A NOTE ON AIG- CLOSURE AND AIG- INTERIOR IN IDEAL TOPOLOGICAL SPACES

D. Vinodhini[a], M. Karpagadevi[b], L. Senthil Kumar[c], and S.Santhiya[d]

[a,] *Department Of Mathematics, Amrita School of Agricultural Sciences, Coimbatore, Tamilnadu-642 109, India*
[b]*Department of mathematics, Sri g. V. G. Vishalakshi College for Women, Udumalpet, Tamilnadu-642 128, india*
[c]*Departent of Mathematics, Dr. Mahalingam College of Engineering and Technology, Pollachi,Tamil Nadu -642130, india,*
[d]*Department of Mathematics, Sri Krishna College of Technology, Coimbatore , Tamil Nadu-641 008, India.*

Abstract: The concepts of αIg- closure, αIg- interior and αIg- boundary of a subset of an ideal topological space (X, τ, I) are introduced in this article. Some of their basic properties are proven. Furthermore, the relationships between these sets are investigated to get the best of them. Also, it is established that αIg- closure is a Kuratowski closure operator on (X, τ, I) under certain conditions.

KEYWORDS.αIg- closed set, αIg- closure, αIg- interior, αIg- boundary.

1. Introduction

Topology is a branch of mathematics, which is believed that topological structure will be an important base for modification of knowledge extraction and processing. Kuratowski's closure complement theorem [1] has been a guiding source of topology. Peleg[2], while investigating the transitive closure of a binary relation, came across several closure operators which do not satisfy some of the four of Kuratowski's closure axioms, though their properties suffice to maintain "closure complement phenomenon". Similar kind of generalized closure operators are generated by the monotonic mappings introduced by A. Csaszar[3]. When several such operators are considered simultaneously and composed, the study of closure complement phenomenon becomes complicated and highly interesting.

The contributions of Hamlett and Jankovic[4-7] in ideal topological spaces initiated the generalization of some important properties in general topology via topological ideals. Kuratowski and Vaidhanathaswamy studied the notion of ideal topological spaces. Dontchev et al, Navaneethakrishnan et al, Jankovic et aletc., were investigated applications to various fields of ideal topology. The properties like decomposition of continuity, separation axioms, connectedness, compactness and resolvability [8-12] have been generalized using the concept of ideals in topological spaces. In [13], O. Njasted investigated the notion of α-closed sets. By using α-open sets, Mashhour et al.[14] defined and studied the concept of α-closed sets, α-closure of a set, α-continuity and α-closedness in topology. In the present note,we provide our investigations regarding all possible compositions of generalized closure operators and their corresponding interior operators in ideal topological spaces.

A topological space is a pair (X, τ) consisting of a set X and family τ of subsets of X satisfying the following conditions: a) $\phi \in$ and X $\in \tau$ b)τ is closed under arbitrary union c) τ is closed under finite intersection.The elements of X are called points of the space, the subsets of X belonging to τ are called open sets in the space, the complement of the subsets of X belonging to τ are called closed sets in the space. The τ-closure of a subset A $\subseteq$ X is denoted by cl(A) which is defined as the smallest closed subset of X which contains A. The τ-interior of a subset A $\subseteq$ X is denoted by int(A) which is defined as the union of all open subsets of X which contained in A. Note that A is open if and only if A = int(A). The boundary of a subset A $\subseteq$ X is denoted by b(A) and is given by

b(A) = cl(A)   int(A).

An ideal I on a set X is a nonempty collection of subsets of X which satisfies:

   a) A $\in$ I and B $\subseteq$ A implies that B $\in$ I and
   b) A $\in$ I, B $\in$ I implies that A $\cup$ B $\in$ I.

An ideal topological space is a topological space (X, τ) with an ideal I on X and it is denoted by (X, τ, I). Given a topological space(X, τ) with an ideal I on X and if ρ(X) is the set of all subsets of X, a set operator (*):ρ(X)$\rightarrow\rho$(X), is called a local function [15] of A with respect to τ and I, is defined as follows: for A $\subseteq$ X, A^*(X, I)= { x $\in$ U $\cap$ A $\notin$ I for every U $\in \tau$(x) where τ(x) = { U $\in \tau$(x)/ x $\in$ U}}. A Kuratowski closure operator cl^*(A) = A $\cup$ A^*(I, τ).

