Effects of Violation of Equivalence Principle on UHE Neutrinos at IceCube in 4 Flavour Scenario

Madhurima Pandey[†,‡] [1]

[†] Theory Division, Saha Institute of Nuclear Physics, HBNI
1/AF Bidhannagar, Kolkata 700064, India
[‡] Department of Physics, School of Applied Sciences and Humanities
Haldia Institute of Technology, Haldia, West Bengal, 721657, India

Abstract

If weak equivalence principle is violated then different types of neutrinos would couple differently with gravity and that may produce a gravity induced oscillation for the neutrinos of different flavour. We explore here the possibility that very small violation of the principle of weak equivalence (VEP) can be probed by ultra high energy neutrinos from distant astrophysical sources. The very long baseline length and the ultra high energies of such neutrinos could be helpful to probe very small VEP. We consider a 4-flavour neutrino scenario (3 active + 1 sterile) with both mass-flavour and gravity induced oscillations and compare the detection signatures for these neutrinos (muon tracks and shower events) with and without gravity induced oscillations at a kilometer scale detector such as IceCube. We find that even very small VEP ($\sim 10^{-42}$) can considerably affect the detected muon yield produced by UHE neutrinos from distant Gamma Ray Bursts (GRBs).

[1]email: madhurima0810@gmail.com

1 Introduction

The oscillation of neutrinos [1] from one flavour to another are now established by several terrestrial experiments with neutrinos of natural origin such as solar and atmospheric neutrinos and man-made neutrinos that include reactor [2, 3, 4] or accelerator neutrinos. Due to the mass-flavour mixing of the neutrino eigenstates, a flavour eigenstate after traversing a distance can oscillate into a different flavour due to the phase difference acquired by the mass eigenstates on its propagation. This phase depends on the mass squared differences and the baseline length. Thus discovery of the phenomenology of neutrino oscillation ensures that the neutrinos are massive. Being massive, The neutrinos should also therefore undergo gravitational interactions. In the event that the weak interaction eigenstates of neutrinos are not the same as those of their gravity eigenstates, neutrino oscillations can again be induced if different neutrinos interact with gravity with different strengths, i.e. the gravitational constant G is different for different types of neutrinos. This situation may occur if the principle of weak equivalence is violated [5, 6].

General consequence of the weak equivalence principle is that there is no difference between the gravitational mass and the inertial mass. This is to say that the force experienced by an object grounded on Earth is the same as the force experienced by the same object at the floor of a spaceship which is moving with an acceleration same as that of the acceleration due to gravity in a no gravity environment. This can lead to the phenomenon of gravitational red shift − under the influence of which the wavelength of a radiation suffers a widening (or the energy of a particle is shifted towards a lower energy) while traversing through a gravitational field. The energies of the neutrinos too from a distant astrophysical object such as Gamma Ray Bursts (GRBs) would experience such a gravitational redshift on their travelling to the Earth. The shifted energy is given by $E' = (1 - \phi)E$ [6, 7], where $\phi (= GM/R)$ is the gravitational potential [8] through which the neutrino is propagating. If the equivalence principle is not violated then the energy shifts for all the types of neutrinos are equal and this will not induce any phase difference between two types of neutrinos during its propagation. But if the equivalence is violated then the energy shifts will be different for different types of neutrinos (since the gravitational coupling $G_i (= G\alpha_i$, say) of the neutrino species i is different from $G_j (= G\alpha_j)$, the coupling for the species j). As a result, a pair of neutrino species (i, j) will acquire a phase $\sim \Delta EL$ ($\Delta E = |E_i - E_j|$, E_i and E_j being the redshifted energies of the species i and j respectively) while traversing a distance L (baseline length) from a distant GRB, say, to Earth. Note that E_i, E_j are the energy eigenstates in gravity basis. This would lead to a gravity induced oscillation between neutrinos of different flavour with the oscillatory part given by $\sim |\Delta EL| = |\Delta f_{ij}|LE$ ($|\Delta f_{ij}| = |f_i - f_j|$) with $f_i = (GM/R)\alpha_i = \phi\alpha_i$.

In general there are no specific signatures of violation of weak equivalence principle (VEP) in nature. But in case this is very weakly violated ($|\Delta f_{ij}|$ very small) then depending on the length of the baseline, neutrinos may probe such small VEP. For distant ultra high energy (UHE) neutrino sources such as GRBs, since the baseline length can be of the order of tens or hundreds of $\sim$ Mpc or more, gravity induced neutrino oscillations can be effective for very small violation of equivalence principle (such that the quantity $|\Delta f_{ij}|LE$ is not very small or very very large).

In this work, we consider the UHE neutrinos from a GRB and estimate its flux on reaching the Earth if they suffer both mass induced oscillations/suppressions and gravity induced oscillations. We then estimate the muon yields for these neutrinos at a kilometer square detector such as IceCube [9] and compare our results with similar estimation when no oscillations are considered. For our estimation we consider a four flavour scenario where an extra sterile [10] neutrino is added with the three flavour families .

The paper is organised as follows. In Section 2 we give the formalism for UHE neutrino fluxes from a single GRB (Subsection 2.1) as well as we discuss about the formalism of gravity and mass induced oscillations in the 4-flavour and 3-flavour scenario (Subsection 2.2). In Section 3 we furnish calculational details and results. A discussion and summary is given in Section 4.

2 Formalism

2.1 Astrophysical neutrino fluxes from a single GRB

Ultra high energy neutrinos may be produced via the highly energetic events in Gamma Ray bursts. Although the detailed mechanism of such high energy phenomena are yet to be understood, a well known model for GRB processes is the relativistically expanding fireball model. The fireball is generated and powered when a failed star or supernova leading to formation of a black hole accretes mass from its surroundings and the gravitationally infalling mass bounces back (like supernova explosion) producing shock wave in the outward direction. This highly accelerated outwardly mobile "fireball" contains protons, γ and carries with it enormous amount of energy. The proton - γ interaction yields pions which decays to produce neutrinos through the process

$\pi^+ \to \mu^+ + \nu_\mu$, $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. For a three flavour case, neutrinos are then in the ratio $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$. If we consider a fourth sterile neutrino ν_s (as is considered in this work), then this ratio will be $\nu_e : \nu_\mu : \nu_\tau : \nu_{\rm s} = 1 : 2 : 0 : 0$

The neutrino spectrum from a GRB can be parametrized as [28, 30] (with E_ν, the neutrino energy and N a normalization constant)

$$\frac{dN_\nu}{dE_\nu} = N \times \min\left(1, \frac{E_\nu}{E_\nu^{\rm brk}}\right) \frac{1}{E_\nu^2} \ . \tag{1}$$

