Fuzzy Parametric Geometric Programming in Process Reliability Based on Fuzzy EPQ Model

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ABSTRACT

This paper is developed under the influence of economic production quantity model with demand dependent unit production cost in fuzzy environment. Development of fuzzy cost coefficient and restriction on storage space has been done under the condition of flexibility and reliability in the production process. The inventory related costs and space parameters are treated as fuzzy numbers and solved by the fuzzy parametric geometric programming technique. The execution of sensitivity analyses caused by different measures performed here.

Keywords: Geometric Programming, Economic Production Quantity (EPQ), Inventory, Reliability, Fuzzy Set, Storage space.

1. INTRODUCTION

Geometric Programming (GP) is an effective method among different non-linear optimization techniques to solve a meticulous type of non-linear programming problem. Geometric Programming [1] is very popular in various fields of science and engineering. Duffin, Peterson and Zener [2] developed the GP with theory and applications in their books. Peterson [3], Rijckaert [4], Jefferson and Scott [5] have presented informative surveys on GP. The parameter used in the GP problem may not be crisp, rather it is more fruitful to consider it as fuzzy. In such a situation we have introduced the concept of fuzzy parametric GP technique, where the parameters are fuzzy. GP in fuzzy environment is a competent optimization method to solve a typical fuzzy optimization problem is called Fuzzy Geometric Programming [6,7].

Use of GP can be observed in many aspects of inventory/production. There are very few papers [8-13] concerned with the inventory problems using GP. Significant amount of research effort have been expended on EPQ leading to the publication [15-17] of many interesting results in the literature. In the classical EPQ model a basic assumption is that the production set-up cost is fixed and also implicitly assumes that items produced are of perfect quality [18]. Practically, product quality is not always perfect but directly affected by the reliability of the production process employed to manufacture the product. Product quality can only consistently achieve high level with substantial investment in improving the reliability of the production process. Furthermore, while the set-up time, hence set-up cost, will be fixed in short term, it will tend to decrease in the long term because of the possibility of investment in new machineries that are highly flexible, e.g. flexible manufacturing system. Different researchers [8,9,19-20] have addressed extensively the issue of flexibility and reliability improvement production and inventory management under various scenarios. Issues of process reliability, quality improvement and set-up time reduction etc. have been discussed by Porteus [21-22], Rosenblatt and Lee [23], Zangwill [24], Islam and Roy [25], Leung [26], Mondal [27], Eshkiki et all [28] and Joyanthi and Ritha [29].

In this paper, an economic production quantity model with demand dependent unit production cost in fuzzy environment has been developed. Flexibility and reliability consideration are introduced in the production process. We have considered the coefficients of the model are fuzzy in nature. The model is developed under fuzzy goal and fuzzy restrictions on storage space. The inventory related costs and storage space parameters are taken as fuzzy in nature. Finally, the EPQ model is solved by fuzzy parametric geometric programming technique.

2. MATHEMATICAL FORMULATION OF EPQ MODEL

Let us consider a single product production system with conventional production process up to a certain level of reliability. Production in smaller batch sizes with flexible process is economical, in that way we can reduce the inventory holding cost. Also, substantial capital expenditure due to illustration of the new production process will give rise to mighty interest changes and great depreciation cost.

2.1 NOTATIONS

We construct a model for this problem using following variables and parameters

S	set-up cost per batch (a decision variable),
D	demand rate (a decision variable),
q	production quantity per batch (a decision variable),
r	production process reliability (a decision variable),
H	inventory carrying cost per item per unit time,
f(S, r)	cost of interest and depreciation for a production process per production cycle,
TC(D, S, q, r)	total average cost,
w	storage space per unit,
W	total storage space.

2.2 ASSUMPTIONS

Following assumptions are made for developing the EPQ model:

(i) The rate of demand D is uniform over time

(ii) The time horizon is infinite

(iii) Shortages are not allowed

(iv) Total cost of interest and depreciation per production cycle is inversely related to a set-up cost and directly related to process reliability according to the following equation

$$f\left(S,r\right)=aS^{-b}r^{c} \tag{2.1}$$

where $a,b,c>0$ are constant chosen to provide best fit of estimated cost function.

