Transient Solution of $M^{[X]}/G/1$ Queue system with Balking and Re service

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Abstract. A batch arrival non-Markovian queue system in where consumers arrival are batches in accordance with Poisson process and customer are treated by first-come-first-served principle. When a customer's service is completed, he or she has the choice of requesting reservice for the same service or exiting the mode. Each customer's service duration follows a genaral (arbitrary) distribution. In addition, after completing service, server takes vacation by having probability p or may be remain with probability $1-p$, if any, to serve a subsequent customer. One of the impatient customers behavior balking, has also been added, indicating that a consumer can choose whether or not to enter the system. Also, we suppose after every period the server could not begin to providing service immediately, but rather requires some startup time before giving service to the first consumer. After discussion of transiant steady state system we can obtain closed-form system performance measurements.

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1. Introduction

Queue system with various vacation rules has been garnered a lot of attention because queueing models have a versatile applications in real world scenarios like production systems, banking services, computer model and so on. In M/G/1 queue system, Yechiali Levy (1975) examined the utilization of idle time. Due to the requirement for some prior work to starting process before start of each service period, the server does not always supply the required service immediately, and this period has been called setup time, and research in queueing models with setup time hasbecome familiar and intriguing. Baker (1973) was the first to propose an exponential startup with server vacation polices. Under modified Bernoulli vacation, Choudhury and Madan(2005) discussed a batch arrival queue model with two phases and random startup time.

Excessive waiting in any queue can lead to frustration, which is crucial for arrivals and departures in queueing models. Authors have recently placed their work out there. Customers conduct such as balking and reneging substantially influenced their interest in researching their notions on queues. Customers may decide whether or not to buy something in a real-life scenario. Due of the significant wait, you can either join a queue or not; because of long waiting time or any other condition called balking. Such type of customer behavior can be seen in hospitals and emergency services are required, communication systems, signals in customer service and many more situations. Haight was the first to study a line with balking (1957). He examined the M/M/1 queue with balking, in which an arrival would not balk at the longest queue length. Barrer (1957) looked into impatient clients and how they were served. Later, Ancker et al. (1963) looked into single server and multiserver queues with balking and reneging. Since then a considerable work has been done on queue systems with customer impatient behavior. Hariri et al.(1992), Boots and Tijms(1999), Choi(2001) studied about queues with impatient customers. Choudhury and Medhi (2010,2011) analyzed the customer behavior for multiserver queuing models. In most of real situations, customers may demand re-service, for instance patients in hospital may be required to meet the doctor after some investigations are taken, in Manufacturing process if the product is found to be faulty it is sent back for reproduction. Hence, Re-service is an important aspect in queueing theory which is used in modeling most of the real situations like telecommunication networks, computer networking, inventory and production etc. Research papers in queueing models with re-service are attracted by many authors.

This paper looks into a batch arrival queue model were customers arrive by following compound Poisson process with batch size, random variable X. Single server queueing model by incorporation Bernoulli model server vacation and an exponential setup time. Also, becomes empty that is when there no customers the server turned off, called turned off period, in this time the server may be in upstate but switched down, or it may be on vacation. Server startup refers to the server's first preparations before launching the service. Here server shall take a random size of time to setup the server in order to serving a new consumer.

2. Definition of Queueing Model

For describe the our model, consider the following

- Let $\lambda c_i dt; i = 1, 2, 3...$ be the first order probability that of arrival 'i' customers in batches in the system during a short period of time (t,t+dt), $\sum_{i=1}^{\infty} c_i = 1, \lambda > 0$, where $0 \leq c_i \leq 1$ is the mean arrival rate of batches.
- A setup time which is random variable followed by exponential distribution With mean setup time $1/\eta$.
- Single server gives service based on a general distribution with B(v) and density b(v). Given that the discontinued service time is u, Let $\mu(u)du$ be the conditional probability density function of service completion during the distance (u,u+du],

$$\mu(u) = \frac{b(u)}{1 - B(u)}. \tag{1}$$

hence

$$b(v) = \mu(v) e^{-\int_0^v \mu(u) du}. \tag{2}$$

- Let $(1 - a_1)$; $0 \leq a_1 \leq 1$ represent probability that an arriving customer balks wheneverserver is busy state , $(1 - a_2)$; $0 \leq a_2 \leq 1$ represent the probability that an arriving customer balks when the server is on vacation.

