Research Article

Effect of Time Dependent Stenosis on Blood Flow through an Arterial Segment

Amit Bhatnagar[1], Swati Agarwal[1], R. K. Shrivastav[2] and Jayantika Pal[3]
[1]Institute of Applied Sciences, Mangalayatan University, Aligarh
[2]Department of Mathematics, Agra College, Agra
[3]Faculty of Engineering & Applied Sciences, Usha Martin University, Ranchi, Jharkhand
Email- amitbh2112@gmail.com

Abstract: The present study deals with the influence of time dependent stenosis and slip velocity on blood flow through an arterial segment. The problem is investigated by a joint effort of analytical and numerical techniques. The expressions of wall shear stress, resistance to flow, axial velocity and volumetric flow rate are obtained. The graphical representations have been made to validate the analytical findings with a view of its applicability to stenotic diseases. It is noticed that the resistance to flow and wall shear stress increases with the stenosis height.
Keywords: Wall shear stress, slip velocity, resistance to flow, time dependent stenosis and volumetric flow rate.

INTRODUCTION

Circulatory disorders are well known to be reasonable in most cases of death, and stenosis is one such case. Stenosis means narrowing of an artery, tube or orifice. It is the abnormal and unnatural growth in arterial wall thickness. The initial thickening of stenotic artery was understood as an early process in the beginning of atherosclerosis. Atherosclerosis is a vascular disease which affects arteries by a sub-endothelial build-up of fatty or lipid material rich in cholesterol. This disease hardens the vessel walls and narrows their lumen which restricts the blood flow to various parts of the body. The presence of stenosis in the vessels supplying blood to the brain can lead to stroke. For the prevention and cure of atherosclerosis a lot of investigations are performed which results in better understanding the nature of this type of disease. Considering cosine-shaped geometry and by taking blood as Newtonian fluid, Srivastava et al. [1], Biswas and Chakraborty [2], and Verma and Parihar [3] has carried out a good deal of studies, both theoretically and experimentally, to comprehend the influence of stenosis on blood flow through and beyond the narrowed arterial segment. Mandal [4] solved, numerically, the problem of non-Newtonian and nonlinear blood flow through a stenosed artery and the non-Newtonian rheology of the flowing blood is characterized by the generalized Power-law model. Mishra et al. [5] assumed stenosis of bell shaped geometry to investigate the various flow characteristics of blood through an arterial segment. The effect of mild stenosis on blood flow, in an irregular axisymmetric artery with oscillating pressure gradient is examined by Jain et al. [6]. The combined influence of an asymmetric shape and surface irregularities of constriction has been explored in the computational study. Singh et al. [7] developed a mathematical model for studying the magnetic field effect on blood flow through an axially non-symmetric but radially symmetric atherosclerotic artery. Johnston et al. [8] investigated a mathematical model to study the wall shear stress in four different human right coronary arteries using non-Newtonian blood model, as well as the usual Newtonian model of blood viscosity. Nanda et al. [9] investigated a mathematical model for analyzing flow characteristics through a multiple stenosed narrow artery. It is observed that the axial velocity of flow and size of constriction effectively influence the shear stress in a stenosed arterial segment.Bessonov et al. [10]talked about the non-Newtonian properties and rheology of blood considering blood made up of cells and plasma and assumed the blood as heterogeneous and explained the procedure of Dissipative Particle Dynamics on blood flow models. Misraet al. [11] explored Herschel-Bulkleyflow of blood through stenosed arteries. The effects of stenosis and catheter have been quantified on the axial velocity of blood, resistance to flow, volumetric flow rate areinspected.Nasrin et al. [12]presented a mathematical analysis of power law fluid showing the non-Newtonian properties of blood flowing through a thinarterial structure. The effect of transverse magnetic field on the characteristics of blood flow has been shown by Ahmed et al. [13]. They unraveled the system of transport equations by the technique of finite element method. Most of the analysis performed earlier related to constricted flow has dealt with stenosis independent of time. And the effects of time dependent stenosis on blood flow have been quantified with conventional condition of no slip velocity at the vessel wall. In view of the possible presence of a red cell slip at the wall, Nubar [14] has proposed the prospect of slip for blood in viscometers. Mishra et al. [15] explored a mathematical model for analyzing flow characteristics through a stenosed arterial segment assuming the velocity slip condition at the arterial wall. The present study deals with the effect of time dependent stenosis on blood flow with slip velocity at the vessel wall. The present analysis would provide assistance to the bioengineers to improvise the design and the construction of artificial organs.

