Mathematical Knowledge to Teach Physics and Teacher Training: The case of Kinematics graphs

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Abstract: It is mentioned in the literature that the theoretical construct *called Pedagogical Knowledge of Content(CPC)* provides methodological elements for the design of teacher training and updating programs. This work presents a proposal for the design of an update course for physics teachers in high school service, in the specific topic of kinematic graphs. The design of this was based on a previous work by the authors, in which they propose to characterize the CPC of the teachers in service, by means of a closed scale Likert test. Based on this instrument, a course consisting of 10 tasks was designed, and where it is promoted in use of two digital tools: GeoGebra and Tracker.

Keywords: Update courses, Pedagogical knowledge content, Cinematic Graphics, GeoGebra, Tracker.

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1. Introduction

Shulman (1986) was the first to coin the term of the Professor's *Professional Knowledge.* To introduce this term, he subdivided it into 7 categories or types of knowledge: *(1)* knowledge of the content, *(2)* general pedagogical knowledge, *(3)* pedagogical knowledge of the content, *(4)* knowledge of the curriculum, *(5) students'* knowledge and characteristics, *(6)* knowledge of educational contexts and *(7) knowledge* of educational purposes. It was the third category, pedagogical knowledge of content (CPC), that became more relevant over time, and has eventually been enriched with input from other researchers, first in the Anglo-Saxon context, but then across borders into the Spanish-speaking field **(Talanquer, 2004; Garritz, 2011; Garritz, 2014; Talanquer, 2014;)**. Shulman's central proposal was that it is actually the teacher who transforms disciplinary knowledge into a new knowledge of the teacher.

In the same sense, the *National Research Council* (NRC, 1996) mentions that this special knowledge, called "pedagogical knowledge of content", distinguishes the scientific knowledge of professors from that of scientists. (p.62). **Bolívar (2005),** defines CPC as the "set or repertoire of pedagogical *constructions*, the result of the wisdom of teaching practice, usually with a narrative structure, referring to a specific topic." (p. 9).

The construct of CPC has evolved since **Shulman (1986)** proposed it to this day, to the extent that, as Talanquer mentions (2014), it is difficult to find today a proposal for the training of science professors that is not based to a greater or lesser degree in this construct.For their part, **Keller, Newmann& Hans, (2017)** claim that recent research in mathematics and scientific subjects suggests pedagogical knowledge of teacher content (CPC), as one of the most influential factors contributing to student learning and performance.

In the same way **Garces, Font & Morales-Maure (2021)** mentioned that:

> Several studies on mathematics teachingin engineering degrees have indicated that the high number of students who failbasic science subjects in these degrees is directly related, among other aspects,to the way in which teachers approach and teach mathematics […] (p. 5)

In this order of ideas, **Campos-Nava and Ramírez-Díaz (2019),** propose to characterize the declarative CPC of the professorsof Physicsby means of categories or dimensions, which allowed them to develop a closed-scale Likert test, with which they could give and identify how developed the CPC of a physics teacher is on a specific topic.

Taking the above as a reference, this work describes how this characterization of the CPC provides important elements for the design or an update course for Physical teachers in the specific topic of kinematic graphs.

2.Significance of The Study

There is consensus among various researchers that, in order to be a good teacher, a solid basis of disciplinary knowledge must be had, but that the above, although necessary to be a teacher, is not enough, as teaching practice goes beyond *transmitting knowledge*.

Therefore, it is natural to wonder how courses should be designed to help teachers develop skills that allow them to amalgamate disciplinary knowledges is and pedagogical, in such a way that it allows them to improve their teaching practice.

In this order of ideas**, Riveros et al, (2004)** mention that:

> [Regarding physics teachers] There isa assumption that it is sufficient to know the discipline in order to teach it. Nothing more false [...] In general, it is necessary to know about didactics and pedagogy to articulate teaching, learning, evaluation and group techniques, in order to develop strategies with the possibility of functioning satisfactorily. (p. 88)

The above vision, while considering that the physics teacher should master the disciplinary aspects, and separately the pedagogical aspects of teaching, implicitly mentions that these two kinds of knowledge in the teacher, should converge on another kind of knowledge, which is the same as the CPC.

3.Review of Related Studies

Merino (2002) states that being a good professor of physicsinvolves not only knowing the discipline in depth, but also possessing skills in didactics and pedagogy, and also having skills and attitudes as a researcher. He claims that the professors of Physics must have solid knowledge in Physics, didactics and pedagogy and, in addition, openness attitudes, which is compatible with the construct of CPC.

NRC (1996) mentions that "in addition to solid knowledge of science, science teachers must have a solid foundation in learning theory, understand how learning occurs, and how it is facilitated." (p.62), which is also consistent with the concepts handled by the CPC construct.

In this order of ideas, **Talanquer (2004)** also states that the various results of teacher training studies have shown that:

> A teacher's ability to create conditions that facilitate learning not only depends on their knowledge of the subject or on various teaching methods, their success seems to depend on their ability to transform the disciplinary knowledge they possess into ways that aremeaningful to their students. (pp. 60- 61).