- Instantly, service is completed the customer can have an option to leave the system or join the system for demanding reservice, if required. We consider that probability of repeating the service as r and leaving the system without reservice as (1-r) by considering the service maybe repeated only once.
- On completing a service, server takes vacation which treated as random variable havingprobability p (or) may stay back in the system $(1 - p)$ to provide service with $0 \leq p \leq 1$.

- Server vacation time is determined by the density function v(s) and general (arbitrary) distribution function. Given that the elapsed vacation time is u, Let $v(u)du$ be the conditional probability of a completion of vacation during interval (u,u+du], We have,

$$\nu(u) = \frac{v(u)}{1 - V(u)}. \tag{3}$$

we have,

$$v(s) = \nu(s)e^{-\int_0^s \nu(u)du}. \tag{4}$$

- The queueing system many stochastic processes are believed to be independent of one another.

2.1. Governing Equations and Definitions

We take,

(i) $P_n(u, t)$ follows Probability that the server is active delivering service at time 't', there are 'n' ($n \geq 0$) consumers in the queue excluding the one being serviced, with an elapsed service time of u. As a result, $p_n(t)$ specifies the probability that the server is active providing service at time t and that there are 'n' consumers in the queue exclution of the one being under service irrespective of the value of u.

(ii) $R_n(u, t)$ represent Probability that at time 't' there are 'n' ($n \geq 0$) (consumers in the queue excluding one being repeating the service and the elapsed service time for this consumer is u. Accordingly $R_n(t)$ represent the probability that at time't' there are 'n' ($n \geq 0$) customers in the queue excluding the one being repeating the service, irrespective of the value of u.

(iii) $V_n(u, t)$ denotes Probability that server is on vacation at time t', with elapsed vacation time u, and there are 'n' ($n \geq 0$) customers waiting in the queue for service. As a result, irrespective of the value of u, $V_n(t)$ reflects the probability that there are 'n' customers in the queue and the server is on vacation at time t.

(iv) $D_n(t)$ represent Probability that server is in startup mode at time t, with 'n' ($n \geq 0$) customers in the queue.

(v) Q(t) follows Probability that there are no consumers in the system at time 't' and serveris idle but accessible.

2.2. Transient state behaviour

Model is regulated by differential-difference equations listed below.

$$\frac{\partial}{\partial t}P_n(u,t)+\frac{\partial}{\partial u}P_n(u,t)+(\lambda+\mu(u))P_n(u,t)=\lambda(1-a_1)P_n(u,t)+\lambda a_1\sum_{i=1}^{n}c_iP_{n-i}(u,t);n\geq 1. \tag{5}$$

$$\frac{\partial}{\partial t}R_n(u,t)+\frac{\partial}{\partial u}R_n(u,t)+(\lambda+\mu(u))R_n(u,t)=\lambda(1-a_1)R_n(u,t)+\lambda a_1\sum_{i=1}^{n}c_iR_{n-i}(u,t);n\geq 1. \tag{6}$$

$$\frac{d}{dt}D_n(t)=-(\lambda+\eta)D_n(t)+\lambda(1-a_1)D_n(t)+\lambda a_1\sum_{i=1}^{n}c_iD_{n-i}(u,t)+\lambda a_1c_nQ(t);n\geq 1. \tag{7}$$

$$\frac{\partial}{\partial t}V_n(u,t)+\frac{\partial}{\partial u}V_n(u,t)+(\lambda+\nu(u))V_n(u,t)=\lambda(1-a_2)V_n(t)+\lambda a_2\sum_{i=1}^{n}c_iV_{n-i}(u,t);n\geq 1. \tag{8}$$

$$\frac{\partial}{\partial t}V_0(u,t)+\frac{\partial}{\partial u}V_0(u,t)+(\lambda+\nu(u))V_0(u,t)=\lambda(1-a_2)V_0(u,t). \tag{9}$$

$$\begin{aligned}\tfrac{d}{dt}Q(t)&=-\lambda Q(t)+\lambda(1-a_1)Q(t)+\int_0^\infty V_0(u,t)\nu(u)du\\&+(1-p)\left\{(1-r)\int_0^\infty P_1(u,t)\mu(u)du+\int_0^\infty R_1(u,t)\mu(u)du\right\}.\end{aligned} \tag{10}$$

