A BRIEF LITERATURE ON PROBLEMS AND PERSPECTIVES OF MATHEMATICAL AND STOCHASTIC MODELLING

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Abstract: This research article mainly explores on problems and perspectives of mathematical and stochastic modelling. There is a large element of compromise in mathematical modelling. The majority of interacting systems in the real world are far too complicated to model in their entirety. In this research paper an extensive discussion has been made on linear models, nonlinear models, static models, dynamic models. A comparative study is done between the pairs explicit and implicit model, discrete and continuous model, deterministic and probabilistic model. In this talk a brief discussion on different types of models has been proposed and the concept of stages of model building is extensively discussed. Problems of stochastic model building are presented in a lucid manner and this literature is highly helpful for young researchers in stochastic modelling.

Keywords: Linear and nonlinear model, Static and dynamic model, Model building, Stochastic modeling, Explicit and Implicit model.

1. Introduction

Modelling is a cyclic process of creating and modifying models of empirical situations to understand them better and improve decisions. The role of modelling and mathematical modelling has received increasing attention as generating authentic learning and revealing the ways of thinking that produced it. We review a subset of the related literature; discuss benefits and challenges in teaching and learning mathematical modeling activities and implications for instruction and assessment as well as for research.

Models describe our beliefs about how the world functions. In mathematical modelling, we translate those beliefs into the language of mathematics. This has many advantages.

(1) Mathematics is a very precise language. This helps us to formulate ideas and identify underlying assumptions.
(2) Mathematics is a concise language, with well defined rules for manipulations.
(3) All the results that mathematicians have proved over hundreds of years are at our disposal.
(4) Computers can be used to perform numerical calculations.

There is a large element of compromise in mathematical modelling. The majority of interacting systems in the real world are far too complicated to model in their entirety. Hence, the first level of compromise is to identify the most important parts of the system. The second level of compromise concerns the amount of mathematical manipulation which is worthwhile. Although mathematics has the potential to prove general results, these results depend critically on the form of equations used. Small changes in the structure of equations may require enormous changes in the mathematical methods. Using computers to handle the models equations may never lead to elegant results, but it is much more robust against alterations.

2. METHODOLOGICAL MODELING PRINCIPLES

Mathematical modelling is a principled activity that has both principles behind it and methods that can be successfully applied. The principles are over-arching or meta-principles phrased as questions about the intentions and purposes of mathematical modelling. These meta-principles are almost philosophical in nature.

Methodological modeling principles are also captured in the following list of questions and answers:

- What are one looking for? Identify the need for the mode.
- What do one wants to know? List the data we are seeking.
- What do one knows? Identify the available relevant data.
- What can one assume? Identify the circumstances that apply.
• How should one look at this model? Identify the governing physical principles.
• What will model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
• Are the predictions valid? Identify tests that can be made to validate the model, i.e., is it consistent with its principles and assumptions?
• Are the predictions good? Identify tests that can be made to verify the model, i.e., is it useful in terms of the initial reason it was done?
• Can one improve the model? Identify parameter values that are not adequately known, variables that have been included, and/or assumptions/restrictions that could be lifted. Implement the iterative loop that one can call “model¬validate¬verify¬improve¬predict.”
• How will one exercise the model? What will one do with the model?

This list of questions and instructions is not an algorithm for building a good mathematical model. However, the underlying ideas are key to mathematical modelling, as they are key to problem formulation generally. Thus, one should expect the individual questions to recur often during the modelling process, and one should regard this list as a fairly general approach to ways of thinking about mathematical modelling.

Having a clear picture of why the model is wanted or needed is of prime importance to the model-building enterprise. Defining the task is the first essential step in model formulation.

If one finds that the model is inadequate or that it fails in some way, then one may enter an iterative loop in which one returns to an earlier stage of the model building and re-examine assumptions, known parameter values, the principles chosen, the equations used, the means of calculation, and so on. This iterative process is essential because it is the only way that models can be improved, corrected, and validated.

3. CLASSIFICATION OF MATHEMATICAL MODELLING

A model which uses a large amount of theoretical information generally describes what happens at one level in the hierarchy by considering processes at lower levels; these are called mathematical models because they take account of the mechanisms through which changes occur. In empirical models, no account is taken of the mechanism by which changes the system occur. Instead it is merely noted that they do occur, and the model tries to account quantitatively for changes associated with different conditions.