Based on the above, it is plausible to consider that the CPC can provide methodological elements to design courses that allow teachers to make their disciplinary knowledge and pedagogical knowledge converge and amalgam in what Shulman (1986) proposed as pedagogical knowledge of content, the kind of knowledge that distinguishes the physicsteacher from a physicist or an educator, that is, help the physics teacher, regardless of his initial training, become an educator of the discipline.

Etkina (2005) states that "Future physics teachers should learn in teaching environments similar to those they should create when they teach." (p. 3); in addition to the fact that "Future physics teachers should learn Physics in the same way that they should teach" (p.4); while these statements are aimed at training new physics teachers and not updating the teachers in service, we can consider it equally valid for those active physics teachers, who, however, have had little didactic training in the discipline, that is, they could be considered novice teachers.

In the same order of ideas, **McDermott, Shaffer & Constantinou (2000)** mention that some of the courses offered to train future physics teachers "perceive discipline as an inert body of knowledge that only needs to be memorized, and not as an active process of inquiry." (p. 412), and ensure that standard courses do not provide the kind of preparation teachers need.

McDermott et al., (2000) also claim that

> Teachers tend to teach in the same way that they were taught, if they were taught through readings, they will like to teach through readings, even if that type of instruction is inappropriate for their students [...] Teachers should be able to distinguish observations of inferences and make necessary reasoning from their observations and assumptions to reach logical conclusions. (p.412)

In joined to the above:

> The development of pedagogical knowledge of content by teachers reflects what we know about student learning; can be developed entirely only through continuous experience. But experience is not enough. Teachers should also have opportunities to participate in the analysis of the individual components of science through pedagogical knowledge of content, learning and pedagogy and establish connections between them. **(NRC, 1996, p. 63)**

In their part **Campos-Nava and Ramírez-Díaz (2109)**, propose that the CPC of physics teachers can be characterized for study in 5 dimensions, regardless of the particular topic spoken of:

> ***(i)*** Knowledge and beliefs on the Physics curriculum; ***(ii)*** Knowledge and beliefs about the difficulties of teaching a specific topic of Physics; ***(iii)*** Knowledge and beliefs about students' preconceptions and previous ideas in a specific topic of Physics; (***iv)*** Knowledge and beliefs about the use of teaching resources and strategies to teach a particular topic of Physics; and ***(v)***Knowledge and beliefs about the most effective ways to evaluate a specific topic of Physics. (pp. 346-347)

It is considered after the revision of the concepts adopted for this research, that the dimensions that were *adopted* to characterize *the declarative* CPC of a specific topic of the discipline, can guide the design of an update course that seeks to promote the development of the construct in physics professors in various topics of Physics, such as the one that has already been chosen (kinematic graphs).

The authors mentioned above clarify that these dimensions can be adjusted to explore the CPC of physics professors in any topic of the curriculum,in this case, we are interested in the subject of cinematic graphs.

This interest is based on the prior identification that the topic of kinematic graphics represents a good example of a content within the physics courses, of which various problems associated with their understanding are reported **(Dolores, Rivera and Tejada, 2016; Cuesta, Escalante y Ruiz, 2016)**. According to these studies, the main difficulties students show when trying to interpret kinematic graphics that represent the movement of diverse objects, include not correctly establishing differences entering acceleration-time and speed-time charts, not properly identifying meanings such as slope or reason for change in a chart, or associating average speed with the graph of distance traveled among others.

This problem is important, if we remember that kinematic graphs are powerful tools if you want to integrate information between two or more variables **(Parmar & Singer, 2005),** or in general the development of variational thinking **(Dolores et al., 2016; Urban, 2015).**

4.Objectives of The Study

- Inquire about the way in which a teacher refresher course can influence de development of the physics teacher´s CPC.
- Promote the development of the CPC of teachers through tasks or activities that include elements such as design of instructional scenarios, use of digital tools and strategies based on reflections and inquisitive processes.

5.Hypotheses of The Study

- The CPC of physics teacher can be reinforced with courses based on activities that promote their CPC.
- Such courses may include elements such as prior knowledge, instructional settings, resources employed and strategies deployed by the teachers.
- It is also important that teachers reflect on their beliefs, their previous experiences, errors and difficulties, when experiencing themselves the activities or tasks provided in the teacher training courses.