REPRESENTATION OF PROBLEM

Here an axisymmetric, one-dimensional, laminar and fully developed flow of blood in a narrow artery is considered. The equation of motion governing this sort of flow in may be written as

$$-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)=0 \qquad (1)$$

$$-\frac{\partial p}{\partial r}=0 \qquad (2)$$

Where u the axial velocity of blood, p stands for the pressure at any point, so $(-\partial p/\partial z)$ is called the pressure gradient, μ the viscosity coefficient of blood.

The stenosis taken here is cosine shaped whose radius varies according to the time parameter T. The radius of artery with such type of stenosis is mathematically expressed as

$$R(z,t)=\begin{cases} R_0-\delta\left(1-e^{-t/T}\right)\left[1+\cos\dfrac{\pi(z-d)}{l_0}\right], & d<z<d+l_0 \\ R_0, & otherwise \end{cases} \qquad (3)$$

where R_0 stands for the radius of the artery outside the stenosis, d denotes the location of stenosis, $R(z, t)$ is the radius of the stenosed portion of the arterial segment under consideration at a longitudinal distance z from the left-end of the segment; l_0 denotes the length of stenotic region with l as length of arterial portion taken here and δ is the depth of the stenosis at the throat.

The velocity slip condition for the problem stated above may be listed as

$$u=u_s \text{ at } r=R(z,t) \qquad (4)$$

$$\frac{\partial u}{\partial r}=0 \text{ at } r=0 \qquad (5)$$

From (1), (4) and (5), the velocity function $\bar{u}$ is obtained as

$$u=u_s-\frac{1}{4\mu}\frac{\partial p}{\partial z}(R^2-r^2) \qquad (6)$$

The volumetric flow rate Q through the artery can be defined as

$$Q=\int_0^R 2\pi rudr \qquad (7)$$

which gives

$$Q=\pi R^2u_s-\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial z} \qquad (8)$$

From (8), we have

$$\frac{\partial p}{\partial z}=\frac{8\mu}{\pi R^4}\left(\pi R^2u_s-Q\right) \qquad (9)$$

Integrating (9) and taking $p=p_1$ at $z=0$ and $p=p_2$ at $z=l$, we get

$$p_1-p_2=\frac{8\mu Q}{\pi R_0^4}\int_0^L\frac{dz}{(R/R_0)^4}-\frac{8\mu u_s}{R_0^2}\int_0^L\frac{dz}{(R/R_0)^2} \qquad (10)$$

Thus the resistance to flow λ defined by

$$\lambda = \frac{p_1 - p_2}{Q} \tag{11}$$

may be expressed as

$$\lambda = \frac{8\mu}{\pi R_0^4}\int_0^L \frac{dz}{(R/R_0)^4} - \frac{8\mu u_s}{QR_0^2}\int_0^L \frac{dz}{(R/R_0)^2} \tag{12}$$

which may be written as

$$\lambda = \frac{8\mu}{\pi R_0^4}\left[l - l_0 + \int_d^{d+L_0} \frac{dz}{(R/R_0)^4}\right] - \frac{8\mu u_s}{QR_0^2}\left[l - l_0 + \int_d^{d+L_0} \frac{dz}{(R/R_0)^2}\right] \tag{13}$$

The wall shear stress is now obtained using the formula

$$\tau = -\frac{R}{2}\frac{\partial p}{\partial z} \tag{14}$$

and from equations (9) and (14), the wall shear stress is expressed as

$$\tau = -\frac{4\mu}{\pi R^3}\left(\pi R^2 u_s - Q\right) \tag{15}$$

DISCUSSION OF RESULTS

The influence of time dependent stenosis and slip velocity on the blood flow characteristics like wall shear stress, resistance to flow, volumetric flow rate and axial velocity has been investigated here. The effects of stenosis height, axial and radial distance on the various flow characteristics of blood aredemonstrated graphically.