The two divisions above namely deterministic and mechanistic/empirical represent extremes of a range of model types. In between lie a whole spectrum of model types. Also, the two methods of classification are complementary. For example, a deterministic model may be either mechanistic or empirical (but not stochastic).

One further type of model, the system model, is worthy of mention. This is built from a series of sub-models, each of which describes the essence of some interacting components. The above method of classification then refers more properly to the sub-models; different types of sub-models may be used in any one system model.

Much of the modelling literature refers to ‘simulation models’

(a) Mathematical models may be classified according to the subject matter of the models. Thus one may have mathematical models (M.M) in chemistry (Theoretical Chemistry); M.M in Biology (Mathematical Biology), M.M in Medicine (Mathematical Medicine), M.M in Economics (Mathematical Economics and Econometrics), M.M in Psychology (Mathematical psychology), M.M in Sociology (Mathematical Sociology), M.M in Engineering (Mathematical Engineering) and so on.

One may have similarly M.M. of transportation, of urban and regional pollutions, of population, of environment, of oceanography, of blood flows, of genetics, of water resources, of optimal utilization of exhaustible and renewable resources, of political systems, of land distribution, of linguistics and so on.

In fact, every branch of knowledge has two aspects one of which is theoretical, mathematical, statistical, and computer based and the other of which is empirical, experimental and observational. Mathematical Modeling is essential to the first of these aspects.

(b) One may also classify mathematical models according to the mathematical techniques used in solving them. Thus one may have mathematical modeling (M.M) through classical algebra, M.M through linear algebra and matrices, M.M through ordinary and partial differential equations, M.M through ordinary and partial difference equation, M.M through functional equations, M.M through
graphs, M.M. through mathematical programming, M.M. through calculus of variations, and M.M. through maximum principle and so on.

(c) Mathematical models may be linear or Non-linear according as the basic equations describing them are linear or nonlinear. Mathematical models may also be classified according to the purpose we have for the model. Thus, one may have Mathematical Models (M.M.) for Description, M.M for Insight, M.M for prediction, M.M for optimization, M.M for control and M.M for Action.

(d) Mathematical models may also be classified according to their nature, Mathematical models are usually composed of relationships among variables. Relationships can be described by operators, such as algebraic operators, functions, differential operators, etc. Variables are abstractions of system parameters of interest, that can be quantified. Several classification criteria can be used for mathematical models according to their structure:

- **Linear vs. Nonlinear Models:** If all the operators in a mathematical model exhibit linearity, the resulting mathematical model is defined as linear. A model is considered to be nonlinear otherwise. The definition of linearity and nonlinearity is dependent on context, and linear models may have nonlinear expressions in them. For example, in a statistical linear model, it is assumed that a relationship is linear in the parameters, but it may be nonlinear in the predictor variables. Similarly, a differential equation is said to be linear if it can be written with linear differential operators, but it can still have nonlinear expressions in it. In a mathematical programming model, if the objective functions and constraints are represented entirely by linear equations, then the model is regarded as a linear programming model. If one or more of the objective functions or constraints are represented with a nonlinear equation, then the model is known as a nonlinear programming model.

- **Static vs. Dynamic Models:** A dynamic model accounts for time-dependent changes in the state of the system, while a static (or steady-state) model calculates the system in equilibrium, and thus is time-invariant. Dynamic models typically are represented by differential equations.

- **Explicit vs. Implicit Models:** If all of the input parameters of the overall model are known, and the output parameters can be calculated by a finite series of computations (known as linear programming, not to be confused with linearity as described above), the model is said to be explicit. But sometimes it is the output parameters which are known, and the corresponding inputs must be solved for by an iterative procedure, such as Newton's method (if the model is linear) or Broyden's method (if nonlinear).

- **Discrete vs. continuous Models:** A discrete model treats objects as discrete, while a continuous model represents the objects in a continuous manner.

- **Deterministic vs. probabilistic (stochastic) Models:** A deterministic model is one in which every set of variables states is uniquely determined by parameters in the model and by sets of previous states of these variables; therefore, a deterministic model always performs the same way for a given set of initial conditions. Conversely, in a stochastic model-usually called a "statistical model"-randomness is present, and variable states are not described by unique values, but rather by probability distributions.

- **Deductive, Inductive, or Floating Models:** A deductive model is a logical structure based on a theory. An inductive model arises from empirical findings and generalization from them. The floating model rests on neither theory nor observation, but is merely the invocation of expected structure.