6.Methodological scheme

6.1. Proposed activities

To design the activities of the refresher course, the operational definition that **Campos-Nava and Ramírez-Díaz (2019)** made to characterize the declarative CPC of the professors of in a specific topic was taken as the basis:

Table 1: Operational definition of the CPCvariable(Campos-Nava and Ramirez- Diaz, 2019, p.350)

CPC Dimension	Parameters	Indicators
1 Knowledge and beliefs about the Physics curriculum.	What are you trying to get students to learn about this topic? Why is it important for students to learn this? What else do you know about this topic, and what do you think students should not yet learn?	1.1 Identifies the cross-sectional and longitudinal relationships between the different courses of the discipline in which the particular topic to be taught appears. 1.2 Identifies the importance of the topic to be taught in the student's curriculum. 1.3 Identify the most and least important thing to learn about the topic in question.
2 Knowledge and beliefs about the difficulties of teaching a specific topic of Physics.	What are the difficulties and/or limitations associated with teaching this topic?	2.1 Identifies the main obstacles the student faces when trying to learn the particular topic. 2.2 Identifies the areas of opportunity you have as a teacher when teaching the particular topic. 2.3 Identifies what would be desirable for students to learn about that particular topic, but cannot teach them because of the limitations that exist.
3 Knowledge and beliefs about students' preconceptions and ideas in a specific physics topic	What kind of ideas, conceptual errors, or preconceptions do students who influence the teaching of this topic have formed? What other factors influence the teaching of this topic?	3.1 Identifies what students have preconceptions about the topic to teach. 3.2 Identifies how students' preconceptions regarding the particular topic make it difficult to misunderstanding them. 3.3 identifies the nature of the preconceptions or misconceptions students have regarding the topic to be taught.
4 Knowledge and beliefs about the use of teaching resources and strategies to teach a particular topic of Physics.	What teaching strategies do you know are effective in teaching this topic and why are they?	4.1 Identifies which type of teaching resources are most suitable for teaching the particular topic. 4.2 Identifies the needs to use different teaching resources to address a particular topic. 4.3 Identifies the most effective strategies for trying to teach a

		particular topic.
5 Knowledge and beliefs about the most effective ways to evaluate a specific topic of Physics.	What are the specific ways to determine students' understanding or confusion regarding this topic?	5.1 Identifies the different evaluation strategies for evaluating a particular topic. 5.2 Identifies the different resources you can use to evaluate a particular topic. 5.3 identifies representative indicators that let students know if they have understood a particular topic.

Activities that are designed for the upgrade course should seek to influence at least one of the five dimensions of the CPC, although it is desirable that they affect as many dimensions aspossible. Parameters and indicators make it easier to choose the activity statement, the instructional scenario, the resources to be available and the strategies for assessing achievement.

In particular dimension four requires that, for the specific topic of kinematic graphs, the appropriate resources be chosen, it was decided that the GeoGebra Dynamic Geometry System (SGD) and that the Video Analysis Software (SVA) Tracker are two appropriate resources for physics teachers to manipulate them.

Even with the difficulties reported in literature that teachers of Physics in training might have to implement GeoGebra in their teaching practice, this SGD offers advantages over other software for teaching sciences, since the syntax used is very similar to that used by teachers in their classes, and no programming knowledge is required to develop physical models that are meaningful to students.

> The beauty of GeoGebra software is that you don't need to learn a programming language to use it to create realistic interactive simulations and animations of all kinds of physical phenomena. **(Walsh, 2017, pp. 316)**.

In this sense, with the fact that a teacher decides to create some physical model in GeoGebra to present it in class, this could provide elements to identify the conceptual difficulties that his own students may face, as well as even understand in greater depth characteristics of that phenomenon. The use of a dynamic geometry system, such as GeoGebra, provides a set of tools that facilitate these processes of mathematical thinking for students (Campos Nava, Torres Rodríguez, Morales Maure, 2021)

> One of my favorite things about creating Physics simulations in GeoGebra is that I often understand more deeply a topic or phenomenon by creating a simulation of it. **(Walsh, 2017, pp. 317).**

In contrast to GeoGebra's original intention that was aimed at teaching mathematics, the SVA Tracker had from the outset the intention to be used in physics courses to explore various phenomena through video analysis, mainly motion, but is not restricted only to that topic.While the idea of video analysis is not new and Tracker is not the only SVA that exists, it has some advantages over others, one of the main ones is that it is a free license software.

Beichner (1996), reports the positive effects of using video-analysis software for the correct interpretation of kinematic graphs by mentioning that with the help of software of this nature (in their research videograb software was used),students can examine the event and its graph simultaneously, which favors the ability to retain in memory due to the limitations that the brain has to remember a lot of information in the short term (pp. 1273), additionally mentions that the presentation of the event and its graph simultaneously favors cognitive functions that allow this memory to pass into long-term memory.

For their part, **Brown & Cox (2009),** mention that the didactic value that the different video analysis software (commercial and free license) has been evident in various researches, in part because working in such environments is familiar to students and because a bridge is built between direct observations and abstract mathematical representations of physical phenomena that can be analyzed. In their proposal they mention that Tracker may favor, among others, the study of topics such as collisions in one and two dimensions from the tracking of the center of mass of objects in a reference system; models of the air resistance of light objects in free fall can also be analyzed, as well as the study of thermal expansion using diffraction grid, as well as the study of non-thermal emission spectra of some gases and some fluorescent lamps.

As described above, these two digital tools are chosen as fundamental to the design of the activities of the refresher course, that is, it is necessary for teachers to interact with them, value their potential, and consequently incorporate them into their teaching practice as relevant resources for teaching the topic of kinematic graphs.

