Estimation of the mixed system reliability function using the Pareto and Rayleigh distributions

Asst.Prof.Dr. Ismael Hadi Challoob

Sundus Noori Kambir
Middle Technical University, Technical College of Management, Baghdad

Abstract

The research dealt with the study and analysis of mixed working systems by employing simulation experiments in Estimation the reliability function of the mixed system by using sample sizes and values of different hypothetical parameters on the assumption that the life time of the compounds are distributed Pareto distribution and Rayleigh distribution. The comparison criterion was adopted on the statistical scale (average of integral error squares) in order to determine the priority of each method through the results that were reached.

Introduction

Depending on the reliability of a specific system may be give misleading results in the case of the expansion of this system and its inclusion of sub-reliability systems, each of which exhibits a specific statistical behavior. Reliability of vehicles and systems and methods for their estimation because of their importance in all fields

The researcher Rahim, 2014 showed that the hierarchical Bayesian estimator is the best estimator after using several Bayesian methods to estimate the reliability function of hybrid systems and improved systems for exponential distribution data by employing the simulation method. The researcher Al-Saadi 2016 modeled the dependency function of the (A-Out of -H) system and the sequential system, where she used the logistic regression model and the exponential failure model, she reached the advantage of the unbiased uniform estimator with the least variance (UMVUE) using the (Monte-Carloe) method, depending on on the statistical scale (MSE). The researcher Al-Suhail, 2016 estimated the reliability of the systems (the 2-out of-3 system, the sequential system, and the parallel system) with Bayesian non-parametric and semi-parametric methods and compared them with traditional methods by means of the mean integral error squares (IMSE) statistical scale, and reached the advantage of Bayesian methods. The researchers (Abushal & AL-Zaydi), 2017, inferred about the parameters of a heterogeneous population using a mixture of two-parameter Pareto distributions and using the Markov-Monte Carlo algorithm (MCMC), where the comparison between Bayes estimators and the method of greatest possibility was done using the simulation of the (Monte- Carloe) method .Al-Nadawy 2017, suggested two methods (compact BASE, developed BASE) to estimate the reliability function of a hybrid system (parallel/sequential) that were compared with Bayesian methods and traditional methods for data with an exponential distribution and Pareto distribution, showing that the developed Bayes method is the best. The research aims to obtain several methods for Estimation the mixed system reliability function by finding the best estimator among the estimators with studying the ability of each method to act when the sample size and parameter values differ, using simulation and using the Monte-Carlo method and depending on the mean of squares Integral error (IMSE).

Statistical Distributions

Statistical distributions include many formulas and names depending on the number of parameters and the nature of the variable and will focus on the following statistical distributions

2.1-(Pareto Distribution)

This distribution is attributed to the Italian-born economist VilFredoPareto, who lived between 1923 and 1884, if the distribution bases were laid in the subject of economics, and this distribution has applications in communications and various engineering sciences, so that (t_1) (random variable follows the distribution of burrito that the probability density function of the random variable (t_1) for the distribution of Barito of the first type with two parameters (θ_1,β) :

$$f(t_1,\theta_1,\beta) = \begin{cases} \dfrac{\theta_1 \beta^{\theta_1}}{t_1^{\theta_1+1}} & t_1 > \beta, \beta > 0, \theta_1 > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

So that:-

θ_1: (shape parameters)

β: (scale parameters)

We impose (1 = β) so the probability density function will be as follows: -

$$f(t_1,\theta_1) = \begin{cases} \dfrac{\theta_1}{t_1^{\theta_1+1}} & t_1 > 1, , \theta_1 > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

The assembly distribution function (c.d.f) is found using formula (2) as follows: -

$$F(t_1,\theta_1) = \int_1^{t1} f(u,\theta)\, du \qquad (3)$$

$$F(t_1,\theta_1) = 1-\left(\frac{1}{t_1}\right)^{\theta_1} \qquad (4)$$

The function of the reliability of this distribution will be as follows:

$$R(t) = \left(\frac{1}{t_1}\right)^{\theta_1} \qquad (5)$$

2.2- (Rayleigh Distribution)

Riley's distribution is a model of failure used in life test research, a special case of the two-parameter Whipple distribution when the shape parameter is equal to (2), discovered by the English physicist(Lord Rayleigh).

