Simulative Investigation on Wavelength Division Demultiplexure with Single mode Directional Couplers

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ABSTRACT

In this paper, we have proposed a directional coupler formed by two non-identical single-mode fibers and derived the closed-form equation of the transaction power of the directional coupler. To find the coupling coefficient of the directional coupler, we have derived the analytical closed-form equations for the phased and the non-phase matched condition under various types of index fiber. Moreover, it also shows the effect of coupling length with coupling coefficient for different fiber indexes with the variation of the operating wavelength. Next, with the help of different tables and a graphical representation, the coupling coefficient variation is shown with operating wavelength for step-index, triangular index and parabolic index under single-mode optical fiber for the directional coupler. The mathematical solution shows that the efficiency of the coupling coefficient has been enhanced with corresponding unconditional frequency and waveguide parameters under every index type of the single-mode optical fibers.

Keywords: *WDM, SMF, Directional Couplers, wave number, coupling coefficient, coupling length*

I. INTRODUCTION

Optical fiber communication is a renowned technology to exchange information from sender to retailer due to the capability of its vast capacity and magnificent data transfer rate [1,9]. Over a few decades, single-mode fiber (SMF) has been extensively used in optical systems to connect the end terminals due to attractive features [2]. Material and waveguide dispersion are the major drawbacks in fiber communication while establishing the communicating path. As a result, pulse broadening and various propagation velocity methods arise in the optical system [3,4]. The establishment of the SMF can resolve these limitations into a communication medium.

By the SMF only fundamental mode is considered to propagate the optical signal and this mechanism is typically depends on well-known parameter called ‘v’ number [5,6].On the other hand, directional couplers are widely used in the optical communication system for various purposes, such as optical modulation, optical splitter, filtering and many more operations. The basic principle of a directional coupler is to avoid the partial power exchange between two fibers; when two fibers having core diameter is very close to each other, the power of one fiber may entirely or partially transfer to other fiber [8]. Moreover, to achieve high accuracy in the optical system in terms of capacity and high data transmission rate, wavelength division multiplexing (WDM) and wavelength division de-multiplexing is the appropriate solution in modern communication systems [7,9].

II. PROPOSED MODEL

This section provides our proposed directional coupler model formed by two non-identical single-mode optical fibers. Fig. 1 shows a simplified inner structure of a directional power splitter with single input and two outputs. Here it shows that light is incident from one source with total incident power, which depends on the operating wavelength of the incident light and is split into two modes. The total power may be equally or partially divided depending on the phase-matched value, which is considered later in this paper. In Fig. 2, we have presented the application of a directional coupler for the wavelength division de-multiplexing scheme.

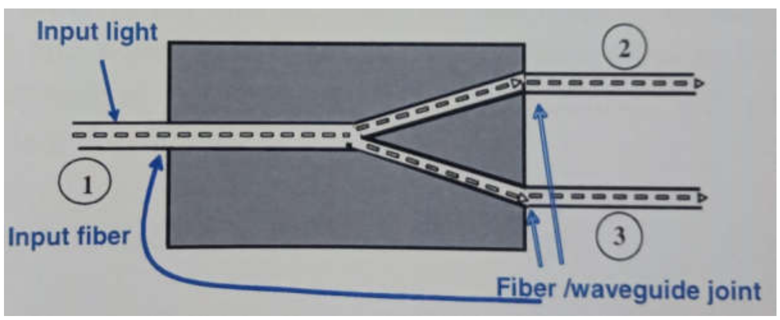

Fig.1.A simplified inner diagram of directional coupler power splitter.

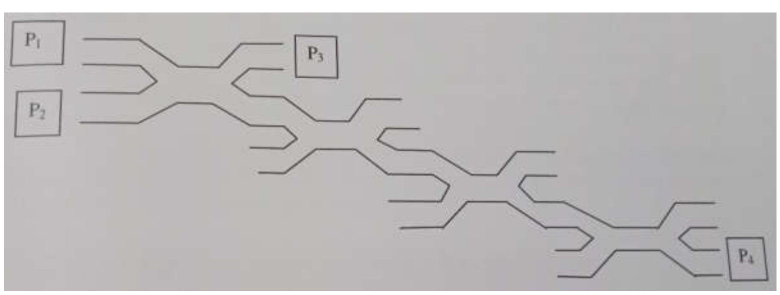

Fig. 2.The schematic diagram of De-multiplexure using directional coupler.