The spectral break energy $E_\nu^{\rm brk}$ in the above is related to the photon spectral break energy ($E_{\gamma,{\rm MeV}}^{\rm brk}$) through the Lorentz boost factor (Γ) as

$$E_\nu^{\rm brk} \approx 10^6 \frac{\Gamma_{2.5}^2}{E_{\gamma,{\rm MeV}}^{\rm brk}} {\rm GeV} \ , \tag{2}$$

where $\Gamma_{2.5}$ is the Lorentz boost factor Γ normalized to $10^{2.5}$ ($\Gamma = \Gamma/10^{2.5}$) and

$$N = \frac{E_{\rm GRB}}{1 + \ln({\rm E}_{\nu{\rm max}}/{\rm E}_\nu^{\rm brk})} \ , \tag{3}$$

where $\nu_{\rm max}$ is the upper cut-off energy of the neutrino spectrum. For a GRB at a redshift z is the observed neutrino energy reaching the Earth would be

$$E_\nu^{\rm obs} = \frac{E_\nu}{(1+z)} \ . \tag{4}$$

and the upper cut-off $E_{\nu_{\rm max}}^{\rm obs}$ will then be

$$E_{\nu_{\rm max}}^{\rm obs} = \frac{E_{\nu_{\rm max}}}{(1+z)} \ . \tag{5}$$

In the absence of neutrino oscillation, the flux reaching the Earth will then be written as,

$$\frac{dN_\nu}{dE_\nu^{\rm obs}} = \frac{dN_\nu}{dE_\nu} \frac{1}{4\pi L^2(z)} (1+z) \ , \tag{6}$$

where the baseline length $L(z)$ for a GRB at a redshift Z is expressed as

$$r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_m (1+z')^3}} \ . \tag{7}$$

where $\Omega_\Lambda = 0.685$, $\Omega_m = 0.315$ are the cosmological parameters representing dark energy density and dark matter density normalized to critical density of the Universe and the

Hubble parameter $H_0 = 67.4$ Km sec^{-1} Mpc^{-1} is adopted for the present calculations. The velocity of light is denoted by c in the above equation.

Assuming no CP violation, the neutrino spectra in Eq. (1) $\mathcal{F}(E_\nu) = \frac{dN_\nu}{dE_\nu^{\rm obs}} = \frac{dN_{\nu+\bar{\nu}}}{dE_\nu^{\rm obs}}$ and therefore, we have the flux for neutrinos only, to be $0.5\mathcal{F}(E_\nu)$.

Since the neutrinos are produced at the source in flavour (e, μ, τ and sterile s) ratio 1 : 2 : 0 : 0, the flux for each neutrino flavour at the source can be written as,

$$\phi^s_{\nu_e} = \frac{1}{6}\mathcal{F}(E_\nu)\ , \phi^s_{\nu_\mu} = \frac{2}{6}\mathcal{F}(E_\nu) = 2\phi^s_{\nu_e}\ , \phi^s_{\nu_\tau} = 0\ , \phi^s_{\nu_s} = 0\ . \tag{8}$$

In the 4-flavour framework considered here, the neutrinos experience four flavour oscillations upon reaching the terrestrial detector from the astronomical extragalactic sources. The flux of neutrino flavours on reaching the Earth can be expressed as

$$\begin{aligned} F_{\nu_e} &= P_{ee}\phi^s_{\nu_e} + P_{\mu e}\phi^s_{\nu_\mu}\ , \\ F_{\nu_\mu} &= P_{\mu\mu}\phi^s_{\nu_\mu} + P_{e\mu}\phi^s_{\nu_e}\ , \\ F_{\nu_\tau} &= P_{e\tau}\phi^s_{\nu_e} + P_{\mu\tau}\phi^s_{\nu_\mu}\ , \\ F_{\nu_s} &= P_{es}\phi^s_{\nu_e} + P_{\mu s}\phi^s_{\nu_\mu}\ , \end{aligned} \tag{9}$$

where $P_{\alpha\beta}(\alpha, \beta = e, \mu, \tau, s)$ is the oscillation probability between the flavours α and β and F_{ν_α} is the flux for the neutrinos $\nu_\alpha(\alpha = e, \mu, \tau, s)$ on reaching the Earth for the four flavour case.

2.2 Contribution of VEP to the neutrino oscillation probability

In case of a nonvanishing rest mass of neutrinos the weak and mass eigenstates are not necessarily identical, a fact wellknown in the quark sector where both types of state are connected by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. This non zero mass nature of the neutrino allows for the phenomenon of neutrino oscillations, first given by Pontecorvo [14, 15], and it can be described by quantum mechanics. They are observable as long as the neutrino wave packets from a coherent superposition of states. Such oscillations among the different neutrino flavours do not conserve individual lepton numbers but only total lepton number. So that neutrino oscillation can be expressed as a quantum mechanical phenomenon wherby a neutrino created with a specific lepton family number ("lepton flavour") can later be measured to have a different lepton family number.

The n flavour eigenstate $|\nu_\alpha\rangle$ (with $\langle\nu_\beta|\nu_\alpha\rangle = \delta_{\alpha\beta}$), where n is an arbitrary number of orthonormal eigenstates, are connected to the nth mass eigenstate (with $\langle\nu_i|\nu_j\rangle = \delta_{ij}$)

via a unitary matrix U

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle \ , \tag{10}$$

with

$$\sum_i U_{\alpha i}U^*_{\beta i} = \delta_{\alpha\beta},\ \sum_\alpha U_{\alpha i}U^*_{\alpha j} = \delta_{ij}\ . \tag{11}$$

For the 4 (3 active +1 sterile) flavour scenario, the neutrino flavour eigenstates are related to the mass eigenstates through a 4×4 unitary matrix given as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = \tilde{U}_{(4\times4)} \begin{pmatrix} \nu_1 \\ \nu_1 \\ \nu_3 \\ \nu_4 \end{pmatrix} = \begin{pmatrix} \tilde{U}_{e1} & \tilde{U}_{e2} & \tilde{U}_{e3} & \tilde{U}_{e4} \\ \tilde{U}_{\mu1} & \tilde{U}_{\mu2} & \tilde{U}_{\mu3} & \tilde{U}_{\mu4} \\ \tilde{U}_{\tau1} & \tilde{U}_{\tau2} & \tilde{U}_{\tau3} & \tilde{U}_{\tau4} \\ \tilde{U}_{s1} & \tilde{U}_{s2} & \tilde{U}_{s3} & \tilde{U}_{s4} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_1 \\ \nu_3 \\ \nu_4 \end{pmatrix} \ , \tag{12}$$

where $\tilde{U}_{\alpha i}$ etc. indicate the elements of the Pontecorvo-Maki-Nakigawa-Sakata (PMNS) matrix [16]. The PMNS matrix $\tilde{U}_{(4\times4)}$ can be generated by considering the successive rotations (R) in terms of mixing angles θ_{14}, θ_{24}, θ_{34}, θ_{13}, θ_{12}, θ_{23} [17, 18]

$$\tilde{U}_{(4\times4)} = R_{34}(\theta_{34})R_{24}(\theta_{24})R_{14}(\theta_{14})R_{23}(\theta_{23})R_{13}(\theta_{13})R_{12}(\theta_{12}) \ , \tag{13}$$

Since we consider no CP violation in the neutrino sector, the CP phases are absent. The successive rotation terms (R) for 4-flavour case is written as

$$\begin{aligned} R_{34}(\theta_{34}) &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix}, R_{24}(\theta_{24}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24} & 0 & c_{24} \end{pmatrix}, \\ R_{14}(\theta_{14}) &= \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix}, R_{12}(\theta_{12}) = \begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\ R_{13}(\theta_{13}) &= \begin{pmatrix} c_{13} & 0 & s_{13} & 0 \\ 0 & 1 & 0 & 0 \\ -s_{13} & 0 & c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, R_{23}(\theta_{23}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 \\ 0 & -s_{23} & c_{23} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{14}$$