Linear, static and deterministic models are usually easier to handle than non-linear, dynamic and stochastic models and in general in any discipline these are the first to be considered.

### 4. STAGES OF MODEL BUILDING

The various steps involving in the model building can be characterized as:

(i) examining the situation and setting up the goals to be accomplished;
(ii) identifying variables in the situation and selecting those that represent essential features;
(iii) formulating a model by creating and selecting geometric, graphical, tabular, algebraic or statistical representations that describe relationships between the variables;
(iv) analyzing and performing operations on these relationships to draw conclusions, if the implementation of the performed operations cannot be complete, then revise the selection of the variables used to formulate the models;
(v) interpreting the results of the mathematics in terms of the original situation;
(vi) validating the conclusions by comparing them with the situation and then either improving the model or if it is acceptable; and
(vii) Applying the model to similar situations for evaluation and refinement.

The process of developing sufficiently useful models for a specific purpose usually involves a series of iterative testing and revision cycles. Also choices, assumptions and approximations are present throughout the modeling cycle.

This characterization of the model building is illustrated in figure.

Setup the goal(s) identify variables and relationships between variables formulate a model

Perform Relationship

Real world problem

Real world solution

Report

The identified

Validati
on

Mathematical problem

Mathematical solution

Interpret the mathematical solution in terms of the original problem evaluate and refine the formulated model.

5. PRINCIPLES FOR EFFICIENT MODEL BUILDING

As much as all steps that are taken during a modelling cycle are important, it makes an important difference whether it is a good model or a bad one if a mathematical model is used to improve decisions. The model one has available may not be good enough to use, or there may be more efficient models available for use in a given situation. The forefront of the thinking about mathematical modelling and suggests six principles to go by in taking the measure of a model: Accuracy, descriptive, realism, precision, robustness, generality and fruitfulness.

Definitions of the six principles:
A model is said to be

(i) Accurate, if the output of the model (the answer it gives) is correct or very near to correct.
(ii) Descriptively realistic, if it is based on assumptions which are correct.
(iii) Precise, if its predictions are definite numbers (or other definite kinds of mathematical entities; functions, geometric figure, etc.). By contrast, if a model’s prediction is a range of numbers (or a set of functions a set of figures, etc.) the model is imprecise.
(iv) Robust, if it is relatively immune to errors in the input data.
(v) General, if it applies to a wide variety of situations.
(vi) Fruitful, if its conclusions are useful or it inspires or points the way to other models.

6. PROBLEMS OF MODEL BUILDING

Based on the completeness and ambiguity of the information composing a problem, modeling problems can be categorized into three types with third type being the most authentic type as follows:
1. Problems under this category are already carefully defined so there is little ambiguity about what needs to be done and how to do it. They contain all the information necessary to formulate a model. They either specifically call for a certain procedure to be used or its use is evident on prior instruction or placement of the task. Researchers are expected to search for the needed information that is hidden in the problem, recall the (implicitly or explicitly) called for procedure and carry it out correctly. There is no need to collect additional data to formulate a model.

2. Problems under this type still have little ambiguity about what needs to be done and how to do it. However, they do not provide all the information needed to successfully complete the task. Although researchers may be given a direction of what data is needed, they need to devise a meaningful way to gather the needed data and test if the gathered data would produce a reasonable answer.

3. These type problems are comprised of information that is pen-ended, incomplete and/or redundant. There is not a well-rehearsed approach or pathway explicitly suggested by the task. Researchers are expected to analyze the task to find what needs to be done and actively examine tasks constraints that may limit or suggest possible solution strategies and solutions.

7. PROBLEMS OF STOCHASTIC MODEL BUILDING

Following are some important problems of stochastic model building which can be frequently arise in the specification stochastic models:

(a) Selection of Stochastic Model
(b) Mis-specification of Stochastic Model
(c) Variables selection for Stochastic Model

(a) Selection of Stochastic Model:
In general, Stochastic Models may be two types namely,
(i) Nested Stochastic Model: If a stochastic model can be described as a particular case of another stochastic model, then the first model is said to be ‘Nested Stochastic Model’ within second stochastic Model.
(ii) Non-Nested Stochastic Models: Two stochastic models are known as ‘Non-Nested Stochastic Models if stochastic model one cannot be derived as a particular case of another stochastic model.