Another element that he considered indispensable to influence the CPC of teachers is the reflection on his teaching practice, in this regard **Caballero(2017)** mentions that:

> There must be, in principle, a process of critical reflection on the work of each teacher, and then move on to reflexive and critical action, followed perhaps by a modification and reorientation in the refresher courses, which serve as a tool for improvement, not as a bureaucratic thing, which, according to these reflections, can be discovered what is going through the teacher's mind while performing some action that helps him improve (p. 3).

This element of reflection is closely related to dimension 2 of the CPC (see Table 1). To encourage initial reflection, starting with Table 1, a question bank was developed to propose in plenary to teachers at the beginning of the refresher course, this element is considered to detonate individual and collective reflection, while allowing superficially exploring the CPC of teachers:

- ❑ What does it take to be a good physics teacher?
- ❑ Can I improve my teaching practice? In what way?
- ❑ Has the way I teach over time changed the way I teach? In what way?
- ❑ Have I incorporated new teaching resources for my Physics classes? Which ones?
- ❑ What are the teaching resources I use most? Why?
- ❑ Do I need to get to know my students better? What kind of students are they?
- ❑ What kind of mistakes do students make most often? Can I detect them? How do I try to keep them from being committed?
- ❑ What physics program topics do I teach consider less important? Why is that?
- ❑ What topics do I consider most important? Why?
- ❑ How do I evaluate my students? Has my way of evaluating them over time changed?

Choosing the resources to develop the activities and developing a list of questions to encourage teacher reflection, we proceed to design the activities that should influence at least one dimension of the CPC.

One way to selectstatements for activities is to search textbooks for problems that, while not explicitly mentioning the development of kinematic graphs as part of the solution, the development of the graphs could implicitly be used for interpretation and solution.

The temporality of the course should be proposed, in this case a total duration of 20 hours is proposed, mainly because for purposes of curricular value, it is the minimum hours required, it was decidedto divide into five sessions of four hours each, with the intention of proposing intermediates of 10 minutes to the middle of each session. In each span of two hours a specific activity will be worked on, for which 10 activities must then be designed for the entire course.Since two digital tools (GeoGebra and Tracker) have been chosen in each session one activity will be worked with each resource, to have at the end 5 activities on kinematic graphs to be addressed with the SGD GeoGebra and another 5 to be worked with the SVA Tracker.

As it cannot be assumed in advance that teachers know both resources, it is proposed that the first two activities be introductory so that participants become familiar with the use of them.

Below is the general structure of the refreshercourse, it was proposed that each session be addressed a thematic unit with a specific objective that address the topics that regularly appear in the highschool physics ministryprograms, the competencies were taken from official documents of the Mexican Ministry of Public Education (SEP), since the teachers to which the course is directed, belong to the Mexican education system:

Table 2: Proposed Agendaand Dosage. Own source

UNIT /TOPIC	PARTICULAR OBJECTIVE	DURATION	GENERIC COMPETENCES (SEP)[1]	DISCIPLINAR COMPETENCES (SEP)[2]
UNIT I: Dynamic Geometry Systems for modeling kinematics problems. (Activities 1 and 2)	For teachers to reflect on and become familiar with the benefits of using Digital Technologies in kinematics learning.	**4 hrs (face-to-face)**	**He organizes** his continuous training throughout his professional career. **Master and** structure knowledge to facilitate meaningful learning experiences. **It brings teaching and** learning processes to practice in an effective, creative and innovative way to its institutional context.	**It** theoretically and practically solves different cases of movement in one and two dimensions for application in situations of daily life. **It** applies the different laws, principles and models of mechanics to explain phenomena that occur in its daily life.
UNIT II: Video analysis systems to analyze kinematics problems. (Activities 4 and 6)	For teachers to reflect on and become familiar with the benefits of using Digital Technologies in kinematics learning.	**4 hrs (face-to-face)**	**He organizes** his continuous training throughout his professional career. **Master and** structure knowledge to facilitate meaningful learning experiences. **It brings teaching and** learning processes to practice in an effective, creative and innovative way to its institutional context.	**It** theoretically and practically solves different cases of movement in one and two dimensions for application in situations of daily life. **It** applies the different laws, principles and models of mechanics to explain phenomena that occur in its daily life.
UNIT III: Particle kinematics in one and two dimensions with GeoGebra. (Activities 3 and 5)	Teachers to use GeoGebra dynamic geometry software to model problems involving graphs of kinematic variables.	**4 hrs (face-to-face)**	**He organizes** his continuous training throughout his professional career. **Master and** structure knowledge to facilitate meaningful learning experiences. **It brings teaching and** learning processes to practice in an effective, creative and innovative way to its institutional context. **Build environments** for autonomous and collaborative learning.	**It** theoretically and practically solves different cases of movement in one and two dimensions for application in situations of daily life. **It** applies the different laws, principles and models of mechanics to explain phenomena that occur in its daily life.
UNIT IV:	Teachers to	**4 hrs (face-**	**He organizes** his	**It** theoretically

[1] Agreement 447 PMI (2008). Teaching Skills Higher middle education schooled.
[2] SEP (2017). Profile Parameters and indicators for entry into teaching functions in higher middle education.