(v) The unit production cost (p) is continuous function of demand (D) and directly related to production reliability (r), so unit production cost is as follows

$$p=\gamma D^{-\beta}r^{\nu} \tag{2.2}$$

where β(>1) is called price elasticity and γ, ν (>0) is a scaling constant.

The first three assumptions are basic assumptions in classical EPQ model. The fourth assumption is based on the fact that to reduce costs of production set-up and scrap and rework on shoddy protects substantial investment in improving the flexibility and reliability of the production process is necessary. The fifth assumption is based on the unit variable production is demand dependent. When demand of an item increase, the production/purchasing cost spread over all items and hence unit purchasing cost reduces and varies inversely with demand. The process reliability level r means of all the items produced in a production run only r percentage are of acceptable quality that can be used to meet demand. The situation of the inventory model is given below:

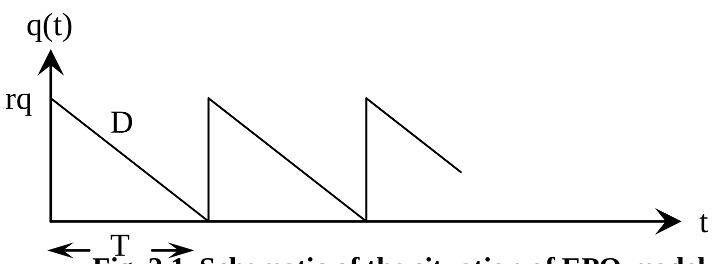

Fig. 2.1. Schematic of the situation of EPQ model

2.3 CRISP MODEL

If q(t) is the inventory level at time t over the time period (0, T), then

$$\frac{dq(t)}{dt} = -D \quad for\ 0 \le t \le T \tag{2.3}$$

with initial and boundary conditions q(0) = rq and q(T) = 0.

The solution of the differential equation (1.3) is $q(t) = rq - Dt$ and $T = (rq)/D$.

Now, inventory-carrying cost is $= H\int_0^T q(t)dt = H\int_0^T (rq - Dt)dt = \frac{1}{2}\frac{Hr^2q^2}{D}$

The length of the production cycle is the sum of set-up, production, inventory carrying and interest and depreciation costs, thus total cost per cycle is

$$= S + pq + \frac{1}{2}\frac{Hr^2q^2}{D} + f(S,r) \tag{2.4}$$

The objective is to minimize the total cost per unit time under limited storage space

So TC (D, S, q, r) = (total cost per cycle) / (qr / D) (2.5)

Substituting (2.1), (2.2) and (2.4) in (2.5) which becomes as follows

$$TC(D,S,q,r) = DSq^{-1}r^{-1} + \gamma D^{1-\beta}r^{\nu-1} + \frac{1}{2}Hqr + aDS^{-b}q^{-1}r^{(c-1)}$$

Let the required storage area per unit be wr^x and the total required storage area is less or equal to W. So we have considered space constraint as follows

$$wr^xq \le W \qquad x \in (0,1)$$

Hence the inventory model can be written as

$$Minimize\ \ TC(D,S,q,r) = DSq^{-1}r^{-1} + \gamma D^{1-\beta}r^{\nu-1} + \frac{1}{2}Hqr + aDS^{-b}q^{-1}r^{(c-1)} \tag{2.6}$$

Subject to $wr^xq \le W \qquad x \in (0,1)$

D, S, q, r >0

The above problem (2.6) can be treated as a Posynomial Geometric Programming problem with zero Degree of Difficulty.