With Eq. (14) $\tilde{U}_{(4\times4)}$ can be expressed as

$$\tilde{U}_{(4\times4)} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ -s_{14}s_{24} & c_{24} & 0 & c_{14}s_{24} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & c_{34} & c_{14}c_{24}s_{34} \\ -c_{24}s_{14}c_{34} & -s_{24}c_{34} & -s_{34} & c_{14}c_{24}c_{34} \end{pmatrix} \times \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & 0 \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & 0 \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (15)$$

$$= \begin{pmatrix} c_{14}U_{e1} & c_{14}U_{e2} & c_{14}U_{e3} & s_{14} \\ -s_{14}s_{24}U_{e1}+c_{24}U_{\mu1} & -s_{14}s_{24}U_{e2}+c_{24}U_{\mu2} & -s_{14}s_{24}U_{e3}+c_{24}U_{\mu3} & c_{14}s_{24} \\ \begin{array}{c} -c_{24}s_{14}s_{34}U_{e1} \\ -s24s34U_{\mu1} \\ +c_{34}U_{\tau1} \end{array} & \begin{array}{c} -c_{24}s_{14}s_{34}U_{e2} \\ -s24s34U_{\mu2} \\ +c_{34}U_{\tau2} \end{array} & \begin{array}{c} -c_{24}s_{14}s_{34}U_{e3} \\ -s24s34U_{\mu3} \\ +c_{34}U_{\tau3} \end{array} & c_{14}c_{24}s_{34} \\ \begin{array}{c} -c_{24}c_{34}s_{14}U_{e1} \\ -s_{24}c_{34}U_{\mu1} \\ -s_{34}U_{\tau1} \end{array} & \begin{array}{c} -c_{24}c_{34}s_{14}U_{e2} \\ -s_{24}c_{34}U_{\mu2} \\ -s_{34}U_{\tau2} \end{array} & \begin{array}{c} -c_{24}c_{34}s_{14}U_{e3} \\ -s_{24}c_{34}U_{\mu3} \\ -s_{34}U_{\tau3} \end{array} & c_{!4}c_{24}c_{34} \end{pmatrix} \quad (1,6)$$

where $U_{\alpha i}$ are the PMNS matrix elements for 3 flavour mixing matrix U, which can be expressed as [19, 20, 21]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13} & c_{12}c_{23}-s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13} & -c_{12}s_{23}-s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \quad (17)$$

In Eqs. (14-17) , $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, where θ_{ij} is the mixing angle between ith and jth neutrinos having mass eigenstates $|\nu_i\rangle$ and $|\nu_j\rangle$.

The time evolution equation of neutrino mass eigen state (for 4 neutrino case) is given by

$$i\frac{d}{dt}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \end{pmatrix} = H_m \begin{pmatrix} \nu_1 \\ \nu_1 \\ \nu_3 \\ \nu_4 \end{pmatrix} = \begin{pmatrix} E_1 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_3 & 0 \\ 0 & 0 & 0 & E_4 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_1 \\ \nu_3 \\ \nu_4 \end{pmatrix}. \quad (18)$$

In the flavour basis with $|\nu_\alpha\rangle = \tilde{U}_{(4\times4)}|\nu_i\rangle$ the evolution equation takes the form

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = H' \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix}, \quad (19)$$

where

$$H' = \tilde{U}_{(4\times4)} H_m \tilde{U}^\dagger_{(4\times4)} \ . \tag{20}$$

For the case of relativistic meutrinos of momentum p, the energy eigen value for $|\nu_i\rangle$ can be expressed as

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2p_i} \simeq p + \frac{m_i^2}{2E} \ , \tag{21}$$

where $p_i \simeq p$, (i = 1,2,3,4) is assumed. Now by using Eq. (21) we can write H_m as

$$H_m = \begin{pmatrix} p & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \frac{1}{2E} \begin{pmatrix} m_1^2 & 0 & 0 & 0 \\ 0 & m_2^2 & 0 & 0 \\ 0 & 0 & m_3^2 & 0 \\ 0 & 0 & 0 & m_4^2 \end{pmatrix} \ . \tag{22}$$

In the above Eq. (22), we can neglect the matrix diag(p, p, p, p) as it does not induce any phase differences between the neutrinos and hence does not contribute to the neutrino oscillations. Substract m_1^2 from all the diagonal elements of the matrix diag$(m_1^2, m_2^2, m_3^2, m_4^2)$, Eq. (22) takes the form

$$H_m = \frac{1}{2E}\text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2, \Delta m_{41}^2) \ . \tag{23}$$

As mentioned earlier, the oscillation of neutrinos can also be induced in case the weak equivalence principle is violated in nature. In such a scenario, the gravitational couplings to different types of neutrinos will be different. Therefore in this case the gravitational constant G no more remains same for different types of neutrinos. If the neutrino eigenstates in gravity basis $|\nu_{G_i}\rangle$ are not the same as the flavour eigenstates $|\nu_\alpha\rangle$ of neutrinos then this can lead to neutrino oscillations even though neutrinos are massless. In the present work however we consider $|\nu_\alpha\rangle \neq |\nu_i\rangle \neq |\nu_{Gi}\rangle$ ($|\nu_{Gi}\rangle$ being the gravity eigenstate for neutrino i) such that both the mass flavour oscillations and gravity induced oscillations are explored in a single framework.

In general, no positive signatures have been found for the violation of the weak equivalence principle. In the event that the equivalence principle is violated by a very small account then this may be detected by studying the gravity induced oscillations of neutrino. The effect can be manifested for the neutrinos with the very long basline ($\sim$ Mpc). The UHE neutrinos from distant high energy extragalactic sources can well be a possibility to test the VEP.

In the theory of general relativity the equivalence principle is the equivalence of gravitational and inertial mass. The gravitational "force" as experienced locally while standing on a massive body (such as the Earth) is the same as the pseudo force experienced by an observer in a noninertial (accelerated) frame of reference. Therefore equivalence principle is violated if the universality of the gravitational constant G is no more valid. A consequence of the equivalence principle is that an object with an energy E in a gravitational field will suffer a shift in energy in the same way as would be observed in an accelerated frame of reference in a no gravity environment. If we assume a weak and static gravitational field then this can be shown that for such a field, the metric is diagonal with $g_{00} = (1 + 2\phi)$. with $\phi(= -\frac{GM}{R})$ is the gravitational potential and where R is the distance over which the gravitation field is operational and M is the mass of the source of the gravitational field. The energy from an object in this potential ϕ will be redshifted by an amount given by $E' = \sqrt{g_{00}}E = E(1 - \frac{GM}{R}) = E(1 + \phi)$ [2].

Let us consider the case where in a neutrino oscillation experiments, the neutrinos from a distant astrophysical object propagate through a gravitational field in addition to the vacuum. With respect to the vacuum, the neutrino energies are redshifted (due to the Doppler effect) by an amount $E \to E' = \sqrt{g_{00}}E$. But because of the universality of the gravitaional coupling, the equivalence principle indicates that for all the neutrino types the energy shifts should be the same and therefore it cann not lead to any neutrino oscillations. Only the non-universality of the coupling of gravity to the neutrino field, because of the consequence of the possible violation of equivalence principle, can contribute to the neutrino flavour oscillations.