The p.d.f function of Riley's single-parameter distribution is: -

$$f(t_2, \theta_2) = \begin{cases} \frac{2}{\theta_2} t_2 e^{-\frac{t_2^2}{\theta_2}} & t_2(0,\infty) \quad \theta > 0 \\ 0 & \text{Otherwise} \end{cases} \qquad (6)$$

θ_2 :(shape parameters)

The aggregate distribution function (c.d.f) is found using formula (6) as follows:

$$F(t_2, \theta_2) = \int_0^{t_2} \frac{2}{\theta_2} u e^{-\frac{u^2}{\theta_2}} du \qquad (7)$$

$$F(t_2, \theta_2) = 1 - e^{-\frac{t^2}{\theta_2}} \qquad (8)$$

The function of the reliability of this distribution is as follows:

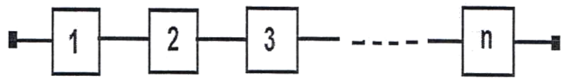
$$R(t_2) = e^{-\frac{t^2}{\theta_2}} \qquad (9)$$

3- (Reliability of Systems)

The system is defined as the link of a set of subsystems and vehicles that are somehow connected so that they are able to achieve the purpose for which they were designed, the system depends on the nature of the work of subsystems and vehicles as the system is a factor as long as the vehicles operate, from the common connectivity systems of vehicles of a particular system: -

3.1- Series System

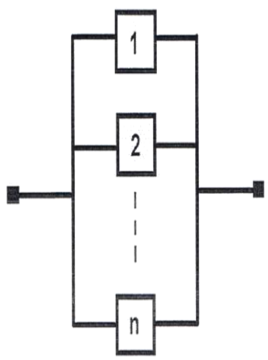
When vehicles are chain-attached, the failure of any component of the system causes the entire system to fail.



the shape (1): shows The Series system

3.2- Parallel System

When the system vehicles are connected in parallel, the system ceases to function if all its vehicles stop.



the shape (2): shows The parallel system

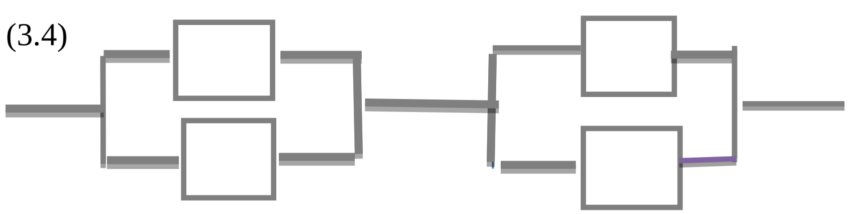
3.3- System Mixed (Parallel \ Series)

This system consists of many partial systems and within each partial system many vehicles,

This system acts as a series of parallel models (3) that explain how vehicles communicate in this system, the system fails if one of the two cases is achieved:

Vehicle failure (2.1)

Vehicle failure (3.4)



the shap(3): show Demonstrates the work of the mixed system

4 - (Reliability function) R (t)

The function of reliability, also known as the "survival function", is the probability of any random variable drawing a set of Events usually associated with the survival or failure of a system according to time, in other words the probability of equipment or machines remaining after time (t) and symbolized by the symbol R (t) either mathematically written as follows:

$$\boldsymbol{R(t) = \Pr(T > t) \ , t \ \geq \ 0} \qquad (10)$$

T is a constant random variable that represents the accumulated time of a particular system's life.

4.1 - (Reliability function properties)

1- dal is always decreasing and its value ranges from zero to one ($0 \leq R(t) \leq 1$).