2.4 FUZZY MODEL

Let the coefficients of objective function and constraint function of (2.6) be fuzzy in nature then crisp model is transformed to the following fuzzy model

$$Minimize\ \ TC(D,S,q,r) = DSq^{-1}r^{-1} + \tilde{\gamma} D^{1-\beta}r^{\nu-1} + \frac{1}{2}\tilde{H}qr + \tilde{a}DS^{-b}q^{-1}r^{(c-1)}$$

$$subject\ to \qquad \tilde{w}r^xq \le W \qquad x \in (0,1) \tag{2.7}$$

D, S, q, r>0.

Where $\tilde{\gamma}, \tilde{H}, \tilde{a}$ and $\tilde{w}$ are fuzzy in nature.

3. PRELIMINARY MATHEMATICS

Definition 3.1 Fuzzy Set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as a set of pairs $\tilde{A}=\{(\mathrm{x}, \mu_{\tilde{A}}(\mathrm{x})):\mathrm{x}\in \mathrm{X}\}$. Here $\mu_{\tilde{A}}: X \to [0,1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(\mathrm{x})$ is called the membership value or degree of membership of x$\in$ X in the fuzzy set $\tilde{A}$.

Definition 3.2 α-Level Set or α-cut of a Fuzzy Set: The α-level set or α-cut of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in $\tilde{A}$ greater than or equal to α i.e. $A_\alpha=\{x:\mu_{\tilde{A}}(x)\geq\alpha, x\in X,\ \alpha\in[0,1]\}$

Definition 3.3 Triangular Fuzzy Number (TFN): A TFN $\tilde{A}=(a_1,a_2,a_3)$ is a fuzzy set of the real line $\Re$ whose membership function $\mu_{\tilde{A}}(x)$ has the following characteristics

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & for\, a_1\leq x\leq a_2\\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2\leq x\leq a_3\\ 0 & otherwise\end{cases}$$

4. MATHEMATICAL ANALYSIS

Consider a particular type of non-linear programming problem

$$\begin{aligned} & Min\ g_0(x) \\ subject\, to\quad & g_i(x)\leq 1 \qquad (1\leq i\leq n) \\ & \mathrm{x} > 0. \end{aligned} \tag{4.1}$$

Its objective and constraint functions are of the form

$g_i(x)=\sum_{k=1}^{T_i} c_{ik}\prod_{j=1}^{m} x_j^{\rho_{ikj}}\quad (0\leq i\leq n)$ where $x_j > 0$; and c_{ik}, ρ_{ikj} are real numbers.

The constraint in (4.1) needs softening and considering the problem of fuzzy objective and constraint with fuzzy coefficients, we transform (4.1) into a fuzzy geometric programming as follows

$$\begin{aligned} & \tilde{Min}\ g_0(x) \\ subject\ to\quad & g_i(x)\lesssim 1 \qquad (1\leq i\leq n), \\ & \mathrm{x} > 0, \end{aligned} \tag{4.2}$$

where $x=(x_1,x_2,...,x_m)^T$ is a variable vector and $g_i(x),(0\leq i\leq n)$ are all posynomials of x in which coefficients c_{ik} are fuzzy numbers. For TFN $\tilde{c}_{ik}=(c_{1ik},c_{2ik},c_{3ik})$ containing the coefficients $\tilde{c}_{ik}(0\leq i\leq n;1\leq k\leq T_i)$, the problem (4.2) can be replaced with α-cut of $\tilde{c}_{ik}(0\leq i\leq n;1\leq k\leq T_i)$ is given by $c_{ik}(\alpha)=[c_{ikL}(\alpha),c_{ikR}(\alpha)]=[c_{1ik}+\alpha(c_{2ik}-c_{1ik}),c_{3ik}-\alpha(c_{3ik}-c_{2ik})]$ according to the following proposition

Proposition: When the coefficients of the fuzzy geometric programming problem are taken as fuzzy numbers.

$$\tilde{M}in\sum_{k=1}^{T_{0i}}\tilde{c}_{0k}\prod_{j=1}^{m}x_j^{\rho_{0kj}} \tag{4.3}$$