Definition 1.1.[17]Let (X, τ) be a topological space and I be an ideal on X. A subset A of X is said to be α-Ideal generalized closed set (αIg closed set) if $A^* \subseteq$ U whenever A $\subseteq$ U and U is α-open. The complement of αIg closed set is called α-Ideal generalized open set(αIg open set)

Definition 1.2.[18]Let (X, τ, I) be an ideal topological space and x $\in$ A $\subseteq$ X. Then A is said to be an Iα-neighborhood of x, if there exist an Iα-open set U such that x $\in$ U $\subseteq$ A, and simply write as IαN(x). If A is Iα-open set then it is Iα-open neighbourhood for any element x $\in$ A.

Definition 1.3.[18]Let (X, τ, I) be an ideal topological space and x ∈ A ⊆ X. Then x is said to be an Iα-interior point of A if A contain an Iα-open neighborhood set for x. The set of all Iα- interior points of A is called Iα-interior set and simply is denoted by Iα-int(A).

Definition 1.4.[18] Let (X, τ, I) be an ideal topological space and A ⊆ X. x ∈X is said to be an Iα-boundary point of A if for every Iα-open neighborhood set for x satisfies that the intersection with A and A^c is nonempty set. The set of all Iα-boundary points of A is called Iα-boundary set of A and simply is denoted by Iα-b(A).

2.αIg-CLOSURE

Definition 2.1. For every set F ⊆ (X, τ, I), αIg-closure of F is defined as the intersection of all αIg-closed sets containing F.(i.e.) αIg-cl(F) = ∩ {A : F ⊆ A, A ∈αIg-cl(X, τ, I)}.

Example 2.2. Let X = {a, b, c} with topology τ = {φ, {a}, {b, c}, X} and I = {φ, {c}}. Then αIg- closure of {b} is given as αIg-cl{b} = ∩{{b}, {a, b}, X/ where {b}, {a, b}, X are αIg- closed set}.

Theorem 2.3. If αIg-cl(X, τ, I) is closed under finite union, then αIg-closure is a Kuratowski operator on (X, τ, I) .

Proof. (i) αIg-cl(φ) = φ and αIg-cl(X) = X also A ⊆αIg-cl(A).

(ii) Suppose that E and F are two subsets of X, then αIg-cl(E) ⊆αIg-cl(E ∪ F) andαIg-cl(F) ⊆αIg-cl(E ∪ F). Hence αIg-cl(E) ∪αIg-cl(F) ⊆αIg-cl(E ∪ F). If x ∉αIg-cl(E) ∪αIg-cl(F), then there exists A, B ∈αIg-cl(X, τ, I) such that E ⊆ A, x ∉ A, F ⊆ B and x ∉ B. Hence E ∪ F ⊆ A ∪ B and x ∉ A ∪ B. By hypothesis, A ∪ B is αIg-closed. Thus, x ∉αIg-cl(A ∪ B) implies that, x ∉αIg-cl(E ∪ F). Hence αIg-cl(E ∪ F) ⊆αIg-cl(E) ∪αIg-cl(F). Thus, αIg-cl(E ∪ F) = αIg-cl(E) ∪αIg-cl(F).

(iii) Let E ⊆ X and A be an αIg-closed set containing E. Then by Definition 2.1., αIg-cl(E) ⊆ A and αIg-cl(αIg-cl(E)) ⊆ A. Since αIg-cl(αIg-cl(E)) ⊆ A, αIg-cl(αIg-cl(E)) ⊆∩ { A : E ⊆ A, A ∈αIg-cl(X, τ, I)}. Hence (αIg-cl(E)) ⊆αIg-cl(αIg-cl(E)) and by Definition 2.1., αIg-cl(αIg-cl(E)) ⊂ (αIg-cl(E)) implies that (αIg-cl(E)) = αIg-cl(αIg-cl(E)). Thus, αIg-closure is a Kuratowski closure operator on X.