In the presence of the gravitational field, the flavour eigenstates $|\nu_\alpha\rangle(\alpha = e, \mu, \tau, s)$ can be expressed as the superpositions of the gravitational eigenstate $|\nu_{Gi}\rangle(i = 1, 2, 3, 4)$ in terms of the mixing angle parameters $\theta'_{ij}(i \neq j), i, j = 1, 2, 3, 4$ in the 4-flavour framework.

$$|\nu_\alpha\rangle \;=\; \tilde{U}'_{(4\times4)}|\nu_{Gi}\rangle \;, \tag{24}$$

[2]The relation $E' = \sqrt{g_{00}}E$ can be realised by noting that the proper time in a curved manifold (presence of gravitation) is $d\tau = \sqrt{g_{\mu\nu}dx^\mu dx^\nu}$. Now the proper time is related to the coordinate time by $d\tau = \sqrt{g_{00}}dt$ (clock is at rest). If N number of waves are emitted from a distant star with frequency $f_{\rm star}$ and proper time interval $\Delta\tau_{\rm star}$ and if the same are detected at Earth with frequency $f_{\rm Earth}$ with proper time interval $\Delta\tau_{\rm Earth}$ then $\frac{f_{\rm star}}{f_{\rm Earth}} = \frac{\Delta\tau_{\rm star}}{\Delta\tau_{\rm Earth}} = \frac{\sqrt{g_{00}(x_{\rm Earth})}}{\sqrt{g_{00}(x_{\rm star})}} = \sqrt{\left(\frac{1+2\phi_{\rm Earth}}{1+2\phi_{\rm star}}\right)} = 1 + |\Delta\phi|$.

where $\tilde{U}^{'}_{(4\times4)}$ represents the flavour-gravity mixing matrix in 4-flavour scenario

$$\tilde{U}^{'}_{(4\times4)} = \begin{pmatrix} c^{'}_{14}U'_{e1} & c^{'}_{14}U'_{e2} & c^{'}_{14}U'_{e3} & s^{'}_{14} \\ -s^{'}_{14}s_{24}U'_{e1}+c^{'}_{24}U'_{\mu1} & -s^{'}_{14}s_{24}U'_{e2}+c^{'}_{24}U'_{\mu2} & -s^{'}_{14}s_{24}U'_{e3}+c^{'}_{24}U'_{\mu3} & c^{'}_{14}s_{24} \\ \begin{array}{c}-c^{'}_{24}s_{14}s^{'}_{34}U'_{e1}\\ -s'24s'34U'_{\mu1}\\ +c^{'}_{34}U'_{\tau1}\end{array} & \begin{array}{c}-c^{'}_{24}s^{'}_{14}s^{'}_{34}U'_{e2}\\ -s'24s'34U'_{\mu2}\\ +c^{'}_{34}U'_{\tau2}\end{array} & \begin{array}{c}-c^{'}_{24}s^{'}_{14}s^{'}_{34}U'_{e3}\\ -s'24s'34U'_{\mu3}\\ +c^{'}_{34}U'_{\tau3}\end{array} & c^{'}_{14}c^{'}_{24}s^{'}_{34} \\ \begin{array}{c}-c^{'}_{24}c^{'}_{34}s^{'}_{14}U'_{e1}\\ -s^{'}_{24}c^{'}_{34}U'_{\mu1}\\ -s^{'}_{34}U'_{\tau1}\end{array} & \begin{array}{c}-c^{'}_{24}c^{'}_{34}s^{'}_{14}U'_{e2}\\ -s^{'}_{24}c^{'}_{34}U'_{\mu2}\\ -s^{'}_{34}U'_{\tau2}\end{array} & \begin{array}{c}-c^{'}_{24}c^{'}_{34}s^{'}_{14}U'_{e3}\\ -s^{'}_{24}c^{'}_{34}U'_{\mu3}\\ -s^{'}_{34}U'_{\tau3}\end{array} & c^{'}_{14}c^{'}_{24}c^{'}_{34} \end{pmatrix} \quad (25)$$

The evolution equation for $|\nu_{Gi}\rangle$ can be written as

$$i\frac{d}{dt}|\nu_{Gi}\rangle = H_{Gi}|\nu_G\rangle , \quad (26)$$

where $H_G = \text{diag}(E_{G1}, E_{G2}, E_{G3}, E_{G4})$ for 4 neutrino framework. Therefore the evolution equation for the flavour eigenstate ($|\nu_\alpha\rangle$) for the case of massless neutrinos is written as

$$i\frac{d}{dt}|\nu_\alpha\rangle = \tilde{U}'_{(4\times4)}H_G\tilde{U}'^{\dagger}_{(4\times4)}|\nu_\alpha\rangle . \quad (27)$$

As discussed earlier, in the absence of any violation of equivalence principle all the gravitational energy eigenvalues ($E_G = \sqrt{g_{00}}E = (1 - \frac{GM}{R})E$) will not induce any phase difference to the neutrino eigenstate after the propagation. But if the equivalence principle is violated, the gravitational coupling G is different for different types of neutrinos and in that case we have $H_G = \text{diag}\left((1-\phi\alpha_1)E, (1-\phi\alpha_2)E, (1-\phi\alpha_3)E, (1-\phi\alpha_4)E\right)$, where $\frac{G_iM}{r} = \frac{GM}{r}\alpha_i = \phi\alpha_i$. Therefore this will induce the phase differences $\Delta E_{ij,G}$, where

$$\Delta E_{ij,G} = \frac{GM}{R}\Delta\alpha_{ij}E = \phi\Delta\alpha_{ij}E , \quad (28)$$

where $\Delta\alpha_{ij} = |\alpha_i - \alpha_j|$. In what follows we use U' and U to signify the mixing matrix $\tilde{U}'_{(4\times4)}$ and $\tilde{U}_{(4\times4)}$ respectively. The effective Hamiltonian of the system, which includes the contribution of both mass and gravitational mixing terms, can then be written as

$$H'' = UH_mU^\dagger + U'H_GU'^\dagger . \quad (29)$$

It may be noted that for the UHE neutrinos with energies TeV and above the matter effect (MSW effect) for the neutrinos passing through the matter will not have only significance on neutrino oscillations. Thus in case equivalence principle is violated, the UHE neutrino from distant astrophysical objects will suffer only vacuum and gravity induced oscillations/suppressions. In our formalism, we assume that the mixing angle between mass and the flavour states and the mixing angle between the flavour and the gravitational eigenstate are same, i.e. $\tilde{U}_{(4\times4)} = \tilde{U}'_{(4\times4)}$. Then the Hamiltonian with this assumption takes the form

$$H'' = U(H_m + H_G)U^\dagger , \tag{30}$$

where

$$H_G = \mathrm{diag}(0, \Delta f_{21}E, \Delta f_{31}E, \Delta f_{41}E) . \tag{31}$$