Particle kinematics in one and two dimensions with Tracker. (Activities 7, 8 and 9)	use Tracker video analysis software to analyze problems involving graphs of kinematic variables.	**to-face)**	continuous training throughout his professional career. **Master and** structure knowledge to facilitate meaningful learning experiences. **It brings teaching and** learning processes to practice in an effective, creative and innovative way to its institutional context. **Build environments** for autonomous and collaborative learning.	and practically solves different cases of movement in one and two dimensions for application in situations of daily life. **It** applies the different laws, principles and models of mechanics to explain phenomena that occur in its daily life.
UNIT V: Design of activities with digital technologies. (Activity 10)	For teachers to reflect on the characteristics of learning activities in the topic of interpreting cinematic graphs and design appropriate tasks.	**4 hrs (face-to-face)**	**He organizes** his continuous training throughout his professional career. **Master and** structure knowledge to facilitate meaningful learning experiences. **It brings teaching and** learning processes to practice in an effective, creative and innovative way to its institutional context. **Build environments** for autonomous and collaborative learning.	**It** theoretically and practically solves different cases of movement in one and two dimensions for application in situations of daily life. **It** applies the different laws, principles and models of mechanics to explain phenomena that occur in its daily life.

The structure of each learning activity is proposed or subsequently, they should contain: a title, a goal, a motivating context, a statement and a guide on how to use the digital tool.

The final version of the seventh activity to be performed with Tracker ispresented as an example below:

Activity 7: Two-dimensional movement with Tracker.

Objective: Teachers will analyze objects moving in two dimensions as projectiles in Tracker to verify the independence of horizontal and vertical movement.

One motivation to introduce the theme is to remember those curious cartoons of a coyote chasing a roadrunner, it was common for the coyote to reach the edge of a cliff in such pursuits, to keep moving forward, to stand still for a few seconds in the air and then fall into free fall.

Figure 1: Coyote Willy falling off the cliff. ™ Warner Bros

Given the above context, teachers are encouraged to analyze in Tracker videos of objects that simulate a similar situation, that is, an object that travels in a straight line on a surface and that suddenly the surface does not hold it anymore, for example, a ball rolling on a table until it reaches the edge. With this analysis it is expected to confront some erroneous preconceptions, such as that if the ball carries a higher initial speed, falling off the table will take less time to reach the ground, or that the horizontal speed changes its value when the ball starts its fall.

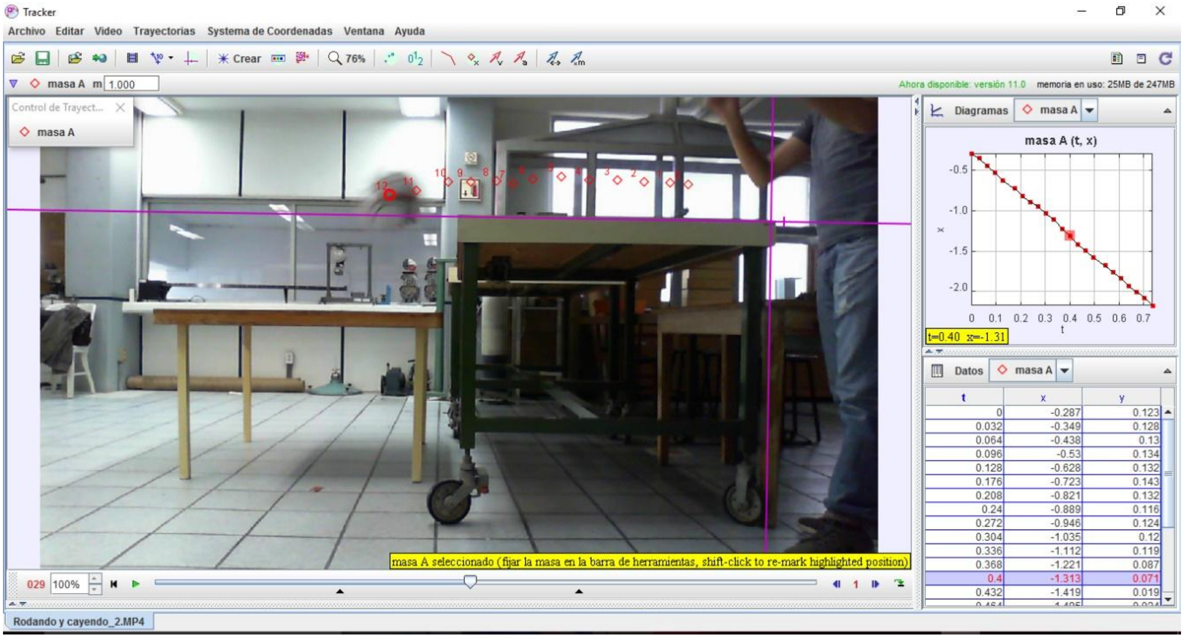

Figure 2: Tracker analysis of a ball rolling on a table and falling to the ground. Own source

After the proposal of the activity, an analysis is carried out to identify *whether a priori* it will be able to positively influence the CPC of the physics teachers, according to the dimensions of Table 1.