2- This indicates that any device reaches failure when. ($t \rightarrow \infty$)

3- ($R(t)=1$) when ($t=0$) this indicates that the device is at the beginning of work

4.2-(Reliability Function of the Mixed System)

The function of the reliability of the system consisting of (n) of partial systems sequentially linked is defined as follows:

$$R(x) = \Pr\left(\varphi\,(X) > (x)\right)$$

The following:

φ (X): Represents a function of random variables, so :

$$R(x) = \Pr(X_1 > x \ , X_2 > x \, , \ldots , Xn > x \,)$$

The system's reliability will be the product of multiplying the individual responsibility of each system: -

$$R(x) = \Pr(X_1 > x) \Pr(\, X_2 > x \,) \ \ldots \ \Pr(Xn > x\,)$$

$$R(x) = [R_1(x) \, R_2(x) \ldots \ R_n(x)]$$

$$R(x) = \prod_{k=1}^{n} R_k\,(x) \qquad (11\,)$$

So that:

$R_k\,(x)$Represents a function of partial system reliability (k) and counts as follows

$$R_k\,(x) = 1 \ \ -\prod_{j=1}^{m} \left[\left(1 - R_{kj}(x)\right)\right] \qquad (12)$$

The following:

: $R_{kj}(x)$ represents a vehicle's (j) function attached parallelly in the partial system (k) i.e. :

$$R_{kj}(x) = 1 - [Pr(X_{kj} \leq x)] \quad (13)$$

We assume that the life time of vehicles within each partial system follows the distribution of barreto in the parameter () in the first vehicle and follows the distribution of Riley with the parameter) in the second vehicle thus being the function of the reliability of each vehicle in the system as follows:

$$R(x_1) = \left(\frac{1}{x_1}\right)^{\theta_1} \quad (14)$$

$$R(x_2) = e^{-\frac{x_2^2}{\theta_2}} \quad (15)$$

That is, the function of the partial system (K) of the equation (12) is as follows:

$$R_K(x) = \left[1 - \left(1 - \left(\frac{1}{x_1}\right)^{\theta_1}\right)\left(1 - e^{-\frac{x_2^2}{\theta_2}}\right)\right] \quad (16)$$

As for the function of the mixed system (parallel/serial) it takes the following formula:

$$R_K(x) = \prod_{k=1}^{n}\left[1 - \prod_{i=1}^{m}\left(1 - R_{kj}(x)\right)\right] \quad (17)$$

Assuming that each partial system contains two vehicles (j=1,2) and the number of partial systems (K=1,2) the final version of the mixed system's reliability is:

$$R_s(x) = \left[1 - \left(1 - \left(\frac{1}{x_1}\right)^{\theta_1}\right)\left(1 - e^{-\frac{x_2^2}{\theta_2}}\right)\right]^2 \quad (18)$$

5- Classical Methods In Estimation Reliability function for Mixed System

These methods are based on the assumption that the parameters to be estimated are fixed values and the methods to be addressed in this research are:

5.1- Maximum Likelihood Method(MLE)

It is one of the important methods of appreciation proposed by R.Q. Fisher in (1920), and its estimate (compared to other methods of appreciation) is good, and this method has characteristics that distinguish it from other methods of appreciation, the most important of which is invariance properties.

The idea of this method is to choose the destiner and let it be) (which maximizes the function of the possibility for viewing.

$$L(\theta, x_1........x_n) = Max\{L(\theta, x_1........x_n), \theta \in \Omega\}$$

Let's say x1,x2,...... xn represents a random sample of views that track the distribution of barreto, so the function of the greatest possibility for random variable views (x) is: -

$$\mathrm{L}(\mathrm{x}_1, \theta_1) = \prod_{\mathrm{i}=1}^{n} f(x_{1i})$$

$$= \prod_{i=1}^{n} \frac{\theta_1}{x1^{\theta_1+1}} \tag{19}$$

The estimator (θ_1) is obtained in the most possible way by taking the natural logarithm of the equation(19),

Then the teacher (θ_1) is partially derived and then the derivative is equal to zero:

$$\hat{\theta}_{1\,MEL} = \frac{n}{\sum_{i=1}^{n} \ln(x_{1i})} \tag{20}$$

Equation (20) is compensated for finding a distribution reliability equation in equation 5 according to the following formula:

$$\hat{R}_{(\mathrm{x}_{1\mathrm{i}})MEL} = \left(\frac{1}{x_{1i}}\right)^{-\frac{n}{\sum_{i=1}^{n} \ln(x_{1i})}} \tag{21}$$