III. SCALAR WAVE EQUATION OF CONVENTIONAL FIBER

In an inhomogeneous medium, the EM field of a conventional optical fiber can be expressed [Ref] by a well-known equation as follows

$$\nabla^2 \bar{E} + \nabla\left(\frac{\nabla n^2}{n^2}\right)\bar{E} - \epsilon_0 \mu_0 n^2 \frac{\delta^2 \bar{E}}{\delta t^2} = 0 \tag{1}$$

where'n' is the refractive index, ϵ_0 and μ_0 are the free space permittivity and magnetic permeability, respectively. At the homogeneous medium, $\nabla\left(\frac{\nabla n^2}{n^2}\right)\bar{E} = 0$, and each electric field in the specific coordinate must follow the scalar wave equation such as,

$$\nabla^2\Psi = \epsilon_0\mu_0 n^2 \frac{\delta^2\Psi}{\delta t^2} \tag{2}$$

At the transverse coordinate, we can derived the following expression

$$\Psi(r,\varphi,z,t) = \Psi(r,\varphi)e^{i(\omega t-\beta z)} \tag{3}$$

where ω is known as angular frequency and β is called propagation constant. Now plugging Eq.(3) into Eq.(1), we have expressed the following mathematical framework as

$$\left(\nabla^2 - \frac{\delta^2}{\delta z^2}\right)\Psi + [\Psi^2 n^2(r,\varphi) - \beta^2] = 0 \tag{4}$$

The term '$n^{2\prime}$ typically depends on the radial part of the cylindrical coordinate for every optical fiber, and it can be derived as

$$\frac{\delta^2\Psi}{\delta r^2} + \frac{1}{r}\frac{\delta\Psi}{\delta r} + \frac{1}{r^2}\frac{\delta^2\Psi}{\delta\varphi^2}\left[{k_0}^2 n^2(r) - \beta^2\right]\Psi = 0 \tag{5}$$

where the term k_0 is called the wave numbers, are the ratio of angular velocity and the speed of light. After some mathematical manipulation, we can express the following equation in terms of waveguide parameter (U) and dimensionless frequency (V) as

$$V^2 = U^2 + W^2 \tag{6}$$

Where, $U^2 = a^2\left({k_0}^2{n_1}^2 - \beta^2\right)$ and $V^2 = a^2\left(\beta^2 - {k_0}^2{n_2}^2\right)$, and for guided mode ${k_0}^2{n_2}^2 < \beta^2 < {k_0}^2{n_1}^2$

IV. COUPLING COEFFICIENTS FOR DIRECTIONAL COUPLER

To discuss the inner properties of the array fiber, the coupling coefficient $\left(C_{pp}^{(i)(j)}\right)$ between i and j fiber for p^{th} mode of operation can be derived as [1]

$$\rho C_{pp}^{(i)(j)} = \sqrt{\delta}\frac{U^2}{V^3}\frac{K_0}{K_1^2}\left[W\left(\frac{d_{i,j}}{\rho}\right)\right] \tag{7}$$

Where'V' is known as dimensionless frequency, defined as $V^2 = \delta\left(\frac{2\pi\rho n_1}{\lambda}\right)^2 = U^2 + W^2$ and λ is the corresponding frequency, $\delta = 1 - \left(\frac{n_1}{n_2}\right)^2$. For the waveguide parameter, we have considered the following mathematical framework, which is the combination of modified Bessel's function (K) and first kind (J) as $UK_0(W)J_1(U) = WK_1(W)J_0(U)$ (8)

With the help of Eq.(7) and the properties of Bessel's function, we have expressed the dimensionless frequency (V) in terms of numerical apertures as

$$V^2 = \left(\frac{2\pi\rho n_1}{\lambda}\right)^2 \frac{{n_1}^2 - {n_2}^2}{{n_1}^2} \tag{9}$$