In Eq. (31) $\Delta f_{ij} = \Delta\alpha_{ij}\phi, i, j = 1, 2, 3, 4$ and $\tilde{U}_{(4\times4)} = \tilde{U}'_{(4\times4)}U$. So finally H'' can be written as

$$\begin{aligned} H'' &= U\,\mathrm{diag}(0, \frac{\Delta m^2_{21}}{2E} + \Delta f_{21}E, \frac{\Delta m^2_{31}}{2E} + \Delta f_{31}E, \frac{\Delta m^2_{41}}{2E} + \Delta f_{41}E)U^\dagger \\ &= \frac{1}{2E}U\,\mathrm{diag}(0, \Delta\mu^2_{21}, \Delta\mu^2_{31}, \Delta\mu^2_{41})U^\dagger , \end{aligned} \tag{32}$$

where

$$\begin{aligned} \Delta\mu^2_{21} &= \Delta m^2_{21} + 2\Delta f_{21}E^2 \\ \Delta\mu^2_{31} &= \Delta m^2_{31} + 2\Delta f_{31}E^2 \\ \Delta\mu^2_{41} &= \Delta m^2_{41} + 2\Delta f_{41}E^2 . \end{aligned} \tag{33}$$

Generally, the oscillation probability from a neutrino $|\nu_\alpha\rangle$ of flavour α to a neutrino $|\nu_\beta\rangle$ of flavour β can be expressed as [22, 23]

$$P_{\nu_\alpha\to\nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}\sin^2\left(\frac{\pi L}{\lambda_{ij}}\right) , \tag{34}$$

with $U_{\alpha i}$, etc. are the elements of PMNS mixing matrix. In Eq. (32) L defines the baseline length for the neutrinos (which in the present case $\sim$ Mpc for UHE neutrinos from distant GRBs) and λ_{ij} is the oscillation length. In the presence of both mass and graviaty induced oscillations, λ_{ij} is given by

$$\lambda_{ij} = \frac{4\pi E}{\Delta\mu^2_{ij}} = \frac{4\pi E}{\left(\Delta m^2_{ij} + 2\Delta f_{ij}E^2\right)} . \tag{35}$$

Eq. (32) reduces to the form

$$P_{\nu_\alpha \to \nu_\beta} = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} S_{ij}^2 \ . \tag{36}$$

With $S_{ij}^2 = \sin^2\left[\left(\frac{\Delta m_{ij}^2}{4E} + \frac{\Delta f_{ij} E}{2}\right) L\right]$. In the 4-flavour scenario, the mass and the gravity induced oscillation probabilities are therefore expressed as

$$\begin{aligned}
P_{ee}^4 &= 1 - 4[|U_{e2}|^2|U_{e1}|^2 S_{21}^2 + (|U_{e3}|^2|U_{e1}|^2 + |U_{e3}|^2|U_{e2}|^2)S_{32}^2 \\
&\quad +(|U_{e4}|^2|U_{e1}|^2 + |U_{e4}|^2|U_{e2}|^2)S_{42}^2 + |U_{e4}|^2|U_{e3}|^2 S_{43}^2] \\
P_{e\mu}^4 &= 4[|U_{e2}||U_{\mu 2}||U_{e1}||U_{\mu 1}|S_{21}^2 + (|U_{e3}||U_{\mu 3}||U_{e2}||U_{\mu 2}| + |U_{e3}||U_{\mu 3}||U_{e1}||U_{\mu 1}|)S_{32}^2 + \\
&\quad (|U_{e4}||U_{\mu 4}||U_{e2}||U_{\mu 2}| + |U_{e4}||U_{\mu 4}||U_{e1}||U_{\mu 1}|)S_{42}^2 + |U_{e4}||U_{\mu 4}||U_{e3}||U_{\mu 3}|S_{43}^2] \\
P_{e\tau}^4 &= 4[|U_{e2}||U_{\tau 2}||U_{e1}||U_{\tau 1}|S_{21}^2 + (|U_{e3}||U_{\tau 3}||U_{e2}||U_{\tau 2}| + |U_{e3}||U_{\tau 3}||U_{e1}||U_{\tau 1}|)S_{32}^2 + \\
&\quad (|U_{e4}||U_{\tau 4}||U_{e2}||U_{\tau 2}| + |U_{e4}||U_{\tau 4}||U_{e1}||U_{\tau 1}|)S_{42}^2 + |U_{e4}||U_{\tau 4}||U_{e3}||U_{\tau 3}|S_{43}^2] \\
P_{es}^4 &= 4[|U_{e2}||U_{s2}||U_{e1}||U_{s1}|S_{21}^2 + (|U_{e3}||U_{s3}||U_{e2}||U_{s2}| + |U_{e3}||U_{s3}||U_{e1}||U_{s1}|)S_{32}^2 + \\
&\quad (|U_{e4}||U_{s4}||U_{e2}||U_{s2}| + |U_{e4}||U_{s4}||U_{e1}||U_{s1}|)S_{42}^2 + |U_{e4}||U_{s4}||U_{e3}||U_{s3}|S_{43}^2] \\
P_{\mu\mu}^4 &= 1 - 4[|U_{\mu 2}|^2|U_{\mu 1}|^2 S_{21}^2 + (|U_{\mu 3}|^2|U_{\mu 1}|^2 + |U_{\mu 3}|^2|U_{\mu 2}|^2)S_{32}^2 \\
&\quad +(|U_{\mu 4}|^2|U_{\mu 1}|^2 + |U_{\mu 4}|^2|U_{\mu 2}|^2)S_{42}^2 + |U_{\mu 4}|^2|U_{\mu 3}|^2 S_{43}^2] \\
P_{\mu\tau}^4 &= 4[|U_{\mu 2}||U_{\tau 2}||U_{\mu 1}||U_{\tau 1}|S_{21}^2 + (|U_{\mu 3}||U_{\tau 3}||U_{\mu 2}||U_{\tau 2}| + |U_{\mu 3}||U_{\tau 3}||U_{\mu 1}||U_{\tau 1}|)S_{32}^2 + \\
&\quad (|U_{\mu 4}||U_{\tau 4}||U_{\mu 2}||U_{\tau 2}| + |U_{\mu 4}||U_{\tau 4}||U_{\mu 1}||U_{\tau 1}|)S_{42}^2 + |U_{\mu 4}||U_{\tau 4}||U_{\mu 3}||U_{\tau 3}|S_{43}^2] \\
P_{\mu s}^4 &= 4[|U_{\mu 2}||U_{s2}||U_{\mu 1}||U_{s1}|S_{21}^2 + (|U_{\mu 3}||U_{s3}||U_{\mu 2}||U_{s2}| + |U_{\mu 3}||U_{s3}||U_{\mu 1}||U_{s1}|)S_{32}^2 + \\
&\quad (|U_{\mu 4}||U_{s4}||U_{\mu 2}||U_{s2}| + |U_{\mu 4}||U_{s4}||U_{\mu 1}||U_{s1}|)S_{42}^2 + |U_{\mu 4}||U_{s4}||U_{\mu 3}||U_{s3}|S_{43}^2] \\
P_{\tau\tau}^4 &= 1 - 4[|U_{\tau 2}|^2|U_{\tau 1}|^2 S_{21}^2 + (|U_{\tau 3}|^2|U_{\tau 1}|^2 + |U_{\tau 3}|^2|U_{\tau 2}|^2)S_{32}^2 \\
&\quad +(|U_{\tau 4}|^2|U_{\tau 1}|^2 + |U_{\tau 4}|^2|U_{\tau 2}|^2)S_{42}^2 + |U_{\tau 4}|^2|U_{\tau 3}|^2 S_{43}^2] \\
P_{\tau s}^4 &= 4[|U_{\tau 2}||U_{s2}||U_{\tau 1}||U_{s1}|S_{21}^2 + (|U_{\tau 3}||U_{s3}||U_{\tau 2}||U_{s2}| + |U_{\tau 3}||U_{s3}||U_{\tau 1}||U_{s1}|)S_{32}^2 + \\
&\quad (|U_{\tau 4}||U_{s4}||U_{\tau 2}||U_{s2}| + |U_{\tau 4}||U_{s4}||U_{\tau 1}||U_{s1}|)S_{42}^2 + |U_{\tau 4}||U_{s4}||U_{\tau 3}||U_{s3}|S_{43}^2] \\
P_{ss}^4 &= 1 - 4[|U_{s2}|^2|U_{s1}|^2 S_{21}^2 + (|U_{s3}|^2|U_{s1}|^2 + |U_{s3}|^2|U_{s2}|^2)S_{32}^2 \\
&\quad +(|U_{s4}|^2|U_{s1}|^2 + |U_{s4}|^2|U_{s2}|^2)S_{42}^2 + |U_{s4}|^2|U_{s3}|^2 S_{43}^2] \ .
\end{aligned} \tag{37}$$

