverse dominating set $S'$is $\{v_{i+2}, v_{i+}, \dots, v_{i-1}\}$for n ≡1 (mod 5) and$\{v_{i+2}, v_{i+7}, \dots, v_{i-3}\}$ for n ≡0, 2, 3 (mod 5), where the suffixes modulo n. Clearly $\delta(\langle S' \rangle)$= 0. Therefore S' is an inverse isolate dominating set of $[G^*(n)]^{v_i}$. Hence $\gamma_0{}^{-1}([G^*(n)]^{v_i})$ = $\left\lceil \frac{n}{5} \right\rceil$.

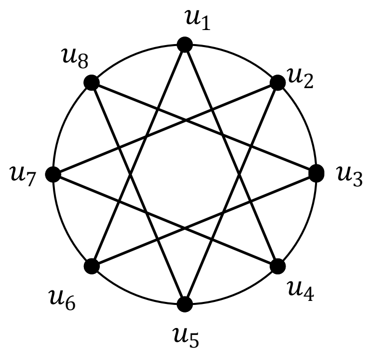
Theorem 3. 4. Let G be a graph obtained by the addition of G*(n) and K_2. Then $\gamma_0{}^{-1}(G)$ = 1.

Proof. Let G = G*(n) + K_2. Let V(G*(n)) = $\{v_1, v_2, \dots, v_n\}$ and V(K_2) = $\{u_1, u_2\}$.Then E(G*(n)) = $\{v_iv_{i+1}, v_iv_{i+3}, v_iv_{i-1}, v_iv_{i-3} / 1 \leq i \leq n\}$ and E(K_2) = $\{u_1u_2\}$.By the definition of additionof two graphs, we have V(G) = $\{v_1, v_2, \dots, v_n, u_1, u_2\}$ andE(G) = $\{v_iv_{i+1}, v_iv_{i+3}, v_iv_{i-1}, v_iv_{i-3}, u_1v_i, u_2v_i, u_1u_2 / 1 \leq i \leq n\}$. Clearly a minimumisolate dominating set S = $\{u_1\}$ and the minimum inverse isolate dominating set with respect to S isS' = $\{u_2\}$. Also $\delta(\langle S' \rangle)$= 0. Hence S' is an inverse isolate dominating set of G. Therefore $\gamma_0{}^{-1}(G)$ = 1.

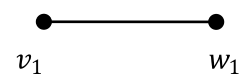
Theorem3. 5. Let G be the corona product of G*(n) and K_2.Then$\gamma_0{}^{-1}(G)$ = n.

Proof. Let G be a graph obtained bythe corona product of G*(n) and K_2. Let V(G*(n)) = $\{u_i / 1 \leq i \leq n\}$ be the vertex set such that $u_1u_2 \dots u_nu_1$ is a cycle. Let $\{v_i, w_i\}$ be the vertex set of i^{th} copy of K_2. Join u_i with v_i and w_i, $1 \leq i \leq n$. The resulting graph is G*(n)$\odot$ K_2.Clearly V(G) = $\{u_i, v_i, w_i / 1 \leq i \leq n\}$ and E(G) = $\{u_iu_{i\pm1}u_iu_{i\pm3}, u_iv_i, u_iw_i, v_iw_i / 1 \leq i \leq n\}$, where the suffixes modulo n. To dominate the vertices v_i and w_i, we must select either u_i or v_i or w_i,$1 \leq i \leq n$. Hence a minimum dominating set must contains n vertices.Clearly S = $\{v_1, u_2, u_3, \dots, u_n\}$ is a minimum isolate dominating set. Now S' = $\{u_1, v_2, v_3, \dots, v_n\}$ is a minimum inverse isolate dominating set with respect to S. Also $\delta(\langle S' \rangle)$= 0. Therefore S' is an inverse isolate dominating set of G. Hence$\gamma_0{}^{-1}(G)$ = n.

Example 3. 6. Consider the graph G*(8) $\odot K_2$ in Fig. 4. 1. Clearly S = $\{u_1, v_2, v_3, v_4, v_5, u_6, v_7, v_8\}$is a minimum isolate dominating set. The corresponding minimum inverse isolate dominating set S'= $\{w_1, u_2, w_3, u_4, w_5, u_6, w_7, u_8\}$. Hence $\gamma_0{}^{-1}(G * (8) \odot K_2) = 8$.



G*(8)