The boundary equations at $u=0$ must be considered when solving the aforementioned equations.

$$\begin{aligned}P_n(0,t)&=(1-p)\left\{\int_0^\infty R_{n+1}(u,t)\mu(u)du+(1-r)\int_0^\infty P_{n+1}(u,t)\mu(u)du\right\}\\&+\int_0^\infty V_n(u,t)\nu(u)du+\eta D_n(t);n\geq 1.\end{aligned} \tag{11}$$

$$R_n(0,t)=r\int_0^\infty P_n(u,t)\mu(u)du;n\geq 1. \tag{12}$$

$$V_n(0,t)=p\left\{(1-r)\int_0^\infty P_{n+1}(u,t)\mu(u)du\right\};n\geq 0. \tag{13}$$

At first Assume that no consumers in system so that, server is idle state.

$$V_0(0)=0;V_n(0)=0;Q(0)=1;P_n(0)=0;R_n(0)=0;n=0,1,2,... \tag{14}$$

3. Generating Functions of Queue Size

Probability generating functions are defined as

$$P_q(u,z,t)=\sum_{n=0}^{\infty} z^n P_n(u,t); P_q(z,t)=\sum_{n=0}^{\infty} z^n P_n(t). \tag{15}$$

$$R_q(u,z,t)=\sum_{n=0}^{\infty} z^n R_n(u,t); R_q(z,t)=\sum_{n=0}^{\infty} z^n R_n(t). \tag{16}$$

$$V_q(u,z,t)=\sum_{n=0}^{\infty} z^n V_n(u,t); V_q(z,t)=\sum_{n=0}^{\infty} z^n V_n(t). \tag{17}$$

$$C(z)=\sum_{n=1}^{\infty} c_n z^n. \tag{18}$$

The above defined probability generating functions are convergent in unit circle and Laplace transform of function is

$$\overline{f}(s)=\int_0^{\infty} f(t)e^{-st}dt. \tag{19}$$

and using equations from (5) to (10) by considering Laplace Transforms

$$\frac{\partial}{\partial u}\bar{P}_n(u,s)+(s+\lambda+\mu(u))\bar{P}_n(u,s)=\lambda(1-a_1)\bar{P}_n(u,s)+\lambda a_1\sum_{i=1}^{n} c_i\bar{P}_{n-i}(u,s). \tag{20}$$

$$\frac{\partial}{\partial u}\bar{R}_n(u,s)+(s+\lambda+\mu(u))\bar{R}_n(u,s)=\lambda(1-a_1)\bar{R}_n(u,s)+\lambda a_1\sum_{i=1}^{n} c_i\bar{R}_{n-i}(u,s). \tag{21}$$

$$(s+\lambda+\eta)\bar{D}_n(s)=\lambda(1-a_1)\bar{D}_n(s)+\lambda a_1\sum_{i=1}^{n} c_i\bar{D}_{n-i}(u,s)+\lambda a_1 C_n\bar{Q}(s). \tag{22}$$

$$\frac{\partial}{\partial u}\bar{V}_n(u,s)+(s+\lambda+\nu(u))\bar{V}_n(u,s)=\lambda(1-a_2)\bar{V}_n(u,s)+\lambda a_2\sum_{i=1}^{n-1} c_i\bar{V}_{n-i}(u,s). \tag{23}$$

$$\frac{\partial}{\partial u}\bar{V}_0(u,s)+(s+\lambda+\nu(u))\bar{V}_0(u,s)=\lambda(1-a_2)\bar{V}_0(u,s). \tag{24}$$

$$(s+\lambda)\bar{Q}(s)=1+(1-p)\left\{(1-r)\int_0^{\infty}\bar{P}_1(u,s)\mu(u)du+\int_0^{\infty}\bar{R}_1(u,s)\mu(u)du\right\}$$
$$+\int_0^{\infty}\bar{V}_0(u,s)\nu(u)du+\lambda(1-a_1)\bar{Q}(s). \tag{25}$$