Subject to $\sum_{k=1}^{T_i}\tilde{c}_{ik}\prod_{j=1}^{m}x_j^{\rho_{ikj}} \lesssim 1, \ \left(1\le i\le n\right)$

$x_j > 0.$

Using α-cut of the fuzzy numbers coefficients, the above problem is reduces to

$$Min\sum_{k=1}^{T_0}\left[c_{0kL}\left(\alpha\right),c_{0kR}\left(\alpha\right)\right]\prod_{j=1}^{m}x_j^{\rho_{0kj}} \tag{4.4}$$

Subject to $\sum_{k=1}^{T_i}\left[c_{ikL}\left(\alpha\right),c_{ikR}\left(\alpha\right)\right]\prod_{j=1}^{m}x_j^{\rho_{ikj}} \lesssim 1, \ \left(1\le i\le n\right)$

$x_j > 0.$

Which is equivalent to

$$Min\sum_{k=1}^{T_0}c_{0kL}\left(\alpha\right)\prod_{j=1}^{m}x_j^{\rho_{0kj}} \tag{4.5}$$

Subject to $\sum_{k=1}^{T_i}c_{ikL}\left(\alpha\right)\prod_{j=1}^{m}x_j^{\rho_{ikj}} \le 1, \quad \left(1\le i\le n\right)$

$x_j \ge 0.$

5. SOLUTION TECHNIQUE OF FUZZY PARAMETRIC GEOMETRIC PROGRAMMING

The solution procedure of constrained posynomial parametric GP problem (4.5) using fuzzy parametric geometric programming problem is discussed in this section. The number of terms in each posynomial constraint function varies and it is denoted by T_r for each r=0,1,2,…,l. Let $T=T_0+T_1+T_2+\ldots+T_l$ be the total number of terms in the primal program. The Degree of Difficulty is =T-(m+1).

The dual problem of the primal problem (4.5) is as follows

Maximize $d(\delta)=\prod_{r=0}^{l}\prod_{k=1}^{T_r}\left(\frac{c_{rkL}\left(\alpha\right)}{\delta_{rk}}\right)^{\delta_{rk}}\left(\sum_{s=1+T_{r-1}}^{T_r}\delta_{rs}\right)^{\delta_{rk}}$ (5.1)

subject to $\sum_{k=1}^{T_0}\delta_{0k}=1$, (Normality condition)

$\sum_{r=0}^{l}\sum_{k=1}^{T_r}\rho_{rkj}\delta_{rk}=0$, (j=1,2,...,m) (Orthogonality conditions)

$\delta_{rk}>0$, (r=0,1,2,…,l; k=1,2,…,T_r). (Positivity conditions)

Case I. For $T\ge m+1$, the dual program presents a system of linear equations for the dual variables. A solution vector exists for the dual variables.

Case II. For T<m+1, in this case generally no solution vector exists for the dual variables.

The solution of the GP problem is obtained by the solving system of linear equations of dual problem (5.1). Ones optimal dual variable vector δ^* are known, the corresponding values of the primal variable vector t is found from the following relations:

$$c_{ikL}\left(\alpha\right)\prod_{j=1}^{m}x_j^{\rho_{ikj}}=\delta_i^*\,d^*\left(\delta^*\right) \qquad (i=0,1,2,\ldots,T_0) \tag{5.2}$$

Taking logarithms in (5.2), T_0 log-linear simultaneous equations are transformed as

$$\sum_{j=1}^{m}\rho_{ikj}(\log x_j)=\log\left(\frac{\delta_i^* d^*\left(\delta^*\right)}{c_{ikL}\left(\alpha\right)}\right) \qquad \text{(i=0,1,2,}\ldots\text{,}T_0\text{)} \qquad (5.3)$$

It is a system of linear equations in t_j (=log x_j) for j=1,2,...,n. Since there are more primal variables x_j than the number of terms T_0, many solutions x_j (j=1,2,...,n) may exist. So, to find the optimal primal variables x_j (j=1,2,...,n), it remains to minimize the primal objective function with respect to reduced m-T_0 ($\neq 0$) variables. When m-T_0=0 i.e. number of primal variables = number of log-linear equations, primal variables can be determined uniquely from log-linear equations. For different value of $\alpha \in [0,1]$, equ.(5.3) will return different solution sets of δ_i^* and hence different solution sets for dual as well as primal problem will be obtained. These solutions sets will help decision-maker to take appropriate decision.