K_2

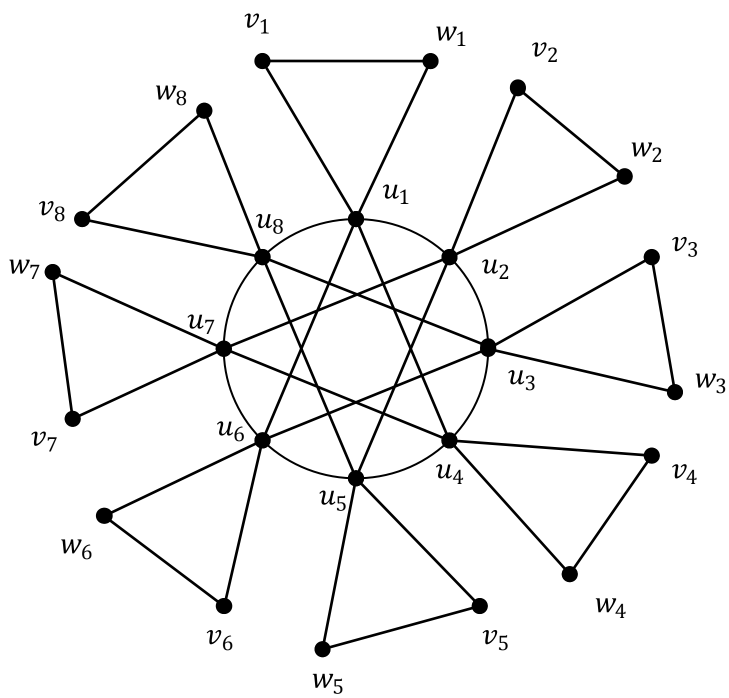


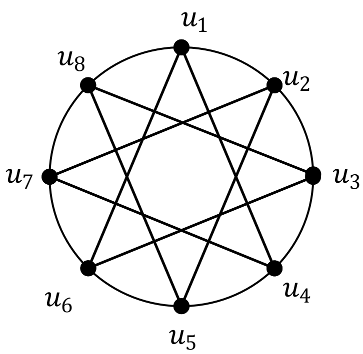
Fig. 4. 1 G*(8) $\odot K_2$

Theorem 3. 7. Let G be the graph obtained by the rooted product of G*(n) and K_2. Then $\gamma_0{}^{-1}(G) = n$.

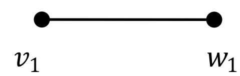
Proof. Let G be the graph obtained by the rooted product of G*(n) and K_2. The vertex set of G*(n) and K_2 are $V(G^*(n)) = \{u_i / 1 \leq i \leq n\}$ and $V(K_2) = \{v_i, w_i\}$,we get the resulting graph G with $V(G) = \{u_i, w_i / 1 \leq i \leq n\}$ and $E(G) = \{E(G^*(n)), u_i w_i / 1 \leq i \leq n\}$.To dominate the vertices u_i and w_i, we must select either u_i or w_i, $1 \leq i \leq n$. Hence a minimum dominating set contains n vertices.Clearly a minimum isolate dominating set S is $\{u_i, w_{i+1}, u_{i+2}, w_{i+3}, \ldots, u_{i-2}, w_{i-1}\}$ if n is odd and $\{u_i, w_{i+1}, u_{i+2}, w_{i+3}, \ldots, w_{i-2}, u_{i-1}\}$ if n is even. The corresponding inverse dominating set S' is $\{w_i, u_{i+1}, w_{i+2}, u_{i+3}, \ldots, w_{i-2}, u_{i-1}\}$ if n is odd and $\{w_i, u_{i+1}, w_{i+2},$

$u_{i+3},\ldots,u_{i-2},w_{i-1}\}$ if n is even,where the suffixes modulo n and $1 \leq i \leq n$. In both cases $\delta(\langle S' \rangle)= 0$. Therefore S' is an inverse isolate dominating set of G. Hence$\gamma_0{}^{-1}(G)= n$.

Example 3. 8. Consider the graph $G^*(8) \times K_2$ in Fig. 4. 2. Clearly S = $\{u_1, w_2, u_3, w_4, u_5, w_6, u_7, w_8\}$ is a minimum isolate dominating set. The corresponding minimum inverse isolate dominating set $S' = \{w_1, u_2, w_3, u_4, w_5, u_6, w_7, u_8\}$. Hence $\gamma_0{}^{-1}(G)= 8$



G*(8)



K_2

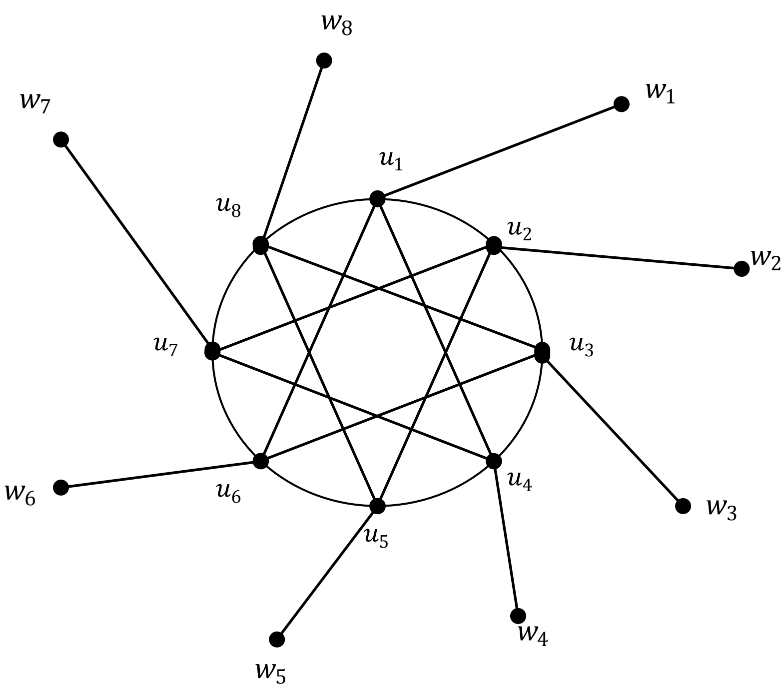


Fig. 4. 2. G

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