6.2. Data Analysis and Interpretation

Following the methodology described in the previous section, 10 activities were proposed for an update course aimed at professors of physics in service, in the specific topic of kinematic graphs, the list of them is presentedbelow, their objective andstatement.

Table 3: List of 10 activities designed. Own source

Activity	Title	Objective	Statement
1	Model in	Introduce teachers to the	Two separate particles a distance

	GeoGebra the collision of two particles with uniform rectilinear motion.	use of GeoGebra dynamic geometry software by creating simulations of Uniform Rectilinear Motion, in particular to simulate the conditions that must be met for two particles to collide.	"d" travel at constant speeds and in a straight line in different directions what angle measured between the two paths will allow the particles to collide? θ (Campos y Torres, 2016, p.22)
2	Analyze in Tracker the movement of a cart with constant speed.	Introduce teachers to the use of Video Tracker Analysis software by analyzing the video of a toy cart that moves at constant speed, generate the graph position against time and speed against time.	Analyze with Tracker the video of a motorized toy vehicle, which regularly travel at constant speed to get yourgraphic position-time andspeed-times.
3	Model the pursuit of two particles, one with uniform straight motion and one with uniformly accelerated straight motion.	Teachers will build in GeoGebra a simulationthat represents the pursuitofa vehicle with Uniform Rectilinear Motion by another vehicle that has Uniformly Accelerated Rectilinear Motion, in addition to showing on the same screen the position versus time plots of both objects.	A compact vehicle and a truck are 50 meters apart. The truck moves with a constant speed of 54 km/h while the compact vehicle, which is initially stopped, starts with an acceleration of1.6 m/s^2 that keeps constant. a. How long will it take for the compact car to catch the truck? b. What position will they be in then? c. How fast will the car be right now?
4	Tracker analysis of an object with constant acceleration.	Compare the kinematic graphs of an object with constant acceleration with those generated by an object with constant velocity.	Analyze the video of a low friction cart that ascends by means of a push and descends freely on a sloping plane, comparing its kinematic graphs with those obtained in activity 2.
5	GeoGebra simulation of the Uniformly Accelerated Rectilinear Movement.	Teachers will build a straight motion model with constant acceleration, with which they can relate the position-to-time plot and the line tangent to it.	Model in GeoGebra a situation similar to that analyzed in the video of activity 4 and verify that the slope of the tangent line at one point of the graph,equals the instantaneous velocity.
6	Obtaining position-time and acceleration-time graphs using GeoGebra.	Teachers will use GeoGebra and math concepts to obtain missing kinematic graphs of an object from a given graph.	A particle moves in a straight line with the speed shown in the figure(omitted). If it is known that x -540 ft in t x 0: a) constructs the curves a-t & x-t for 0 < t <50 s; determine b) the total distance traveled by the particle when t'50 s, and c) the two times at which x'0. Generate in GeoGebra the part function that provides the problem for speed and from it with the

			software commands get the position and acceleration commands and answer the questions.**(Beer et al., 2010, p.635)**
7	Two-dimensional movement with Tracker.	Teachers will analyze in Tracker objects that move in two dimensions as projectiles do, toverify the independence of horizontal movementfrom the vertical.	Analyze in Tracker videos of objects that travel in a straight line on a surface and that suddenly the surface does not hold it anymoreandfall, for example, a ball rolling on a table until it reaches the edge.
8	Projectiles modeled with GeoGebra and analyzed with Tracker.	Teachers will solve a problem about projectiles by having only one video to analyze in Tracker as data, then build a theoretical model in GeoGebra to compare results.	Given a video of a free-kick pitch in Basketball, and a distance reference within it, how fast did the player throw the ball and at what angle of inclination? Model GeoGebra with that information and compare it with the data provided by the video.
9	Circular motion modeled with GeoGebra and analyzed with Tracker.	Teachers will build a circular motion model in GeoGebra and analyze a circular motion video with Tracker to compare the kinematic graphs of some variables.	Analyze the video of a circular motion object, for example, of a vinyl disc, to corroborate what the angular offset graph is like against time and the angular velocity. Compare to a model generated in GeoGebra using the parameters of the circular motion (angular velocity and radius)
10	GeoGebra model of a gun's recoil mechanism.	Use GeoGebra to model a very specific application of using kinematic graphics.	The brake mechanism used to reduce the recoil of certain types of guns essentially consists of a plunger attached to a cannon moving in a fixed cylinder filled with oil. When the barrel recoils at an initial speed, the plunger moves and the oil is forced through the plunger holes, causing the plunger and the barrel to decelerate at a reason proportional to its speed, that is. Please say: a) "v" in terms of "t"; b) "x" in terms of "t"; c) "v" in terms of "x". Draw the corresponding motion curves. $v_o a = -kv$**(Beer et al., 2010, p611)**

The activities proposed in Table 3 were *analyzed a priori*, based on the experience of the authors as physics teachers doing a thoughtful analysis and taking as reference the fact that they themselves resolved them, in order to be able to identify whether they have an impact on one or moredimensions of the CPC, as proposed in Table 1, table 4 presents as an example, the result of this analysis for two particular activities.