Using Eq.(9) and some mathematical calculation $\left(C_{pp}^{(i)(j)}\right)$ can be represented in terms of separated distance $\left(d_{ij}\right)$as

$$C_{pp}^{(i)(j)} = \frac{\lambda}{2\pi n_1}\frac{U^2}{a^2V^2}\frac{K_0\left[W\left(\frac{d_{ij}}{a}\right)\right]}{K_1^2(W)} \tag{10}$$

where Eq.(10) typically is the coupling coefficient between two fibers with spacing distance. Moreover, the coupling coefficient (k) also depends on the coupling length (Lc). Suppose the step-index single-mode fiber only makes the directional coupler. In that case, we can solve the waveguide parameter and unconditional frequency from Eq.(10) with any values of (V) and vice versa. In this regard, we have utilized the following relation between grading parameter and profile parameter (q) as

$$n^2(r) = \begin{cases} {n_1}^2\left[1-\delta\left(\frac{r}{a}\right)^q\right], r \le a \\ {n_1}^2[1-\delta], r \ge a \end{cases} \tag{11}$$

V. POWER EXCHANGE BETWEEN TWO FIBERS

Two non-identical single-mode fibers prepare a directional coupler to find the amount of exchange power between two fibers in the directional coupler, as shown in Fig.1. The fibers are supported LP_{01} modes having propagation constant β_1 and β_2 respectively. Then the resultant power propagation through the fibers can be derived by following the mathematical framework [1].

$$\frac{P_1(z)}{P_1(0)} = 1 - \frac{k^2}{\gamma^2}\sin^2(\gamma z) \tag{12}$$

and, $$\frac{P_2(z)}{P_1(0)} = 1 - \frac{k^2}{\gamma^2}\sin^2(\gamma z) \tag{13}$$

where $\gamma^2 = k^2 + \frac{(\Delta\beta)^2}{4}$ and, $\Delta\beta = \beta_1 - \beta_2$, k is the measure of the interaction strength between two corresponding fibers. Its value depends on the fiber parameters, the separation distance between two fibers, and the operating wavelength. The term $\Delta\beta$is called the phase mismatch. At the normal separation condition, the relation between power propagation can be expressed as

$$p_1(z) + p_2(z) = p_1(0) \tag{14}$$

On the other hand, if the separation between two fibers is comparatively large enough, then there is no interaction between them. For this condition, the amount of measure of the interaction strength between fibers becomes zero, which leads$p_1(z) = p_1(0)$and$p_2(z) = 0$.

A. POWER EXCHANGE FOR PHASE MATCHED CONDITION

In the phase matched condition $\Delta\beta$ becomes zero, means the propagation constant for two non identical fiber is approximately same or equal. For this regime we have represented the power relation between two fibers as

$$p_1(z) = p_1(0)(1 - \sin^2(kz))$$

or,$$p_1(z) = p_1(0)\cos^2(kz) \tag{15}$$

and, $$p_2(z) = p_1(0)\sin^2(kz) \tag{16}$$

Eq.(15) and Eq.(16) shows that there is periodic power exchange phenomenon is happen between two non-identical fibers and the amount of power is becomes $p_2(z) = p_1(0)$ and $p_1(z) = 0$.

Now, the minimum distance for which total is completely transferred from one to another is called as coupling length and it can be derived as

$$z = L_C = \frac{\pi}{2k} \quad (17)$$

Then the power launched into the fiber can be calculated at any value of z as

$$p_1(z) = p_1(0)\cos^2(kz) \quad (18)$$

With the help of Eq.(17,18) and after some mathematical manipulation we have expressed the amount of launched power for two non identical power into the directional coupler as

$$p_1(z) = p_2(z) = \frac{1}{2}p_1(0) \quad (19)$$

B. POWER EXCHANGE FOR NON PHASE MATCHED CONDITION

In the case of non phase matched condition $\Delta\beta \neq 0$, and power is evaluated in both fibers as function of coupling length, means there is some incomplete power transfer between two non identical fibers occurs. Due to this abnormality the fractional power is transfer from one to another fiber, and this amount of power can be expressed as

$$\eta_{max} = \frac{(p_2)_{max}}{p_1(0)} = \left\{\frac{k^2}{\gamma^2}\sin^2(\gamma z)\right\}_{max} \quad (20)$$