Similar probability equations cn be easily written for 3-flavour case considering the 3-flavour PMNS matrix Eq. (17).

2.3 Detection of secondary muons produced from neutrino-nucleon interactions of diffuse GRB sources

We consider here a kilometer square detector such as IceCube for the detection of ultra high energy neutrinos from the GRBs. Upward going muons observed by the Super - Kamiokande detector are produced by the interactions between high energy atmospheric neutrinos, such as UHE neutrinos from distant extragalactic sources namely GRBs and the rock around the detector. For the case of detection of UHE neutrinos at a Km^2 detector like IceCube, we are looking for these upward going muons, whose production depends on neutrino (ν_μ) charged current interactions ($\nu_\mu + N \to \mu + X$). The most promising advantage of considering upward going muons is that it cannot be misidentified as muons created in cosmic ray showers in the atmosphere.

The rate of upward going muon event from single GRB neutrino depends on $\nu_\mu N$ cross-sections in two different ways namely

i) The interaction length, which is a function of the total cross-section, that leads the attenuation of the neutrino flux due to interactions in the Earth and

ii) The probability that the neutrino induced muon arriving at the detector with an energy larger than the threshold energy $E_\mu^{\rm min}$.

For the UHE neutrino flux, we can represent the attenuation of the neutrinos, reaching the terrestrial detector being unabsorbed by the Earth, by a shadow factor ($S_{\rm shadow}(E_\nu)$). For a particular GRB, the zenith angle θ_z is fixed. Thus the shadow factor for a single GRB is given by

$$S_{\rm shadow}(E_\nu) = \exp[-z(\theta_z)/L_{\rm int}(E_\nu)] \ , \tag{38}$$

where $z(\theta_z)$ is the column depth for the incident zenith angle θ_z of the neutrinos

$$z(\theta_z) = \int \rho(r(\theta_z, l))dl \ . \tag{39}$$

In Eq. (39) $\rho(r(\theta_z, l))$ (l is the path length of neutrino in the Earth) indicates the matter density profile inside the Earth. We have taken Preliminary Earth Model (PREM) [24] to express the matter density profile of the Earth in a more convenient way as we consider Earth as a spherically symmetric ball in our work (dense inner and outer core and a lower mantle having medium density).

The interaction length ($L_{\rm int}(E_\nu)$) in Eq. (38) can be expressed as

$$L_{\rm int}(E_\nu) = \frac{1}{\sigma^{\rm tot}(E_\nu) N_A} \ , \tag{40}$$

where $\sigma^{\rm tot}$ corresponds to the total (charge current ($\sigma_{\rm CC}$) + neutral current ($\sigma_{\rm NC}$)) cross-section and N_A represents the Avogadro number N_A ($= 6.023 \times 10^{23}{\rm mol}^{-1} = 6.023 \times 10^{23}{\rm gm}^{-1}$).

The probability $P_\mu(E_\nu; E_\mu^{\rm min})$ for a muon, produced due to charge current interactions of neutrinos, reaching the detector having energy above $E_\mu^{\rm min}$ is expressed as

$$P_\mu(E_\nu; E_\mu^{\rm min}) = N_A \sigma^{\rm cc}(E_\nu)\langle R(E_\nu; E_\mu^{\rm min})\rangle \ , \qquad (41)$$

where the average range of muon in rock ($\langle R(E_\nu; E_\mu^{\rm min})\rangle$) is given as [25]

$$\langle R(E_\nu; E_\mu^{\rm min})\rangle \ = \ \frac{1}{\sigma_{\rm CC}} \int_0^{(1-E_\mu^{\rm min}/E_\nu)} dy R(E_\nu(1-y); E_\mu^{\rm min}) \times \frac{d\sigma_{\rm CC}(E_\nu, y)}{dy} \ . \qquad (42)$$

We can write E_μ in the place of $E_\nu(1-y)$ in Eq. (42) as $y(= (E_\nu - E_\mu)/E_\nu)$, defines the fraction of energy lost by a neutrino having energy E_ν in the production of secondary muons having energy E_μ via charge current interactions. The muon range $R(E_\mu; E_\mu^{\rm min})$ in Eq. (42) can be wriiten as

$$R(E_\mu; E_\mu^{\rm min}) = \int_{E_\mu^{\rm min}}^{E_\mu} \frac{dE_\mu}{\langle dE_\mu/dX\rangle} \simeq \frac{1}{\beta}\ln\left(\frac{\alpha + \beta {\rm E}_\mu}{\alpha + \beta {\rm E}_\mu^{\rm min}}\right) \ . \qquad (43)$$

The energy loss rate of muon having energy is expressed as [26]

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu \ , \qquad (44)$$

where the constant α stands for the energy losses and β describes the catastrophic losses (namely bremsstrahlung, pair production and hadron production) respectively. Now these two constants we have considered in our work are for $E_\mu \ \leq \ 10^6$ GeV [27]

$$\begin{aligned} \alpha &= 2.033 + 0.077 \ \ln[E_\mu(GeV)] \times 10^3 \ {\rm GeV\ cm^2\ gm^{-1}} \ , \\ \beta &= 2.033 + 0.077 \ \ln[E_\mu(GeV)] \times 10^{-6} \ {\rm GeV\ cm^2\ gm^{-1}} \ , \end{aligned} \qquad (45)$$

and otherwise [28]

$$\begin{aligned} \alpha \ &= \ 2.033 \times 10^{-3} \ {\rm GeV\ cm^2\ gm^{-1}} \, , \\ \xi \ &= \ 3.9 \times 10^{-6} \ {\rm GeV\ cm^2\ gm^{-1}} \, . \end{aligned} \qquad (46)$$

As has already been mentioned, the detection of ν_μ's from a distant GRB sources can be estimated from the tracks of the secondary muons. The total number of secondary muon yields, which is a function of both $S_{\text{shadow}}(E_\nu)$ and $P_\mu(E_\nu; E_\mu^{\text{min}})$, can be detected in a detector such as IceCube of unit area is ([29], [26], [30])

$$S = \int_{E_\mu^{\text{min}}}^{E_{\nu_{\text{max}}}} dE_\nu S_{\text{shadow}}(E_\nu) P_\mu(E_\nu; E_\mu^{\text{min}}) \frac{dN_\nu}{dE_\nu} \ . \tag{47}$$

We replace $\frac{dN_\nu}{dE_\nu}$ in Eq. (47) by F_{ν_μ}, mentioned in Eqs. (3), (33). We also consider the production of muons via the decay channel $\nu_\tau \to \tau \to \bar{\nu_\mu}\mu\nu_\tau$ with probability 0.18. In such cases we can compute the muon events by solving Eqs. (34)-(43) numerically, where $\frac{dN_\nu}{dE_\nu}$ in Eq. (43) is equivalent to F_{ν_τ} (Eqs. (3),(33)).