considering the terminal conditions we get,

$$\bar{P}_n(0,s) = (1-p)\left\{(1-r)\int_0^\infty \bar{P}_{n+1}(u,s)\mu(u)du + \int_0^\infty \bar{R}_{n+1}(u,s)\mu(u)du\right\}$$
$$+\int_0^\infty \bar{V}_n(u,s)\nu(u)du + \eta\bar{D}_n(s); n \geq 1. \quad (26)$$

$$\bar{R}_n(0,s) = r\int_0^\infty \bar{P}_n(u,s)\mu(u)du \quad (27)$$

$$\bar{V}_n(0,s) = p\left\{(1-r)\int_0^\infty \bar{P}_{n+1}(u,s)\mu(u)du\right\}; n \geq 0. \quad (28)$$

multiply equation (20) by z^n, putting n=1 to ∞, using PGF defined above, then

$$\frac{\partial}{\partial x}\bar{P}_q(u,z,s) + (s + \lambda a_1(1-C(z)) + \mu(u))\bar{P}_q(u,z,s) = 0. \quad (29)$$

Using same methods to equations (21) to (24)

$$\frac{\partial}{\partial u}\bar{R}_q(u,z,s) + (s + \lambda a_1(1-C(z)) + \mu(u))\bar{R}_q(u,z,s) = 0. \quad (30)$$

$$(s + \lambda a_1(1-C(z)) + \eta)\bar{D}_q(z,s) = \lambda a_1 C(z)\bar{Q}(s). \quad (31)$$

$$\frac{\partial}{\partial u}\bar{V}_q(u,z,s) + (s + \lambda a_2(1-C(z)) + \nu(u))\bar{V}_q(u,z,s) = 0. \quad (32)$$

Given boundary conditions multiply eqn (26)by z^{n+1} and taking sum over 1 to ∞ and using probability generating function on them, we have

$$z\bar{P}_q(0,z,s) = (1-p)\left\{(1-r)\int_0^\infty \bar{P}_q(u,z,s)\mu(u)du + \int_0^\infty \bar{R}_q(u,z,s)\mu(u)du\right\}$$
$$+z\int_0^\infty \bar{V}_q(u,z,s)\nu(u)du + \eta z\bar{D}_q(z,s) + z(1-s\bar{Q}(s)) - \lambda a_1 z\bar{Q}(s). \quad (33)$$

Similarly multiply equation(27)by z^n, sum over n from 0 to ∞

$$\bar{R}_q(0,z,s) = r\int_0^\infty \bar{P}_q(u,z,s)\mu(u)du. \quad (34)$$

Multiply equation (28) by z^{n+1}, summing over 0 to ∞ and using the above defined generating function

$$z\bar{V}_q(0,z,s) = p\left\{(1-r)\int_0^\infty \bar{P}_q(u,z,s)\mu(u)du\right\}. \quad (35)$$

Integrating equation(29) from 0 to u yields

$$\bar{P}_q(u,z,s)=\bar{P}_q(0,z,s)e^{-(s+\lambda a_1(1-C(z)))u-\int_0^u \mu(t)dt}. \tag{36}$$

where $\bar{P}_q(0,z,s)$ is given by equation(33). Again integrating equation (36) by parts with respect to u yields

$$\bar{P}_q(z,s)=\bar{P}_q(0,z,s)\left[\frac{1-\bar{B}(s+\lambda a_1(1-C(z)))}{(s+\lambda a_1(1-C(z)))}\right]. \tag{37}$$

where

$$\bar{B}[s+\lambda a_1(1-C(z))]=\int_0^\infty e^{-(s+\lambda a_1(1-C(z)))u}dB(u). \tag{38}$$

is LST of the service time $B(u)$. Now multiplying both sides of equation (36) by $\mu(u)$ and integrating over u, we get

$$\int_0^\infty \bar{P}_q(u,z,s)\mu(u)du=\bar{P}_q(0,z,s)\bar{B}(s+\lambda a_1(1-C(z))). \tag{39}$$