Table 4: A priori analysisof the incidenceof two specific activities the CPC of **teachers. Own source**

Activity	Dimensions of CPC	Affects?	Reason

1 Model in GeoGebra the collision of two particles with uniform rectilinear motion.	(i) Knowledge and beliefs about the Physics curriculum.	Yes	Although the teacher may assume that the interpretation of graphs has nothing to do with the topic of collisions, it may reflect that through the analysis of kinematic graphs you can study the topic of collisions.
	(ii) Knowledge and beliefs about the difficulties of teaching a specific topic of Physics.	Yes	Teachers often consider that the topic of Uniform Rectilinear Motion is very *simple*, that you only have to give the expression of the position regarding time and solve problems of simple clearing.
	(iii) Knowledge and beliefs about the preconceptions and previous ideas students have in a specific topic of Physics.	Yes	The same teachers are exposed to their preconceptions about what needs to happen for the two particles to collide?
	(iv) Knowledge and beliefs about the use of teaching resources and strategies to teach a particular topic of Physics.	Yes	Teachers use a resource he probably hadn't considered before to study this topic.
	v) Knowledge and beliefs about the most effective ways to evaluate a specific topic of Physics.	Yes	Teachers may consider that one form of assessment is for students to be able to build dynamic models in GeoGebra.
9 Circular movement modeled with GeoGebra and analyzed with Tracker.	(i) Knowledge and beliefs about the Physics curriculum.	Yes	The teacher identifies that to build a dynamic model in GeoGebra ofthe Circular Uniform Motion, requires having previous notions of topics of geometry and trigonometry.
	(ii) Knowledge and beliefs about the difficulties of teaching a specific topic of Physics.	Yes	The teacher may notice that with the simulation in GeoGebra you can give a physical interpretation to the sign of angular velocity, and that corroborates it graphically in the analysis with Tracker.
	(iii) Knowledge and beliefs about the preconceptions and previous ideas students have in a specific topic of Physics.	Yes	Usually considered to constant speeds in uniform movements as scalar, with analysis in Tracker and model in GeoGebra, the sign of angular velocity acquires a vector connotation. In addition, cases where different radius are available can be inspected, but the same angular speeds can be inspected to confront misconceptions.
	(iv) Knowledge and beliefs about the use of teaching resources and strategies to teach a particular topic of Physics.	Yes	It is common for the relationship of the periodicity of the the Circular Uniform Motion not to be emphasized in class, with the use of GeoGebra and Tracker that relationship can be fostered graphically.

	v) Knowledge and beliefs about the most effective ways to evaluate a specific topic of Physics.	Yes	The student can propose prior to Tracker analysis what the angular velocity versus time graph is like, and confront their ideas with the analysis, this is a feature of the evaluation.

7.Recommendations

- Teacher training courses can enhance the knowledge and skills of the physics teacher to improve their performance.
- The teaching update courses can contemplate the development of the CPC as an articulating axis of content.
- Then development of CPC can be related in turn to the acquisition of better and more varied teaching strategies.

8.Conclusion

In an *a priori analysis*of the incidents shown in Table 4, shows that the design of activities based on the different dimensions of the conceptual construct of the CPC, can have a positive impact on the development of the physics teachers' own CPC, in the specific case of the topicof kinematic graphics, according to the dimensions of Table1

It is also relevant that the teaching practice of physics teachers considers the thoughtful use of theoretical-methodological constructs that contribute to the development of better and varied teaching strategies that the teacher can implement in the classroom. In this sense, teacher training courses can enhance the need to address the specific niche of teaching disciplines such as physics, supporting their design in the theoretical and methodological bases available from the field of specific didactics.

In practical terms, it would be impractical to think of training programs that included courses that addressed each topic of the Physics curriculum. In the case of this proposal, the topic of kinematic graphs turned out to be plausible, as it is content that appears recurrently in the agendas and textbooks of high school mechanics courses; In addition to the analysis of kinematic graphs, you can connect with topics that appear later in mechanics curriculum, for example, with energy conservation or with collisions in one and two dimensions.

The suggestion for those who intend to follow some strategy similar to that set out in this work is that they identify central topics of thephysics agenda, in which a set of previous knowledge, as well as some others of a cross-cutting nature or belonging to other disciplines, could converge, in orderto form a robust topic, so thatconsequently courses involving theoretical elements such as the CPC or others could be designed in order to contribute to the didactic training and discipline of physics teachers of the upper middle level.

References (APA)

Beer, F. Johnston, E. and Cornwell, P. (2010). *Mecánica Vectorial para Ingeniería: Dinámica*. Mc Graw Hill, Mexico.

Beichner, R. (1996). The impact of video motion analysis on kinematics graph interpretation skills. *American JournalofPhysics*, 10 (64), 1272-1277.