3 Calculations and Results

This work explores the possibility that the UHE neutrinos from distant GRBs may be effective in probing even a very small violation of equivalence principle. As discussed earlier, if equivqlence principle is violated, the different coupling strengths of different types of neutrinos with gravity can in turn induce a gravity induced oscillation among neutrino flavours. In such a scenario we compute here the neutrino induced muon yields for thesse neutrinos from single GRBs, in a square kilometer Cerenkov detector such as IceCube and compare with the computed values for the same in case when the equivalence principle is not violated. We consider a 4-flavour (3 flavour + 1 sterile) neutrino framework for all our estimations and then compare our results with similar calculations considering the usual three active flavour neutrino oscillation scenario.

In the presence of gravity induced oscillations with the usual mass flavour oscillations, we can calculate the neutrino induced secondary muon yield at a Km^2 IceCube detector for 4-flavour UHE neutrinos by using Eqs. (1) - (37) in section 1.1 and 1.2 and Eqs. (38) - (47) in section 1.3. For this purpose, we consider upward going muons, which are produced due to charged current interactions of UHE neutrinos in the detector or in the rocks during their passage to the detector. The threshold energy of the detector has been taken as $E_\mu^{\text{min}} = 1$ TeV.

In the present calculations, we have chosen the best fit values of the three active mixing angles given by $\theta_{12} = 33.48°, \theta_{23} = 45°, \theta_{13} = 8.5°$. Different neutrino experimental groups such as MINOS [31]-[42], Daya Bay [43]-[49], Bugey [50], NOVA [51]-[56] have put limits on the flavour mixing angles $(\theta_{14}, \theta_{24}, \theta_{34})$ in 4-flavour scheme. The upper limits of the four flavour neutrino mixing angles have obtained by NOVA as $\theta_{24} \leq 20.8^0$ and $\theta_{34} \leq 31.2^0$ for $\Delta m_{41}^2 = 0.5$ eV^2. For the same value of Δm_{41}^2, MINOS has proposed the upper limits on θ_{34} and θ_{24} as $\theta_{24} \leq 7.3^0$ and $\theta_{34} \leq 26.6^0$. In addition to the above mentioned experimental groups the IceCube-Deepcore results [57] suggest $\theta_{24} \leq 19.4^0$ and $\theta_{34} \leq 22.8^0$ for $\Delta m^2_{41} = 1$ eV^2. In the present work we adopt the ranges for θ_{24}, θ_{34} and θ_{14} to be $2^0 \leq \theta_{24} \leq 20^0$, $2^0 \leq \theta_{34} \leq 20^0$ and $1^0 \leq \theta_{14} \leq 4^0$ respectively and the limits on θ_{14} is consistent with the combined results obtained from MINOS, Daya Bay and Bugey experiments. In our calcuation we consider θ_{14}, θ_{24} and θ_{34} as $4^0, 6^0$ and 15^0 respectively.

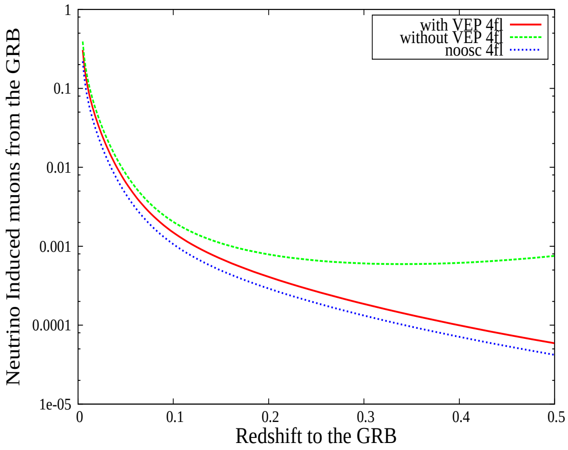
The main motivation of our work is to demonstrate the effects of the gravity induced oscillations, in case of a possible violation of weak equivalence principle. Thus our approach is to consider both the usual mass-flavour oscillation/suppression and the gravity induced oscillation in the same framework. We have also considered a 4 flavour scenario where an extra sterile neutrino is included in the oscillation formalism. We then demonstrate the effect of both the 4-flavour scenario and VEP with no VEP case vis-a-vis the similar effects for the usual three flavour scenario.

The calculations are performed for the representative values (of Δf_{ij}, the amount of VEP) $\Delta f_{21} = 10^{-43}, \Delta f_{32} = 10^{-42}, \Delta f_{41} = 10^{-43}, \Delta f_{43} = 10^{-42}$ and we have considered Δm^2_{32} and Δm^2_{21} as $\Delta m^2_{32} = 7.0 \times 10^{-5}$eV^2 (from solar neutrino oscillation experimental results) and $\Delta m^2_{32} = 2.4 \times 10^{-3}$eV^2 (from atmospheric neutrino oscillation results) respectively. The values of the mass square differences in the 4-flavour cases such as Δm^2_{41} lie within the range $0.2\,\text{eV}^2 \leq \Delta m^2_{41} \leq 2\,\text{eV}^2$ and we assume that $\Delta m^2_{32} \simeq \Delta m^2_{31} \simeq 2.4 \times 10^{-3}$eV^2 and $\Delta m^2_{32} \simeq \Delta m^2_{31} \simeq 1\,\text{eV}^2$.

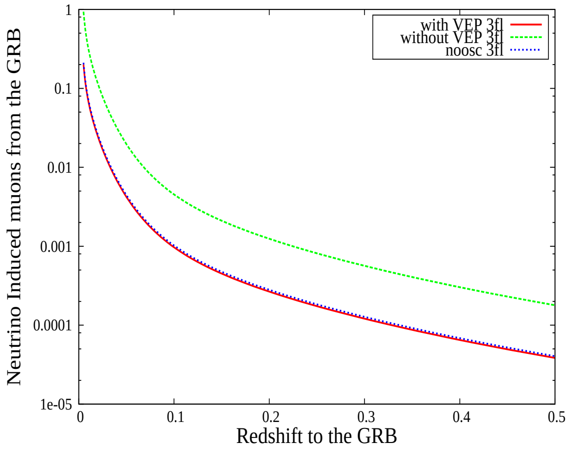
In Fig. 1, we furnish the variations of upward going muon yields at the IceCube like detector with different single GRBs at different redshift distances. The results are compared with the cases for only mass-flavour oscillation (no gravity induced oscillations (i.e. no VEP)) and no oscillation (neither gravity induced nor mass induced). One can note from Fig. 1 that the muon yield with the gravity induced coupling differ with that of no VEP scenario. Also the difference increses for GRBs at larger redshift distances and at $z \sim 50$, the muon yields for the cases with no VEP differ by about 1 order of magnitude. We also show the effect of VEP oscillations while 3-flavour cases are considered in Fig 1(b). Similar feature is observed for the 3-flavour case too.