As for the random variable that follows Riley's distribution, the teacher's ability (): θ_1) is obtained by taking the natural logarithm for the function of the possible equation and then the partial derivation of the teacher (): θ_1) and then the parity of the derivative to zero for the equation (22):

$$\mathrm{L}(\mathrm{x}_2, \theta_2) = \prod_{i=1}^{n} f(x_{2j})$$

$$=\prod_{i=1}^{n}\frac{2}{\theta_2}x_{2i}e^{-\frac{x_{2i}^2}{\theta_2}} \tag{22}$$

$$\hat{\theta}_{2MEL}=\frac{\sum_{i=1}^{n}(x_{2i})^2}{n} \tag{23}$$

Equation (23) is compensated for finding a distribution reliability equation (9) according to the following formula:

$$\hat{R}_{(x_{2i})MEL}=e^{-\frac{n(x_{2i})^2}{\sum_{i=1}^{n}(x_{2i})^2}} \tag{24}$$

Equations (21) and (24) in equation (18) are compensated for an estimate of the function of the mixed system's reliability according to the method of maximum possibility:

$$\hat{R}_{x(MEL)}=\left[\left(1-\left((\frac{1}{x_{1i}})^{-\frac{n}{\sum_{i=1}^{n}\ln(x_{1i})}})(1-e^{-\frac{nx_{2i}^2}{\sum_{i=1}^{n}x_{2i}^2}})\right)\right)\right]^2 \tag{25}$$

5.2- Method of Moments (MOM)

It is one of the methods commonly used in Estimation the parameters proposed by Johan and Bernoolle (1748-1667), which is used as an initial value of other methods, this method depends on the function generating the package where the computational medium of the sample (sample mean) represents the first torque and the basic idea of this method is by equating the determination of society with the determination

of the sample

$$M_1(\theta)=\hat{M}_1\left(x_1\ \ .\ \ x_2\ \ .....\ \ x_n\right)$$

$$\overline{X} = \frac{1}{n}\sum x_i \tag{26}$$

$$E(x) = \overline{X} \tag{27}$$

$$\mu = E(X) = \frac{\theta_1}{\theta_1 - 1} \tag{28}$$

$$\theta^{\wedge}_{1(MOM)} = \frac{\bar{X}}{\bar{X} - 1} \tag{29}$$

The distribution reliability function is estimated by compensating the equation (29) in equation (5) according to the following formula:

$$R^{\wedge}(x) = \left(\frac{1}{x_1}\right)^{\frac{\bar{x}}{\bar{x}-1}} \tag{30}$$

As for the random variable that follows the Riley distribution, the parameter estimate (θ_2) is obtained by following the same previous steps to equate the resolve of the community with the determination of the sample we get

(31)

$$\mu = \mathrm{E}(X) = \frac{\sqrt{\pi\theta_2}}{2}$$

$$\theta^{\wedge}_{2\,(MOM)} = \frac{4\overline{X}^2}{\pi} \tag{32}$$

The distribution reliability function is estimated by compensating the equation (32) in equation (9) according to the following formula:

$$R^{\wedge}(x) = e^{-\frac{\pi x_2^2}{4(\bar{x})^2}} \tag{33}$$

Equations (30) and (33) in equation (18) are compensated for an estimate of the function of the mixed system reliability according to the method of determination:

$$\hat{R}_{x(MOM)} = \left[\left(1-\left((1-(\frac{1}{x_1})^{\frac{\bar{x}}{(\bar{x})-1}})(1-e^{-\frac{\pi x_2^2}{4(\bar{x})^2}})\right)\right)\right]^2 \quad (34)$$

5.3- White's Method (WH):

This method of application depends on the distribution function to be assessed primarily as the function formula is converted into a formula similar to the linear regression equation.