The various diagnostic tests can be used for the selection of good stochastic models. These diagnostic tests have been available in the literature separately for Nested and Non-Nested Stochastic Models. Some of them are given by:

(i) F-test for Nested statistical models.
(ii) Exhaustive search methods.
(iii) Stepwise, Backward and Forward selection Techniques.
(iv) R² or coefficient of multiple Determinations as Model Selection Criterion.
(v) Adjusted R² or \( \bar{R}^2 \) as a Model selection criterion.
(vi) Conditional mean Squared Prediction Error criterion (C_p-Criterion) for Model Selection.
(vii) Amemiya’s unconditional MSE criterion.
(viii) Ullah criteria for model selection.
(ix) Stopping Rules for model selection using Mean squared error of prediction.
(x) Cox Modified Likelihood Ratio test for model selection.
(xii) Fisher and Mc Aleer JA test for model selection.
(xiii) Davidson, Godfrey and Mac Kinnon omitted variables test for model selection.
(xiv) Wu t-test for Model Selection based on Recursive residuals.
(xv) Cross-validation Technique for linear model selection.
(xvi) Berger and pericchi Intrinsic Bayes Factor of model selection.
(xvii) Akaike Bayesian Information criterion for model selection.
(xviii) Fisher Information criterion for model selection.
(xix) The chow Forecast Test of parameter constancy for model selection.
(xxi) CUSUM and CUSUMSQ Residuals Test of constancy for Linear Model Selection.
(xxii) Chow test of structural change for Model Selection.
(b) Mis-Specification of Stochastic Model:

In general, researcher faces frequently the problems of mis-specification of stochastic model. This problem creates many consequences with regard to the estimates of the regression coefficients of the stochastic model.

Mis-specification can arise either because of omission of a variable specified by the truth, the case of the left out variables or because of inclusion of a variable not specified by the truth, the case of irrelevant variables.

Following are some important mis-specification tests for stochastic model available in the literature.

(i) Ramsey Regression Specification Error Test (RESET).
(ii) Utts Rainbow Test.
(iii) Plosser, Schwart and White (PSW) Differencing Test.
(iv) White’s Information Matrix (IM) Test.
(v) Hausman’s Misspecification Test.
(vi) J-Test.
(vii) Fisher and Mc Aleer JA Test.
(viii) The Rank Specification Error Test (KOMSET).
(ix) The Kolmogorov Specification Error Test (KOMSET).
(x) Bartkett’s M Specification Error Test (BAMSET).
(xi) Harvey and Collier Test for functional misspecification in Regression analysis.
(xii) Farebrother Grouping Test for Misspecification.
(xiii) Davids on an Mac Kinnon specification Error Test.
(xiv) Bera and Mc Aleer Exact Tests for specification error.
(xv) White test for functional form.
(xvi) Chow Test for Model specification.
(xvii) Ramsey and Schmidt modified RESET.
(xviii) Mac Kinnon, White and Davidson Projection Extended (PE) Test for Misspecification.

(c) Variables selection for Stochastic Model:

The goodness of prediction in the applied regression analysis has been based on the selection of relevant regressors for specification of stochastic model.

In practice, a large number of regressors usually are introduced at the initial stage of modeling and researchers use stepwise deletion and subset selection.

The various criteria of variables selection for stochastic model existing in the literature are:

(i) The $R^2$ and $\overline{R}^2$ criteria.
(ii) Mallows $C_p$ Criterion.
(iii) Amemiya’s unconditional MS prediction Errors criterion.
(iv) Breiman and Friedman $S_p$ criterion.
(v) Akaike Information criterion.
(vi) Sawa’s BIC criterion.
(vii) Reformulated Akaike Information criterion.
(viii) Jeffreys – Bayes posterior odds Ratio criterion.
(ix) Stein – Rule Formulation criterion.
(x) General Stein Rule criterion.
(xi) The BIVAR criterion.
(xii) Stopping Rule for selection of Regressions.
(xiii) Stepwise, Forward and Backward selection criterion.
(xiv) Average Estimated variance (AEV) criterion
(xv) Mx Cabe U – Statistics for variable selection in Discriminant analysis.
(xvi) Influence measures for selection of criterion.
Research Article

8. Conclusion

In the above talk a brief discussion on different types of models has been proposed and the concept of stages of model building is extensively discussed. Problems of stochastic model building are presented in a lucid manner and the above literature is highly helpful for young researchers in stochastic modelling.

REFERENCES