Integrating equation(30) from 0 to u yields

$$\bar{R}_q(u,z,s)=\bar{P}_q(0,z,s)e^{-(s+\lambda a_1(1-C(z)))u-\int_0^u \mu(t)dt}. \tag{40}$$

where $\bar{R}q(0,z,s)$ is given by equation(34). Again integrating equation (40) by parts with respect to u yields

$$\bar{R}_q(z,s)=\bar{R}_q(0,z,s)\left[\frac{1-\bar{B}(s+\lambda a_1(1-C(z)))}{(s+\lambda a_1(1-C(z)))}\right]. \tag{41}$$

where

$$\bar{B}[s+\lambda a_1(1-C(z))]=\int_0^\infty e^{-(s+\lambda a_1(1-C(z)))u}dB(u). \tag{42}$$

LST of the reservice time $B(u)$. Multiplying by $\mu(u)$ on both sides of equation (40) and integrating over u, we arrive

$$\int_0^\infty \bar{R}_q(u,z,s)\mu(u)du = \bar{R}_q(0,z,s)\bar{B}(s+\lambda a_1(1-C(z))). \tag{43}$$

Similarly integrate equation (32) from 0 to u, we get

$$\bar{V}_q(u,z,s) = \bar{V}_q(0,z,s)e^{-(s+\lambda a_2(1-C(z)))u-\int_0^u \nu(t)dt}. \tag{44}$$

Again Integrating by parts with respect to u equation(44)

$$\bar{V}_q(z,s) = \bar{V}_q(0,z,s)\left[\frac{1-\bar{V}(s+\lambda a_2(1-C(z)))}{(s+\lambda a_2(1-C(z)))}\right]. \tag{45}$$

where

$$\bar{V}[s+\lambda a_2(1-C(z))] = \int_0^\infty e^{-(s+\lambda a_2(1-C(z)))u}dV(u). \tag{46}$$

LST of the vacation time V(u). Multiplying by $\nu(u)$ both sides of equation(44) and integrating over u, we have

$$\int_0^\infty \bar{V}_q(u,z,s)\nu(u)du = \bar{V}_q(0,z,s)\bar{V}(s+\lambda a_2(1-C(z))). \tag{47}$$

getting from equation (31)

$$\bar{D}_q(z,s) = \frac{\lambda a_1 C(z)\bar{Q}(s)}{(s+\lambda a_1(1-C(z)+\eta)}. \tag{48}$$

Now using (43), (47) and (48) in equation (33) and solving for $\bar{P}_q(0,z,s)$ we get

$$\bar{P}_q(0,z,s) = \frac{[z(1-s\bar{Q}(s))] + \left[\frac{\eta z\lambda a_1 C(z)\bar{Q}(s)}{([f_1(z)]+\eta)} - \lambda a_1 z\bar{Q}(s)\right]}{z-[(1-p)+p\bar{V}[f_2(z)]][(1-r)\bar{B}[f_1(z)]+(1-p)r(\bar{B}[f_1(z)])^2]}. \tag{49}$$

where
$f_1(z) = [s+\lambda a_1(1-C(z))]$ and $f_2(z) = [s+\lambda a_2(1-C(z))]$

From equation (49) substituting the value of $\bar{P}_q(0,z,s)$ in to equations (37),(41) and (45)

$$\bar{P}_q(z,s) = \frac{\left\{[z(1-s\bar{Q}(s))] + \left[\frac{\eta z\lambda a_1 C(z)\bar{Q}(s)}{([f_1(z)]+\eta)} - \lambda a_1 z\bar{Q}(s)\right]\right\}\left[\frac{1-\bar{B}[f_1(z)]}{[f_1(z)]}\right]}{z-[(1-p)+p\bar{V}[f_2(z)]][(1-r)\bar{B}[f_1(z)]+(1-p)r(\bar{B}[f_1(z)])^2]}. \tag{50}$$

$$\bar{R}_q(z,s) = \frac{\left\{[z(1-s\bar{Q}(s))] + \left[\frac{\eta z\lambda a_1 C(z)\bar{Q}(s)}{([f_1(z)]+\eta)} - \lambda a_1 z\bar{Q}(s)\right]\right\}\bar{B}[f_1(z)\left[\frac{1-\bar{B}[f_1(z)]}{[f_1(z)]}\right]}{z-[(1-p)+p\bar{V}[f_2(z)]][(1-r)\bar{B}[f_1(z)]+(1-p)r(\bar{B}[f_1(z)])^2]}. \tag{51}$$