This method applies to distribution by taking the natural logarithm of both sides of formula (4) so we get the following formula:

$$\ln[1-F(x_1)] = -\theta_1 \ln(x_1) \quad (35)$$

By analogizing formula (36) with the following linear regression equation:

$$Y = A + BX$$

$$\bar{y} = \frac{\sum_{i=1}^{n} \ln[1-F(x_1)]}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} \ln(x_1)}{n}$$

$$-B = -b = \theta_1$$

$$\hat{\theta}_{1\,(WH)} = -b \quad (36)$$

The estimated function of the dependant is as follows-:

$$\hat{R}(x_1)_{WH} = \left(\frac{1}{x_1}\right)^{-b} \quad (37)$$

As for the random variable that follows the distribution of Riley, the parameter estimate (θ_2) is obtained by taking the natural logarithm of both sides of the formula (8) and we get the following formula:

$$\ln[1-F(x_2)] = -\frac{x_2^{\ 2}}{\theta_2} \tag{38}$$

By multiplying the equation above with negative signal and taking the natural logarithm again, it produces the following:

$$\ln[-\ln[1-F(x_2)]] = 2\ln(x_2) - \ln(\theta_2) \tag{39}$$

The formula (39) is similar to the following linear regression equation:

$$Y = A + BX$$

So:

$$\bar{y} = \frac{\sum_{i=1}^{n} \ln[-\ln(1-F(x_2))]}{n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} \ln(x_2)}{n}$$

$$-A = \ln\theta_2$$

$$\hat{\theta}_{2(WH)} = e^{-a} \tag{40}$$

The estimated function of the reliability is as follows:

$$\hat{R}(x_2)_{WH} = e^{-x_2^2 e^{a}} \tag{41}$$

Equations (37) and (41) in equation (18) are compensated for an estimate of the mixed system reliability function according to the White method:

$$R_{x(WH)}^{\wedge} = \left[\left(1-\left((1-(\frac{1}{x_1})^{-b})(1-e^{-x_2^2 e^{a}})\right)\right)\right]^2 \tag{42}$$

6 experimental side

The empirical side is reviewed to compare the estimation methods that have been addressed by using the simulation method to find the best estimator among those estimators by means of the mean integral error squares (IMSE) criterion.

6.1 Simulation Concept

Simlation is a process of designing a mathematical model that depicts the general features and characteristics of the real system for the purpose of understanding the behavior of the system, or it is a method aimed at evaluating the preferences of alternative courses of action. θ2) for nine different models and sample sizes (n) were chosen for the repetition size (L=1000) for each experiment, after which the random variables that follow the Pareto distribution and Rayleigh distribution are generated to estimate the parameters of the distribution and the mixed system reliability function using the three estimation methods used in this research. On the mean integral error squares (IMSE) to compare the estimations of the methods used according to the formula:

$$IMSE[\hat{R}(t)] = \frac{1}{L}\sum_{i=1}^{L}\left\{\frac{1}{n_t}\sum_{j=1}^{n_t}(\hat{R}_i(t_j) - R(t_j))^2\right\}$$

6.1 Experimental results

After applying previous simulation experiments according to different estimation methods, different sample sizes and different estimation parameters, the following results appeared, as shown in the following tables:

Table (1) shows the estimated values of the mean integral error squares (IMSE) for all methods when the sample size (20)

θ_1	θ_2	MLE	MOM	WH	Best
0.25	0.5	1.56E-05	3.06E-08	3.06E-08	MOM,WH
	1	6.55E-04	1.12E-08	2.48E-07	MOM
	1.5	6.78E-05	1.09E-09	7.61E-08	MOM
0.5	0.5	1.37E-04	1.60E-07	3.45E-06	MOM
	1	2.05E-05	8.50E-08	5.25E-08	MOM
	1.5	5.35E-05	3.47E-09	4.50E-09	MOM
0.75	0.5	3.20E-05	8.30E-07	6.46E-10	WH
	1	1.36E-04	1.55E-08	3.57E-07	MOM
	1.5	5.29E-04	1.19E-09	1.19E-09	MOM.WH

From Table (1) and in the model (θ_2=0.5, θ_1=0.75), the preference was given to the (WH) method.

Table (2) shows the estimated values of the mean integral error squares (IMSE) for all methods when the sample size is (40).

θ_1	θ_2	MLE	MOM	WH	Best
0.25	0.5	1.15E-05	2.09E-10	1.60E-09	MOM
	1	5.40E-06	2.20E-06	1.26E-10	WH
	1.5	1.57E-06	1.60E-10	5.75E-11	WH
0.5	0.5	4.25E-06	1.52E-10	3.83E-08	MOM
	1	2.71E-07	1.03E-11	6.31E-10	MOM
	1.5	9.60E-10	3.03E-10	1.14E-10	WH
0.75	0.5	7.71E-06	1.11E-10	4.60E-09	MOM
	1	1.47E-04	7.64E-12	8.88E-10	MOM
	1.5	5.00E-06	1.41E-11	3.29E-10	MOM

From Table (2) and in the model (θ2 = 1, θ1 = 0.75), the preference was given to the (MOM) method.