Theorem 2.4. Let A be the subset of the ideal topological spaces (X, τ, I). Then x ∈αIg-cl(A) if and only if A ∩ U ≠ φ for every αIg-open set U containing x.

Proof. Suppose that x ∈αIg-cl(A). Let U be an αIg-open set containing x such that A ∩ U = φ and so A ⊂ X   U. But X   U is an αIg-closed set and so αIg-cl(A) ⊂ X   U. Since x ∉ X   U, x ∉αIg-cl (A) which is contrary to the hypothesis.

Conversely, suppose that every αIg-open set of X containing x such that A ∩ U ≠ φ. If x ∉αIg-cl(A), then there exists an αIg-closed set F of X such that A ⊆ F and x ∉ F. Hence x ∈X   F and X   F is an αIg-open set containing x. Thus, (X   F) ∩ A = φ which is a contradiction.

3.αIg-INTERIOR

Definition 3.1. For every set A ⊆ X, αIg-interior of A is defined to be the union of all αIg-open sets contained in A.(i.e.) αIg-int(A) = ∪ { U : U ⊆ A, U ∈αIg-int(X, τ, I)}.

Example 3.2. Let X = {a, b, c} with topology τ = {φ, {a}, {a, c}, X} and I = {φ, {b}}. Then αIg- interior of {a, b} is given as αIg-int{a, b} = ∪ {{b}, {a, b}/{b}, {a,b} are αIg-open set}

Theorem 3.3. Let (X, τ, I) be an ideal topological space and A be a subset of X, then the following statements are true.

(i) X − (αIg-int(A)) = αIg-int(X −A).

(ii) αIg-int(A) = X − (αIg-int(X −A)).

(iii) αIg-cl(A) = X − (αIg-int(A)).

Proof. (i) Let x ∈X − (αIg-int(A)). Thenx ∉ (αIg-int(A)). That is, every αIg-open set U containing x is such that U is not a proper subset of A. Thus, U ∩ (X − A) ≠φ, for everyαIg-open set U containing x. Hence, x ∈αIg-int(X − A) and so X − (αIg-int(A)) ⊆αIg-int(X − A). Conversely, suppose x ∈αIg-int(X − A). Then, U ∩ (X   A) ≠ φ, for every αIg-open set U containing x. By Definition3.1., x ∉ (αIg-int(A)). Thus, x ∈X − (αIg-int(A)) and so (αIg-int(X −A)) ⊆ X − (αIg-int(A)). Therefore,X − (αIg-int(A)) = αIg-int(X − A).

(ii) From (i), X − (αIg-int(A)) = αIg-int(X −A). By taking the complement on both sides, we get (X − (X − αIg-int(A))) = (X − αIg-int(X − A)) implies that αIg-int(A) = X − (αIg-int(X −A)).

(iv) From (i), X − (αIg-int(A) = αIg-int(X − A). Replace A by X − A, we get X − (αIg-int(X −A)) = X − αIg-int((X −A)) which implies that αIg-cl(A) = X − (αIg-int(X − A)).

4.αIg-BOUNDARY

Definition 4.1. Consider an ideal topological space (X, τ, I). Let A be any subset of (X, τ, I). Then, the αIg-boundary of A is defined as αIg-Bd(A) = αIg-cl(A) ∩αIg-cl(X − A).

Theorem 4.2. For any sets A and B in ideal topological space (X, τ, I), the following conditions hold:

(i) αIg-Bd(A) = αIg-Bd(X − A).

(ii) αIg-Bd(A) ⊆αIg-cl(A) .

(iii) αIg-Bd(A) ⊆αIg-cl(X − A) .

(iv) If A is an αIg-closed set, then αIg-Bd(A) ⊆ A.

(v) If A is an αIg-open set, then αIg-Bd(A) ⊆ X − A.

(vi) Let A ⊆ B and B ∈αIg-cl (X, τ) (resp. B ∈αIg-O(X, τ)). Then, αIg-Bd(A) ⊆ B (resp. αIg-Bd(A) ⊆Bᶜ), where αIg-cl(X, τ) denotes the class of αIg-closed (resp. αIg-O(X, τ) denotes the class of αIg-open) sets in X.