$$\bar{V}_q(z,s) = \frac{p\left\{[(1-s\bar{Q}(s))] + \left[\frac{\eta\lambda a_1 C(z)\bar{Q}(s)}{([f_1(z)]+\eta)} - \lambda a_1 z\bar{Q}(s)\right]\right\}\bar{B}[f_1(z)]\bar{B}_2[f_1(z)]\left[\frac{1-\bar{V}[f_2(z)]}{[f_2(z)]}\right]}{z-[(1-p)+p\bar{V}[f_2(z)]][(1-r)\bar{B}[f_1(z)]+(1-p)r(\bar{B}[f_1(z)])^2]}. \tag{52}$$

equations (48),(50),(51) and (52) presents the pgf of the system's various states in the transient state.

4. The Steady State Analysis

To derive the probability distribution function the model in steady state define for these steady state probabilities by Suppress the argument 't' whenever it present in transiant state,

$$Lt_{s\to 0}s\bar{f}(s) = Lt_{t\to\infty}f(t). \tag{53}$$

multiplying both sides of equation (50),(51),(52) by s and applying equation(54) and simplifying, we get

$$P_q(z) = \frac{\frac{1}{(\lambda a_1(1-C(z))+\eta)}\left[(\lambda a_1+\eta)zQ[\bar{B}(\lambda a_1(1-C(z)))-1]\right]}{z-[(1-p)+p\bar{V}(\lambda a_2(1-C(z)))][(1-r)\bar{B}(\lambda a_1(1-C(z)))+(1-p)r(\bar{B}(\lambda a_1(1-C(z))))^2]}. \tag{54}$$

$$R_q(z) = \frac{\frac{1}{(\lambda a_1(1-C(z))+\eta)}\left[(\lambda a_1+\eta)zQ\bar{B}(\lambda a_1(1-C(z))[\bar{B}(\lambda a_1(1-C(z)))-1])\right]}{z-[(1-p)+p\bar{V}(\lambda a_2(1-C(z)))][(1-r)\bar{B}(\lambda a_1(1-C(z)))+(1-p)r(\bar{B}(\lambda a_1(1-C(z))))^2]}. \tag{55}$$

$$V_q(z) = \frac{p\left(\frac{a_1}{a_2}\right)\frac{1}{((\lambda a_1(1-C(z))+\eta)}\left[(\lambda a_1+\eta)Q\bar{B}(\lambda a_1(1-C(z)))[\bar{V}(\lambda a_2(1-C(z)))-1]\right]}{z-[(1-p)+p\bar{V}(\lambda a_2(1-C(z)))][(1-r)\bar{B}(\lambda a_1(1-C(z)))+(1-p)r(\bar{B}(\lambda a_1(1-C(z))))^2]}. \tag{56}$$

$$D_q(z) = \frac{\lambda a_1 QC(z)}{\lambda a_1(1-C(z))+\eta}. \tag{57}$$

By using condition for normalizing we able to get find Q

$$P_q(1)+R_q(1)+V_q(1)+D_q(1)+Q=1. \tag{58}$$

$$P_q(1) = \frac{\lambda a_1(\lambda a_1+\eta)QE(I)E(S)}{dr}. \tag{59}$$

$$R_q(1) = \frac{r\lambda a_1(\lambda a_1+\eta)QE(I)E(S)}{dr}. \tag{60}$$

$$V_q^{(1)}(1) = \frac{p(1-r)\lambda a_1(\lambda a_1+\eta)QE(I)E(V)}{dr}. \tag{61}$$

$$D_q^{(1)}(1) = \frac{\lambda a_1 Q}{\eta}. \tag{62}$$

$$dr = \eta \left\{1 - \lambda(1-r)a_1 E(I)(E(S) - p\lambda a_2(1-r)E(V)] - 2(1-p)r\lambda a_1 E(I)E(S)\right\}. \quad (63)$$

$$Q = \frac{1}{1+\frac{\lambda a_1}{\eta}} \left\{ \frac{1 - \lambda(1-r)a_1 E(I)(E(S) - p\lambda a_2(1-r)E(V) - 2(1-p)r\lambda a_1 E(I)E(S)}{1 + \lambda p(1-r)(a_1 - a_2)E(I)E(V) + 2rp\lambda a_1 E(I)E(S)} \right\}. \quad (64)$$

and ρ represent utilization factor of the system is obtained by ρ= 1-Q so that $\rho < 1$ satisfies the stability condition for which existence of steady state . From eqn(64),Substitute Q in eqns (54), (55), (56) and (57) $P_q(z), R_q(z), V_q(z)$ and the probability generating function of queue length $D_q(z)$ has been explicitly determined.