6. FUZZY PARAMETRIC GEOMETRIC PROGRAMMING TECHNIQUE ON EPQ MODEL

Let the fuzzy parameters of fuzzy EPQ model (2.7) be triangular fuzzy number given by $\tilde{\gamma}\equiv\left(\gamma_1,\gamma_2,\gamma_3\right)$, $\tilde{H}\equiv\left(H_1,H_2,H_3\right)$, $\tilde{a}\equiv\left(a_1,a_2,a_3\right)$ and $\tilde{w}\equiv\left(w_1,w_2,w_3\right)$. According to section 5, the fuzzy EPQ model (2.7) reduces to a fuzzy parametric geometric programming by replacing $\tilde{\gamma}=\gamma_1+\alpha\left(\gamma_2-\gamma_1\right)$, $\tilde{H}=H_1+\alpha\left(H_2-H_1\right)$, $\tilde{a}=a_1+\alpha\left(a_2-a_1\right)$ and $\tilde{w}=w_1+\alpha\left(w_2-w_1\right)$ where $\alpha\in\left[0,1\right]$. The model takes the reduces form as follows

$$\textit{Minimize}\ \ TC(D,S,q,r)=DSq^{-1}r^{-1}+\left(\gamma_1+\alpha\left(\gamma_2-\gamma_1\right)\right)D^{1-\beta}r^{\nu-1}+\frac{1}{2}\left(H_1+\alpha\left(H_2-H_1\right)\right)qr$$
$$+\left(a_1+\alpha\left(a_2-a_1\right)\right)DS^{-b}q^{-1}r^{(c-1)} \qquad (6.1)$$

$$\textit{subject to}\ \ \left(w_1+\alpha\left(w_2-w_1\right)\right)r^x q\le W \qquad x\in\left[0,1\right]$$

D, S, q, r>0.

Applying GP technique the dual programming of the problem (6.1) is

$$\textit{Max}\ d(\delta)=\left(\frac{1}{\delta_1}\right)^{\delta_1}\left(\frac{\gamma_1+\alpha\left(\gamma_2-\gamma_1\right)}{\delta_2}\right)^{\delta_2}\left(\frac{H_1+\alpha\left(H_2-H_1\right)}{2\delta_3}\right)^{\delta_3}\left(\frac{a_1+\alpha\left(a_2-a_1\right)}{\delta_4}\right)^{\delta_4}\left(\frac{w_1+\alpha\left(w_2-w_1\right)}{W}\right)^{\delta_5} \qquad (6.2)$$

subject to

$$\begin{aligned}
&\delta_1+\delta_2+\delta_3+\delta_4=1\\
&\delta_1+(1-\beta)\delta_2+\delta_4=0\\
&\delta_1-b\delta_4=0\\
&-\delta_1+\delta_3-\delta_4+\delta_5=0\\
&-\delta_1+(\nu-1)\delta_2+\delta_3+(c-1)\delta_4+x\delta_5=0
\end{aligned} \qquad (6.3)$$

Solution of the above system of linear equations gives the optimal solution of dual problem. The values of D, S, q, r is obtained by using the primal dual relation as follows