In Eq.(20) provide the highest value if and only if $\sin^2(\gamma z) = 1$, then Eq.(20) becomes rewritten as follows

$$\eta_{max} = \frac{k^2}{\gamma^2} \quad (21)$$

Taking the help of the linked between k^2 and γ^2 as, $\gamma^2 = k^2 + \frac{(\Delta\beta)^2}{4}$ and Eq.(21) we have developed the following equation as

$$\eta_{max} = \frac{1}{1+\left(\frac{\Delta\beta}{2k}\right)^2} \quad (22)$$

VI. NUMERICAL RESULT

In this section we have provided some numerical results with graphical and Table format based on the above mathematical framework. In Table-1 we have provided the numerical values of waveguide parameter and unconditional frequency for single mode step index and triangular index fiber with various fiber lengths. From the Table-1 it is a show that for any wavelength unconditional frequency remains constant for both step index and triangular index optical fibers while the waveguide parameters are widely varies with operating wavelength for both fibers. The variations of coupling coefficient (k) with different operating wavelength (I) have been reported in Table-2 under the directional coupler. Here we have consider our proposed directional coupler made by two non-identical optical fibers, and in Table-2 we also provided the numeral values of coupling coefficients for step triangular and parabolic index optical fiber. From Table-2 it is confirmed that the values of coupling coefficient is decreases with increases of operating wavelength of the optical fiber for all regime. In Fig.3, we have represented the graphical representation of coupling length (L_C) with coupling coefficient (k) under various types of index fiber. In this representation it is shows that the amount of coupling

Table-1

Step index			Triangular index		
I(µm)	V (Unconditional Frequency)	U (Waveguide Parameter)	I(µm)	V(Unconditional Frequency)	U (Waveguide Parameter)
1.67	1.6	1.36	1.67	1.6	1.58
1.64	1.62	1.37	1.64	1.62	1.61
1.62	1.65	1.39	1.62	1.65	1.63
1.59	1.67	1.40	1.59	1.67	1.65
1.57	1.7	1.41	1.57	1.7	1.68
1.55	1.72	1.42	1.55	1.72	1.70
1.52	1.75	1.43	1.52	1.75	1.72
1.50	1.77	1.44	1.50	1.77	1.75
1.48	1.8	1.45	1.48	1.8	1.77
1.46	1.82	1.46	1.46	1.82	1.49
1.44	1.85	1.47	1.44	1.85	1.81
1.42	1.87	1.48	1.42	1.87	1.83
1.40	1.9	1.49	1.40	1.9	1.86
1.38	1.92	1.50	1.38	1.92	1.88
1.37	1.95	1.51	1.37	1.95	1.90
1.35	1.97	1.51	1.35	1.97	1.92
1.33	2.0	1.52	1.33	2.0	1.94

length is gradually increases from triangular index to step index fiber via parabolic index fiber. This graphical representation shows that the coupling length suddenly increased for step-index fiber under the same coupling coefficient, unlike optical fiber's triangular and parabolic index types.

Moreover, on Fig.4 we have reported the bar chart for the same metric with similar fiber index types. Here it is clearly observed that the coupling length of step index fiber increases nearly 78% than triangular index fiber at coupling coefficient 5µm^{-1}, while for parabolic index fiber it is nearly 45% than triangular index fiber. At 15µm^{-1}coupling coefficient coupling length is enhance ≅ 148% for step index than parabolic index and ≅ 350% than triangular index fiber.

Table-2

Step index		Triangular index		Parabolic Index	
I(μm)	k(μm^{-1})	I(μm)	k(μm^{-1})	I(μm)	k(μm^{-1})
1.67	0.0002036	1.67	0.009216	1.67	0.005064
1.64	0.001905	1.64	0.008646	1.64	0.004750
1.62	0.001794	1.62	0.008118	1.62	0.004459
1.59	0.001687	1.59	0.007628	1.59	0.004191
1.57	0.001588	1.57	0.007175	1.57	0.003942
1.55	0.001496	1.55	0.006754	1.55	0.003712
1.52	0.001410	1.52	0.006364	1.52	0.003498
1.50	0.001330	1.50	0.005998	1.50	0.003298
1.48	0.001257	1.48	0.005663	1.48	0.003115
1.46	0.001188	1.46	0.005349	1.46	0.002943
1.44	0.001123	1.44	0.005056	1.44	0.002784
1.42	0.001063	1.42	0.004783	1.42	0.002634
1.40	0.001007	1.40	0.004528	1.40	0.002495
1.38	0.000954	1.38	0.004290	1.38	0.002365
1.37	0.000905	1.37	0.004068	1.37	0.002244
1.35	0.000859	1.35	0.003860	1.35	0.002130
1.33	0.000816	1.33	0.003665	1.33	0.002023