(vii) X − (αIg-Bd(A)) = αIg-int (A) ∪αIg-int (X − A).

Proof. (i) αIg-Bd(A) = αIg-cl(A) ∩αIg-cl(X − A) = αIg-cl(X − A) ∩αIg-Cl(A) = αIg-cl(X − A) ∩ (X − αIg-cl(X − A)) = αIg-Bd(X − A).

(ii) By definition 4.1, αIg-Bd(A) = αIg-cl(A) ∩αIg-cl(X − A). Hence αIg-Bd(A) ⊆αIg-cl(A).

(iii) By definition 4.1, αIg-Bd(A)=αIg-cl(A) ∩αIg-cl(X − A). ThusαIg-Bd(A) ⊆αIg-cl(X − A).

(iv) αIg-Bd(A) = αIg-cl(A) ∩αIg-cl(X − A) ⊆αIg-cl(A) = A.

(v) If A is αIg-open then, X − A is αIg-closed. By (iv), αIg-Bd(X − A) ⊆ X − A. Again by (i), αIg-Bd(A) ⊆ X − A.

(vi) Since A ⊆ B, αIg-cl(A) ⊆αIg-cl(B). Now, αIg-Bd(A) = αIg-cl(A) ∩αIg-cl(X − A) ⊆αIg-cl(A) ⊆αIg-cl(B) = B. Hence αIg-Bd(A) ⊆ B.

(vii) X − (αIg-Bd(A)) = X − (αIg-cl(A) ∩αIg-cl(X − A)) = X − (αIg-cl(A)) ∪ X − (αIg-cl(X − A)) = (αIg-int(X − A)) ∪ X − (αIg-int(X − A)) = αIg-int(X − A)) ∪αIg-int(A).

Definition 4.3. Let (X, τ, I) be an ideal topological space and A ⊆ X. Then, αIg-border of A is defined as bαIg(A) = A   αIg-int(A).

Theorem 4.4. For a subset A of an ideal topological space (X, τ, I), the following conditions hold:

(i) αIg-Bd(A) ⊆Bd(A), where Bd(A) denotes the boundary of A.

(ii) αIg-cl (A) = αIg-int(A) ∪αIg-Bd(A)

(iii) αIg-int(A) ∩αIg-Bd(A) = φ

(iv) bαIg (A) ⊆αIg-Bd(A)

(v) αIg-Bd(int (A)) ⊆αIg-Bd(A)

(vi) αIg-Bd(cl (A)) ⊆αIg-Bd(A) .

Proof. (i) αIg-Bd(A) = αIg-cl (A) ∩αIg-cl (X − A) ⊆ cl(A) ∩ cl(X − A) = Bd (A).

(ii) αIg-int(A) ∪αIg-Bd(A) = αIg-int(A) ∪ (αIg-cl (A) ∩αIg-cl (X − A)) = αIg-cl (A).

(iii) αIg-int(A) ∩αIg-Bd(A) = αIg-int(A) ∩ (αIg-cl (A) ∩αIg-cl (X − A)) = φ.

(iv) bαIg (A) = A   αIg-int(A) ⊆αIg-cl (A)   αIg-int(A) ⊆αIg-cl (A) ∩ X − (αIg-int(A)) ⊆αIg-cl (A) ∩αIg-cl(X − A) = αIg-Bd(A).

(v) αIg-Bd(int (A)) = αIg-cl(int (A)) ∩ X − αIg-cl(int (A)) ⊆αIg-cl((A)) ∩ X − (αIg-int (A))   = αIg-Bd(A).

(vi) αIg-Bd(Cl(A)) = αIg-cl(cl(A)) ∩αIg-cl(cl(A))c = αIg-cl(A) ∩ (αIg-int(cl(A))) ⊆αIg-cl(A) ∩ X − (αIg-int(A)) = αIg-Bd(A).

5. CONCLUSION

In this present work, we introduced the new concepts αIg- closure, αIg- interior and αIg- boundary of a subset of an ideal topological space (X, τ, I). Also we studied and proved the basic properties of these concepts

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