From primal dual relation we get

$$DSq^{-1}r^{-1}=\delta_1^*\times d^*\left(\delta\right), \qquad \left(\gamma_1+\alpha\left(\gamma_2-\gamma_1\right)\right)D^{1-\beta}r^{\nu-1}=\delta_2^*\times d^*\left(\delta\right)$$

$$\frac{1}{2}\left(H_1+\alpha\left(H_2-H_1\right)\right)qr=\delta_3^*\times d^*\left(\delta\right), \qquad \left(a_1+\alpha\left(a_2-a_1\right)\right)DS^{-b}q^{-1}r^{(c-1)}=\delta_4^*\times d^*\left(\delta\right)$$

$$\frac{\left(w_1+\alpha\left(w_2-w_1\right)\right)}{W}r^x q=\delta_5^*$$

The optimum solution of the model (2.7) through fuzzy parametric geometric programming approach is given by

$$d^*(\delta)=\left(\frac{1}{\delta_1^*}\right)^{\delta_1^*}\left(\frac{\left(\gamma_1+\alpha\left(\gamma_2-\gamma_1\right)\right)}{\delta_2^*}\right)^{\delta_2^*}\left(\frac{H_1+\alpha\left(H_2-H_1\right)}{2\delta_3^*}\right)^{\delta_3^*}\left(\frac{a_1+\alpha\left(a_2-a_1\right)}{\delta_4^*}\right)^{\delta_4^*}\left(\frac{w_1+\alpha\left(w_2-w_1\right)}{W}\right)^{\delta_5^*}$$

$$q^*=\frac{2\delta_3^* d^*(\delta)}{\left(H_1+\alpha\left(H_2-H_1\right)\right)}\left(\frac{\delta_5^* W\left(H_1+\alpha\left(H_2-H_1\right)\right)}{2\delta_3^* d^*(\delta)\left(w_1+\alpha\left(w_2-w_1\right)\right)}\right)^{\frac{1}{1-x}}$$

$$r^*=\left(\frac{2\delta_3^* d^*(\delta)\left(w_1+\alpha\left(w_2-w_1\right)\right)}{\delta_5^* W\left(H_1+\alpha\left(H_2-H_1\right)\right)}\right)^{\frac{1}{1-x}}$$

$$D^*=\left[\frac{\delta_2^* d^*(\delta)}{\left(\gamma_1+\alpha\left(\gamma_2-\gamma_1\right)\right)}\left(\frac{\delta_5^* W\left(H_1+\alpha\left(H_2-H_1\right)\right)}{2\delta_3^* d^*(\delta)\left(w_1+\alpha\left(w_2-w_1\right)\right)}\right)^{\frac{\nu-1}{1-x}}\right]^{\frac{1}{1-\beta}}$$

$$S^*=\left[\frac{\delta_1^*\left(a_1+\alpha\left(a_2-a_1\right)\right)}{\delta_4^*}\left(\frac{2\delta_3^* d^*(\delta)\left(w_1+\alpha\left(w_2-w_1\right)\right)}{\delta_5^* W\left(H_1+\alpha\left(H_2-H_1\right)\right)}\right)^{\frac{c}{1-x}}\right]^{\frac{1}{1+b}}$$

Hence optimal solution of GP technique in parametric approach is depends on α.

7. A NUMERICAL EXAMPLE OF EPQ MODEL

Let us considered that a manufacturing company produces a machine such that the inventory carrying cost of the machine is \$ 9 per unit per year. The production cost of the machine varies inversely with the demand. Let the production cost of the machine is $9000D^{-3.25}\ r^{0.65}$, where D and r are demand rate and production process reliability respectively. The total cost of interest and depreciation per production cycle is $950S^{-1.75}\ r^{0.7}$, where S is set-up cost per batch. Let required storage area per unit is $12\,r^{0.65}$ and total available storage space (W) is 99 sq. units. Determine the demand rate (D), set-up cost (S), production quantity (q), production process reliability (r), and optimum total average cost (TC) of the production system.

Formulation of the proposed model is presented as follows

$$Min\ TC(D,S,q,r)=DSq^{-1}r^{-1}+9000D^{1-3.25}r^{0.65-1}+\frac{9qr}{2}+950DS^{-1.75}q^{-1}r^{(0.7-1)}$$

$$subject\ to \qquad 12\ r^{0.6}\,q\le 99 \qquad (7.1)$$

$$D,S,q,r>0$$

The optimum solution of the problem (7.1) by Non-Linear programming (NLP) and Geometric Programming (GP) are presented in Table 1.