Let $P(z)$ represents the probability generating function of queue length irrespective of state of the system. Adding the equation (55),(56), (57) and (58) we obtain,

$$P(z) = P_q(z) + R_q(z) + V_q(z) + D_q(z). \quad (65)$$

Defining $W_q(z)$ as follows

$$W_q(z) = P_q(z) + R_q(z) + V_q(z). \quad (66)$$

Let L_q express the Avarage number of customers in the queue

$$L_q = \frac{d}{dz} P(z) \mid_{z=1}. \quad (67)$$

$$(i.e) L_q = \frac{d}{dz} W_q(z) \mid_{z=1} + \frac{d}{dz} D_q(z) \mid_{z=1}. \quad (68)$$

Hence, if z=1 then $W_q(1)$ is indefinite of 0/0 form, applying L-Hospitals rule two times then we write as $W_q(z) = \frac{N(z)}{D(z)}$ and equation (68) becomes

$$L_q = \frac{D'(1)N''(1) - N'(1)D''(1)}{2[D'(1)]^2} + \lambda E(I) a_1 Q \left[\frac{\lambda a_1 + \eta}{\eta^2}\right]. \quad (69)$$

where notations primes, double primes in above equation (69) represent the first and second derivative at z=1 respectively, we have

$$N'(1) = (\lambda a_1 + \eta)\lambda a_1 E(I) Q \{(1+r)E(S) + p(1-r)E(V)\}. \quad (70)$$

$$N''(1) = (\lambda a_1 + \eta)Q \{(\lambda a_1 E(I))^2[(1+r)E(S^2) + 2(1-r)pE(S)E(V) + r(E(S))^2]$$
$$+2(1+r)\lambda a_1 E(I)E(S) + 2(1-r)\lambda a_1 E(I)E(V) + p(1-r)\lambda^2 a_1^2 (E(I))^2 E(V^2)$$
$$+\lambda a_1 E(I(I-1))[(1+r)E(S) + (1-r)pE(V)] \}. \quad (71)$$

$$D'(1) = \eta \{1 - \lambda(1-r)a_1 E(I)(E(S) - p\lambda a_2(1-r)E(V)] - 2(1-p)r\lambda a_1 E(I)E(S)\}. \quad (72)$$

$$D''(1) = \eta \{(\lambda a_1 E(I))^2 E(S^2)[(1-r) + 2(1-p)r] + 2(1-p)r(\lambda a_1 E(I))^2(E(S))^2$$
$$+\lambda^2 E(I))^2 a_1 a_2 p(1-r)E(S)E(V) + (\lambda a_2 E(I))^2 p(1-r)E(V^2)$$
$$+\lambda a_1 E(I(I-1)) \{[(1-r) + 2(1-p)r]E(S) + p(1-r)E(V)\}\}$$
$$-2\lambda a_1 E(I) \{1 - \lambda(1-r)a_1 E(I)(E(S) - p\lambda a_2(1-r)E(V)] - 2(1-p)r\lambda a_1 E(I)E(S)\}. \quad (73)$$

With help of Little's formula as following we can get average waiting time.

$$W_q = \frac{L_q}{\lambda} \quad (74)$$

5. Conclusion

We analyzed a batch arrival non-markovian queueing system by considering both server and customers related aspects like server setup time, vacation policy and on the customer impatience form balking. Both transient and steady state solution an obtained for the model accompanied with queue characteristics.

6. References

[1] Abou El-Ata,M. O. and Hariri, A.A.M,(1992), The M/M/C/N queue with balking and reneging,Computers and Operations Research Vol.19, pp.713-716.

[2] Ancker,Jr. C. J. and Gafarian,A. V, (1963), Some queueing problems with balking and reneging, I. Operations Research, Vol.11, pp. 88-100.