The changes in resistance to flow with the presence of stenosis are shown in figures 1 and 2. The results exemplify that resistance to flow increases as stenosis height δ/R_0 and the dimensionless time t/T increases. The curve witht/T = 0.6 revealed the fact that in a long arterial segment, the effect of stenosis height on resistance to flow is very small until the value of δ/R_0 exceeds 0.2. Beyond this critical value of δ/R_0, the existence of the stenosis rapidly becomes noteworthy. The curve labeledt/T = 1 shows openly the influence of the stenosis on the resistance to flow. It may be noted that for δ/R_0 =0.1, the resistance to flow increases for rigid tube by about 20%. It should be accentuated that the change in the real pressure at a point in the blood vessel due to the mild stenosis will still be small in comparison to the mean arterial pressure. Figure 2 demonstrates that the resistance to flow increases as the length of stenosis increases.

Figure 3confirms the divergence in wall shear stress all through the axial length of the tube with time. The wall shear stressincreases rapidly from its approached value i.e., $z = d$ to its peak value at stenosis first throat i.e., at $z = d + l_0/2$. It steeply decreases from its peak value at stenosis first throat to its magnitude at critical stenosis height i.e., at $z = d + 3l_0/4$ and further increases steeply from its value at critical stenosis height to its peak value at stenosis second throat i.e. at $z = d + l_0$. Wall shear stress, then, decreases rapidly from its peak value at stenosis second throat to the end point of the stenotic region i.e., at $z = d + 3l_0/2$. One observes that, the wall shear stressassumes same magnitude at stenosis two throats and increases as time increases.

The fact that the axial velocity decreases as time parameter increases and slip velocity enhances the axial velocity of blood, is revealed in figures 4 and 5. In these figures, the variations in axial velocity of blood along with the radial length of arterial segment have been shown. The changes in axial velocity are the greatest near the axis of tube and least at the boundary of vessel. It is observed that the axial velocity attaining the maximum magnitudes at the axis and minimum at the boundary.

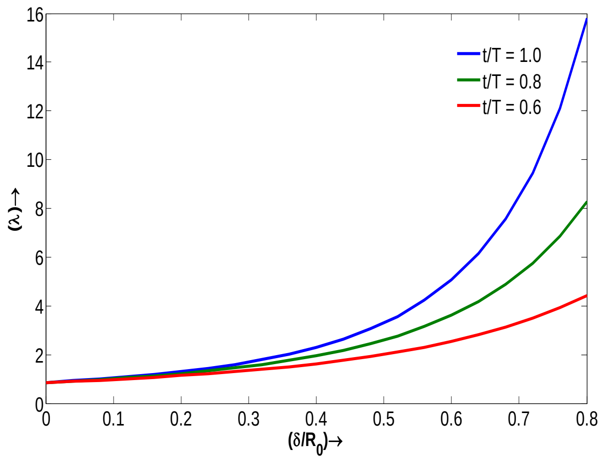

Figure 1. Resistance to flow with stenosis height for different parameters of time

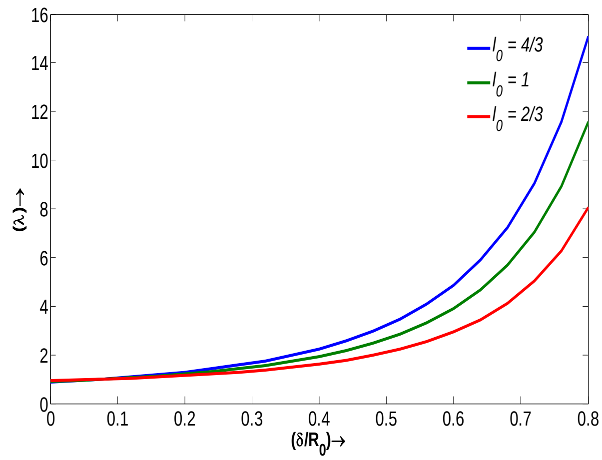

Figure 2. Resistance to flow with stenosis height for different lengths of stenosis