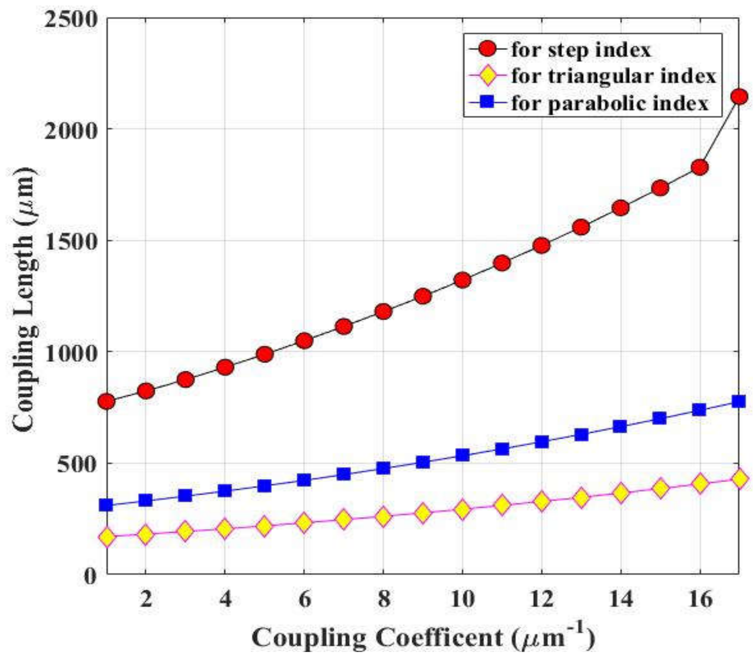

Fig. 3.The graphical representation of coupling length (L_C) with coupling coefficient (k) under various types of index fiber.

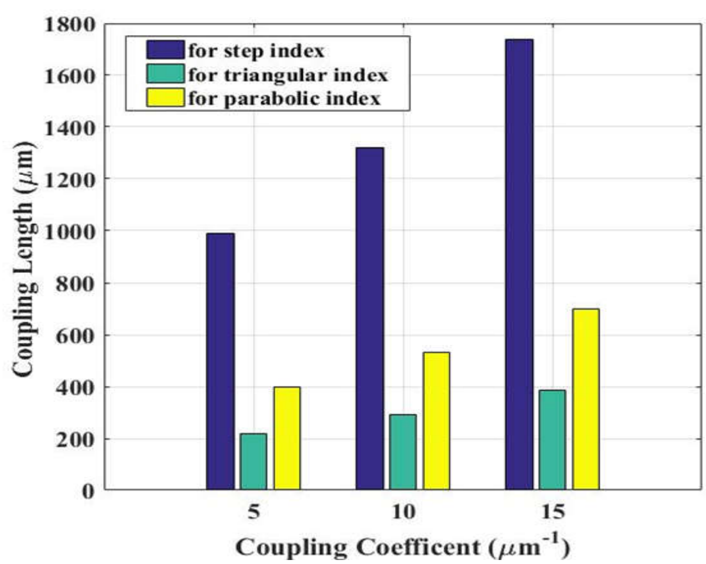

Fig. 4.The bar chart of coupling length (L_C) with coupling coefficient (k) under various types of index fiber.

V. CONCLUSION

This work concludes that the calculated numeral values based on the above mathematical framework of the fiber parameters discussed here, like waveguide parameter (U) and unconditional frequency (V), typically depend on the operating wavelength and corresponding wave number. These numeral values are relatively lower for step-index fibers than the triangular index fibers under the same operating wavelength for single-mode optical fiber, as shown in Table-1. On the other hand, the coupling coefficient values are smaller to the same wavelength region than that of other index types presented in Table-2. Moreover, the values of coupling length are high enough for step-index fiber than the rest of others types of index fiber.

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