[3] Altman,E. and Yechiali, U., (2006), Analysis of customer's impatience in queue with server vacations ,(queueing systems), pp. 261 -279.

[4] Baker,K.R., (1973), A note on operating policies for the queue M/M/1 with exponential startup, (INFOR,11), pp. 71-72.

[5] Barrer,D.Y., (1957), Queuing with impatient customers and ordered service, (Oper. Res, 5) 650-656.

[6] Choudhury.A and Medhi.P., (2010), Balking and Reneging in Multi server Markovian Queuing Systems,,(International Journal of Mathematics in Operational Research, 3:4), pp. 1377-394.

[7] Choudhury,G., (2000), An M^X/G/1 queueing system with a setup period and a vacation period, (Queueing Systems,36), 23-38.

[8] Choudhury,G., (2002), A batch arrival queue with a vacation time under single vacation policy, (Computers and Operations Research, 29:14), pp. 1941-1955.

[9] Choudhury,G.and Madan, K. C., (2005), A Two-Stage Batch Arrival Queuing System with A random Set up Time under Modified Bernoulli Schedule Vacation, (Revista Investigacion Operacional, 26:3) pp. 238-250.

[10]Jeyakumar.S and Arumuganathan, (2011), A Non-Markovian Queue with Multiple Vacations and Control Policy on Request for Re-service, (Quality Technology of Quantitative Management, 8:3), 253-269.

[11]Levy,Y. and Yechiali,U., (1975), Utilization of idle time in an M/G/1 queueing system, (Management Science, 22). PP. 202-211.

[12]Madan,K.C. and Abu Al-Rub, A.Z.,2004, On a single server queue with optional phase type server vacations based on exhaustive deterministic service and a single vacation policy, (Applied Mathematics and Computation, 149), pp. 723-734.

[13]Madan,K.C.,Al-Nasser, A.D.and Al-Masri, A.Q., 2004, On $M^{[x]}$/(G1,G2)/1 queue with optional re-service, (Applied Mathematics and Computation, 152),pp. 71-88.

[14]Madan,K.C.,(2001), On a single server queue with two stage heterogeneous service and deterministic server vacations, (International Journal of Systems Science, 31), pp. 113-125.

[15]Monita Baruah, Kailash C. Madan, Tillal Eldabi.,(2012), Balking and Re-service in a Vacation Queue with Batch Arrival and Two Types of Heterogeneous Service, (journal of Mathematics Research, 4:4), pp. 114-124.

[16]Ke.J. C., 2009, Modified T vacation policy for an M/G/1 queueing system with an un-reliable server and startup, (Mathematical and Computer Modeling, 41), pp. 1267-1277.

[17]L. Tadj and J.C. Ke. A, (2008), A Hysteretic Bulk Queue with a Choice of a Service and Optional Re-service,(Qualitative Technology of Quantitative Management, 5:2), pp. 161-178.

[18]Rajadurai, P., Varalakshmi, M., Saravanarajan, M.C., Chandrasekaran, V.M., (2015), Analysis of $M^{[X]}$/G/1 retrial queue with two phase service under Bernoulli vacation schedule and random breakdown. Int. J. Math. Oper. Res. 7(1), pp. 1941 .

[19]Rajadurai P, Chandrasekaran VM, Saravanarajan MC., (2015), Analysis of an $M^{[X]}$/G/1 unreliable retrial G-queue with orbital search and feedback under Bernoulli vacation schedule. OPSEARCH http://dx.doi.org/10.1007/s12597-015-0226-5.

[20]Madan, K.C., Hadjar, K., (2016), Time dependent and steady state solution of an M(x)/G/1 Queueing system with servers long and short vacations. J. Math. Comput. Sci. 6(3), pp. 486506.

[21]Vignesh, Maragathasundari., (2016), S.: Analysis of non Markovian single server batch arrival queueing system of compulsory three stages of service and fourth optional stage service interruptions and deterministic server vacations. Int. J. Oper. Res. (in press).

[22]Ayyappan G, Karpagam S., (2018), An $M^{[X]}$ /G (a, b)/1 queueing system with breakdown and repair, standby server, multiple vacation and control policy on request for re-service. Mathematics 6(6):101. http:// www.mdpi.com/2227-7390/6/6/101.