Table 1. Optimal solution of crisp model (7.1)

Method	TC^*(\$)	D^*	S^*(\$)	q^*	r^*
GP	99.39587	15.79417	13.57684	10.37070	0.7066305
NLP	99.43568	15.76709	13.61105	10.27278	0.7136518

When the coefficient are taken as TFN, i.e. $\tilde{9000}\equiv(8900,9000,9050)$, $\tilde{9}\equiv(8.75,9,9.5)$, $\tilde{950}\equiv(925,950,980)$ and $\tilde{12}\equiv(11,12,13)$, the optimal solutions of the fuzzy model by fuzzy parametric geometric programming is presented in table 2.

Table 2. Optimal solution of fuzzy model of (7.1)

α	TC^*($)	D^*	S^*($)	q^*	r^*
0.1	97.00980	16.42509	12.75492	12.82113	0.5721587
0.2	97.28188	16.34787	12.85054	12.49780	0.5869352
0.3	97.55331	16.27172	12.94605	12.18553	0.6019459
0.4	97.82409	16.19664	13.04142	11.88387	0.6171904
0.5	98.09423	16.12259	13.13666	11.59235	0.6326712
0.6	98.36374	16.04955	13.23178	11.31056	0.6483886
0.7	98.63263	15.97751	13.32678	11.03809	0.6643443
0.8	98.90091	15.90643	13.42166	10.77454	0.6805394
0.9	99.16859	15.83630	13.51642	10.51955	0.6969750
1.0	99.43568	15.76709	13.61105	10.27278	0.7136518

8. SENSITIVITY ANALYSIS

Table-3 presents the change of optimal solutions of the problem for fuzzy model with small change of tolerance of constraint goal. Table 3 shows that when W increases the total average cost of the given problem slightly decreases, which is expected. So it is clear from the sensitivity analysis that if the management stresses on process reliability then demand and production quantity will be less. It is also noted that the demand and order quantity are increasing with increasing tolerance of storage space but setup cost and process reliability decreases when tolerance increases.

Table 3. Change of objective function and decision variables for change of α

α	Tolerance of W	TC^*($)	D^*	S^*($)	q^*	r^*
0.1	0.5	96.94587	16.47156	12.70218	13.02295	0.5629210
	1	96.88231	16.51792	12.64993	13.22689	0.5538780
	1.5	96.81911	16.56418	12.59814	13.43300	0.5450240
0.3	0. 5	97.48903	16.31775	12.89252	12.37733	0.5922274
	1	97.42511	16.36368	12.83948	12.57118	0.5827128
	1.5	97.36156	16.40951	12.78692	12.76706	0.5733983
0.5	0.5	98.02959	16.16820	13.08235	11.77482	0.6224565
	1	97.96532	16.21371	13.02853	11.95923	0.6124569
	1.5	97.90141	16.25912	12.97519	12.14558	0.6026664
0.7	0.5	98.56764	16.02271	13.27168	11.21183	0.6536184
	1	98.50301	16.06781	13.21708	11.38742	0.6431180
	1.5	98.43875	16.11281	13.16297	11.56487	0.6328369
0.9	0.5	99.10324	15.88110	13.46053	10.68515	0.6857210
	1	99.03827	15.92580	13.40515	10.85247	0.6747061
	1.5	98.97366	15.97040	13.35028	11.02157	0.6639206

9. CONCLUSION

In this paper, an economic production quantity model with investment costs for set-up reduction and reliability consideration is formulated. The model has involved storage space as constraint function. The problem is solved by Fuzzy parametric GP method. The fuzzy parametric geometric programming provides an alternative solution of the problem. The illustrated method is efficient, reliable and also a decision maker may obtain the optimal results according to his expectation. The method presented is quite general and can be applied to the problem in various fields of operations research and engineering.

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