4 Summary and Discussions

In this work, we explore the possibility that very small violation of equivalence principle can be probed via the gravity induced neutrino oscillation. We demonstrate such



(a)



(b)

Figure 1: Variation of the muon yields from single GRB with different redshifts at a fixed zenith angle $\theta_z = 10^0$ for both 4-flavour (a) and 3-flavour (b) cases. See text for details.

a possibility by calculating the muon neutrino flux and consequently muon yields for such neutrinos at a kilometer square detector such as IceCube. We then compare our results for no oscillation case. We consider here a 4 neutrino scenario and calculated the oscillation formalism with mass induced and gravity induced oscillations. We compare our results for gravity induced oscillations with a representative value of the VEP with those where no VEP but only mass-flavour oscillation (suppression) is considered and also with no oscillation case for both 4-flavour and 3-flavour scenario. From comparisons of the muon yields we find that UHE neutrinos from distant sources could be an effective way to probe very small violation of weak equivalence principle.

Acknowledgments : MP thanks the DST-INSPIRE fellowship grant by DST, Govt. of India.

References

[1] S.M. Bilenky and B. Pontecorvo, Phys. Rep. **41**, 225 (1978).

[2] T.A. Mueller *et al.*, Phys. Rev. C **83**, 054615 (2011).

[3] P. Huber, Phys. Rev. C **84**, 024617 (2011).

[4] G. Mention *et al.*, Phys. Rev. D **83**, 073006 (2011).

[5] M. Gasperini, Phys. Rev. D **38**, 2635 (1988)

[6] M. Gasperini, Phys. Rev. D **39**, 12 (1989)

[7] M. Gasperini, Phys. Rev. d **38**, 9, (1988)

[8] L. Landau and E. Lifchitz, *Theorie du Champ* (MIR, Moscow, 1966), Sec. 89.

[9] J. Ahrens *et al.* (IceCube Collaboration), in Proceedings of the 27th International Cosmic Ray Conference, Hamburg, Germany, 2001 (unpublished), p. 1237, http://icecube.wisc.edu.

[10] P. Coloma, D.V. Forero and S.J. Parke (2017)

[11] E. Waxman , J. Bahcall, Phys. Rev. Lett. **78**, 2292 (1997).

[12] E. Waxman , J. Bahcall, Phys. Rev. Lett. **59**, 023002 (1998).

[13] R. Gandhi, C. Quigg, M.H. Reno, and I. Sarcevic,, Phys. Rev. D **58**, 093009 (1998).

[14] B. Pntecorvo, Sov. Phys. JETP **6**, 429 (1958).

[15] B. Pntecorvo, Sov. Phys. JETP **7**, 172 (1958).

[16] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theo. Phys., **28**, (1962).

[17] S.K. Kang *et al.* Hinsawi Publishing Corporation **2013**, 138109 (2013).

[18] M. Pandey, JHEP **01**, 066, (2019).

[19] C. Giunti, C.W. Kim and M. Monteno, Nucl. Phys. B **521**, 3 (1998).

[20] E.Kh. Akhmedov, arXiv:hep-ph/0001264.

[21] A.D. Banik and D. Majumdar, Mod. Phys. Lett. A **30**, 1550001 (2015).

[22] D. Majumdar, A. Ghosal, Phys. Rev. D **75**, 113004 (2007).

[23] M. Pandey, D. Majumdar and A.D. Banik, Phys. Rev. D **97**, 103015 (2018).

[24] A. Dziewonski, Earth Structure, Global, *The Encyclopedia of Solid Earth Geophysics*, D. E. James, ed. (Van Nostrand Reinhold, New York, 1989) p. 331.

[25] P. Lipary and T. Stanev, Phys. Rev. d **44**, 3543 (1991).

[26] T.K. Gaisser, *Cosmic Rays and Particle Physics* (Cambridge University Press, Cambridge, England, 1992); T.k. Gaisser, F. Halzen, and T.Stanev, Phys. Rep. **258**, 173 (1995).

[27] A. Dar, J.J. Lord, and R.J. Wilkes, Phys. Rev. D **33**, 303 (1986).

[28] D. Guetta, D. Hooper, J.A. Muniz, F. Halzen, and E. Reuveni, Astropart. Phys. **20**, 429 (2004).

[29] R. Gandhi, C. Quigg, M.H. Reno, and I. Sarcevic, Astropart. Phys. **5**, 81 (1996); arXiv:hep-ph/9512364.

[30] N. Gupta, Phys. Rev. D **65**, 113005 (2002).

[31] P. Adamson *et al.* (MINOS Collaboration), Nucl. Instrum. Meth. A **806**, 279 (2016).

[32] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **117**, 151803 (2016).

[33] P. Adamson *et al.* (MINOS Collaboration), arXiv:1710.06488[hep-ex].

[34] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **110**, 171801 (2013).

[35] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **107**, 011802 (2011).

[36] D.G. Michael *et al.* (MINOS Collaboration), Phys. Rev. Lett. **97**, 191801 (2006).

[37] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **101**, 221804 (2008).

[38] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. Lett. **112**, 191801 (2014).

[39] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. D **81**, 052004 (2010).

[40] D.G. Michael *et al.* (MINOS Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **596**, 190 (2008).

[41] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. D **77**, 072002 (2008).

[42] P. Adamson *et al.* [Daya Bay and MINOS Collaborations], Phys. Rev. Lett. **117**, no. 15, 151801 (2016).

[43] F.P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).

[44] F.P. An *et al.* (DAYA-BAY Collaboration), following Letter, Phys. Rev. Lett. **117**, 171802 (2016).

[45] F.P. An *et al.* (DAYA-BAY Collaboration), Nucl. Instrum. Methods Phys. Res., Sect. A **811**, 133 (2016).

[46] F.P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **112**, 061801 (2014).

[47] F.P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **116**, 061801 (2016).

[48] F.P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **115**, 111802 (2015).

[49] F.P. An *et al.* (DAYA-BAY Collaboration), Phys. Rev. Lett. **113**, 141802 (2014).

[50] B. Achkar *et al.* (BUGEY Collaboratrion), Nucl. Phys. B **434**, 503 (1995).

[51] P. Adamson *et al.* (NOvA Collaboration), Phys. Rev. D **96**, 072006 (2017).

[52] D. S. Ayres *et al.* (NOvA Collaboration), arXiv:hep-ex/0503053.

[53] P. Adamson *et al.* (NOvA Collaboration), Phys. Rev. Lett. **116**, 151806 (2016).

[54] P. Adamson *et al.* (NOvA Collaboration), Phys. Rev. Lett. **118**, 231801 (2017).

[55] P. Adamson *et al.* (NOvA Collaboration), Phys. Rev. D **93**, 051104(R) (2016).

[56] P. Adamson *et al.* (NOvA Collaboration), Phys. Rev. Lett. **118**, 151802 (2017)

[57] M. G. Aartsen *et al.* [IceCube Collaboration], Phys. Rev. D **95**, no. 11, 112002 (2017).

[58] The IceCube Collaboration,*35th International Cosmic Ray Conference - ICRC2017*, PoS (ICRC2017) 981.