Bolivar, A. (2005). Conocimiento Didáctico del Contenido y Didácticas Específicas. *Revista de currículum y formación del profesorado,*.9 (2), 1-39.

Brown, D. & Cox, A. (2009). Innovative Uses of Video Analysis. *The Physics Teacher*, 47, 145-150.

Caballero, E. (2017). Percepción de los docentes sobre la actualización. *Revista Vinculando*. Availableon: http://vinculando.org/educacion/percepcion-los-docentes-la-actualizacion.html.

Campos-Nava, M. y Torres-Rodríguez, A. (2016). Física y Matemáticas Experimentales en Escenarios Virtuales. *Suma: Revista sobre Enseñanza y Aprendizaje de las Matemáticas*, 82, 19-26.

Campos Nava, M., Torres Rodríguez, A., & Morales Maure, L. (2021). Geogebra As A MeansToIdentifyPatterns In Linear Algebra Class: A Concrete Proposal. Revista Universidad y Sociedad, 13(2), 528-537.

Campos Nava, M., & Ramírez Díaz, M. H. (2019). Diseño de un instrumento el Conocimiento Didáctico del Contenido de profesores de física sobre un tópico específico.*Góndola, enseñanza y aprendizaje de las ciencias*, *14*(2), 340–359. https://doi.org/10.14483/23464712.13900

Córdova, W. G., Moll, V. F., & Maure, L. M. M. (2021). Criteria That Guide the Professor's Practice to Explain Mathematics at Basic Sciences Courses in Engineering Degrees in Peru. A Case Study. *Acta Scientiae*, 23(3).

Cuesta, A., Escalante, J.E. and Ruiz, J.F. (2016). Velocidad, Significadosmanifestados por estudiantesuniversitarios a partir de representacionesgráficas.*Avances de Investigación en Educación Matemática,*9,105-125.

Dolores, C.; Rivera, M. y Tejada, Y. (2016). Una experiencia didáctica con incidencia en la interpretación de gráficas cinemáticas. Revista de la Escuela de Ciencias de la Educación, 2(11), 129-154. DOI: https://doi.org/10.35305/rece.v2i11.264

DOF, (2008) (a). Acuerdo 444 por el que se establecen las competencias que constituyen el marco curricular común del sistema nacional de bachillerato. Disponible en: http://www.sems.gob.mx/work/models/sems/Resource/10905/1/images/Acuerdo_444_marco_curricular_comun_SNB.pdf

DOF, (2008) (b). Acuerdo 447 por el que se establecen las competencias docentes para quienes impartan educación media superior en modalidad escolarizada. Disponible en: http://www.sems.gob.mx/work/models/sems/Resource/10905/1/images/Acuerdo_447_competencias_docentes_EMS.pdf

Etkina, E. (2005) Physics teacher preparation: Dreams and reality. *JournalofPhysicsTeacherEducation*, 3 (2), 3-8.

Garritz, A. (2011). Conocimiento Didáctico del Contenido. Mis últimas Investigaciones CDC en lo afectivo, sobre la estequiometria y la indagación. *TED Tecné, Espistemé y Didaxis*, Specialnumber, 43-56.

Garritz, A. (2014). Creencias de los Profesores, su importancia y cómo obtenerlas. *EducaciónQuímica*, 25 (2), 88-92.

Keller, M., Neumann, K. & Fischer, H. (2017). The Impact of Physics Teachers Pedagogical Content Knowledge and Motivation on Students Achievement and Interest. *Journal of Research in Science Teaching*. 54(5), 586-614.

McDermortt, L., Shaffer, P. &Counstantinou, CP. (2000). Preparing Teachers to Teach Physics and Physical Science by Inquiry. *PhysicsEducation*, 35 (6),411-416.

Merino, M. (2002). Los retos de la enseñanza de la Física en el siglo XXI, *Alambique: Didáctica de las Ciencias Experimentales*, 33.

NRC (1996). *National Science Education Standards*. National Academy Press, USA.

Parmar, R. y Signer, B. (2005). Sources of Error in Constructing and Interpreting Graphs: A Study of Fourth-and Fifth-Grade Students with LD. *Journal of learning disabilities*, 38(3), 250–261.

Riveros, H., Jiménez, E., Riveros, D. (2004). *Cómo mejorar mi clase de Física: nivel medio superior*. Editorial Trillas: México.

Shulman, L. (1986). Those who understand: knowledge growth in teaching. *EducationalResearch*, 15 (2), 4-14.

Talanquer, V. (2014). Razonamiento Pedagógico Específico sobre el Contenido (RPEC). *Educación Química*, 25(3), 391-397.

Talanquer, V. (2004). Formación Docente: ¿Qué distingue a los buenos maestros de química? *EducaciónQuímica*, 15(1), 52-58.

Urban, H. (2015). Motion sensors in mathematics teaching: learning tools for understanding general math concepts?. *International Journal of Mathematical Education in Science and Technology*, 46(4), 584-598.

Walsh, T. (2017). Creating interactive physics simulations using the power of GeoGebra. *The physics Teacher.* 55 (5), 316-317.