Table (3) shows the estimated values of the mean integral error squares (IMSE) for all methods when the sample size is (60).

θ1	θ2	MLE	MOM	WH	Best
0.25	0.5	1.39E-05	5.72E-11	1.69E-09	MOM
	1	3.90E-07	1.54E-11	1.97E-07	MOM
	1.5	4.61E-07	3.07E-08	5.62E-08	MOM
0.5	0.5	2.83E-04	4.10E-11	1.10E-06	MOM
	1	2.95E-06	2.11E-09	9.63E-07	MOM
	1.5	5.75E-07	4.29E-07	1.43E-08	WH
0.75	0.5	1.06E-05	2.74E-11	9.20E-09	MOM
	1	2.81E-06	1.89E-09	1.10E-07	MOM
	1.5	4.14E-07	1.25E-08	3.63E-09	WH

From Table (3) and in the model (θ2 = 1, θ1 = 0.25), the preference was given to the (MOM) method.

Table (4) shows the estimated values of the mean integral error squares (IMSE) for all methods when the sample size is (80)

θ_1	θ_2	MLE	MOM	WH	Best
0.25	0.5	3.92E-08	2.37E-09	5.51E-11	WH
	1	5.07E-08	1.79E-11	3.55E-09	MOM
	1.5	8.80E-09	1.05E-09	4.21E-10	WH
0.5	0.5	2.13E-06	2.92E-10	1.46E-09	MOM
	1	1.25E-08	1.44E-07	5.84E-09	WH
	1.5	7.84E-10	3.71E-08	1.09E-09	MLE
0.75	0.5	1.06E-07	6.73E-09	1.68E-09	WH
	1	3.52E-07	2.46E-10	3.87E-08	MOM
	1.5	1.09E-07	8.11E-14	1.49E-11	MOM

From Table (4), and in the model (θ2 = 1.5, θ1 = 0.75), the preference was given to the (MOM) method.

Table (5) shows the estimated values of the mean integral error squares (IMSE) for all methods when the sample size is (100)

θ_1	θ_2	MLE	MOM	WH	Best
0.25	0.5	1.64E-06	3.35E-12	1.01E-08	MOM
	1	1.42E-10	2.22E-10	2.14E-10	MLE
	1.5	3.30E-08	3.85E-13	1.19E-09	MOM
0.5	0.5	8.67E-09	9.48E-10	2.53E-07	MOM
	1	1.54E-07	2.65E-12	1.55E-07	MOM
	1.5	3.22E-08	1.69E-09	1.37E-12	WH
0.75	0.5	4.14E-07	1.03E-11	2.19E-09	MOM
	1	1.22E-06	2.67E-12	1.50E-08	MOM
	1.5	1.09E-07	2.88E-09	3.46E-11	WH

From Table (5) and in the model (θ2 = 1.5, θ1 = 0.25), the preference was given to the (MOM) method.

7 - Conclusions and Recommendations

After applying a series of simulation experiments, a number of conclusions and recommendations emerged, the most important of which are:

7.1 Conclusions

1- From the simulation experiments, it was found that the moment method ranked first in terms of preference compared to the rest of the other methods, depending on the statistical scale, mean squares of integral error (IMSE).

2- The white method outperformed at the sample size (20), and at the sample sizes (40,60,80,100) the preference was given to the moment method as it gave less IMSE than the rest of the methods.

3- Simulation experiments showed that the method of greatest possibility is less efficient among the methods used in this research, depending on the statistical scale (IMSE).

4- The method of moments outperformed in the first place at the sample size (80) over the rest of the methods for all models in terms of preference.

7.2 Recommendations

1- The researcher suggests adopting the torque method in the process of Estimation the reliability function, due to its preference in estimation compared to the rest of the methods.

2- The researcher suggests conducting future research for mixed systems of compounds with different distributions.

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