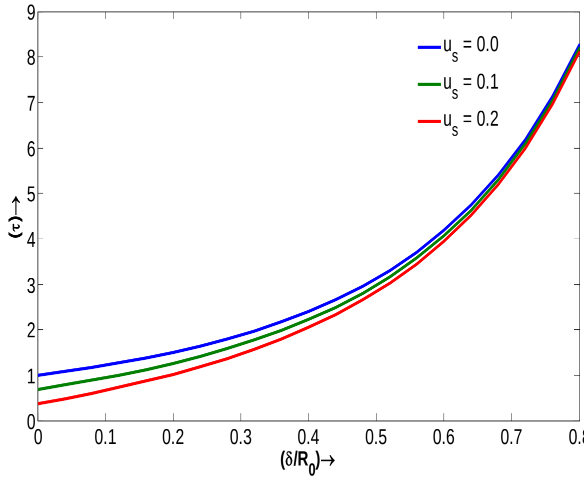

Figure 3. Wall shear stress with stenosis height for different slip velocities

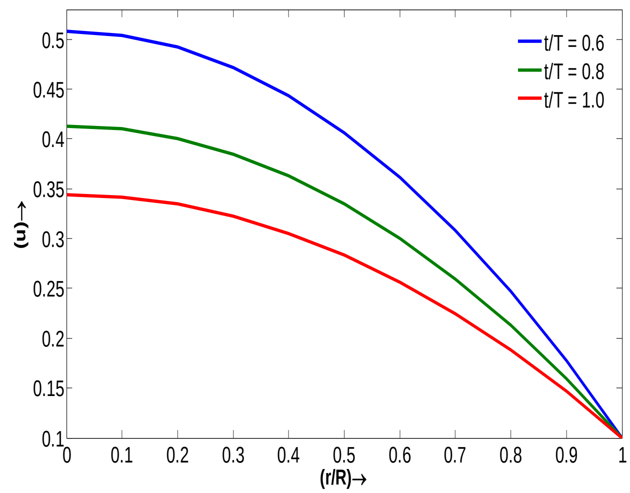

Figure 4. Axial velocity with radial distance for different parameters of time

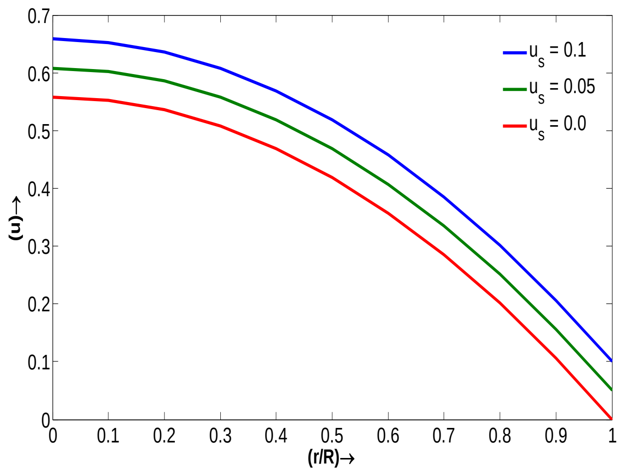

Figure 5. Axial velocity with radial distance for different slip velocities

CONCLUDING REMARKS:

A cosine shaped stenosis dependent on time with mild, moderate and severe size is assumed here to analyze the influence of stenosis on blood flow with slip velocity as one of the boundary condition. It is observed from the above discussion that the highest values of resistance to flowcan be seen when the arterial blockage becomes maximum.The changes in wall shear stress clearly depend on the parameter of time and the size of the constriction. An increase in size of stenosis raises the shear stress at wall. But it could be reduced by the slip velocity at the vessel wall.The volumetric flow rate becomes inversely proportional to the resistance to flow arising out of the stenotic flow. The nature of the axial velocity reflects very narrowly the outline of the stenosis. The slip velocity at the arterial and the parameter of time causes the variations in axial velocity of blood considerably.

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