Design and Implementation of Numerical Based PI Controller for Non Linear Spherical Tank System

Geetha T.[a*] & Somasundaram S.[b]

[a,b]Department of Electronics and instrumentation Engineering, Annamalai university, Chidambaram, Tamil Nadu, India.
Email: geetha22.1988@gmail.com[a*]

Article History: Received: 10 August2020; Revised 12 October 2020;Accepted: 07November 2020; Published online: 27November 2020

Abstract: Mostly in the process control industries, the main issue well-known in the literature is to control the non-linear process. The major problem lies on control of the liquid level in a spherical tank, which is non-linear in nature because of the variation in cross section area resulting in the change of shape. System identification is done using black box model and it is nonlinear along with the worst case model, parameter criterion which is approximated to first order plus dead time delay. Newton's Forward interpolation based Proportional Integral controller is implemented in the control of the non-linear process. The real time work is done in MATLAB via VMAT-01 module. The servo and regulatory responses of NF based proportional integral controller for different operating region is noted and results are compared with the other numerical methods such as Dimensional Analysis method and weighted geometric Centre method. The better controlling method is determined based on the indices such as reduced overshoot, faster settling time, better set point tracking, disturbance rejection and lower performance indices.

Keywords: PI Controller, Nonlinear system, Numerical methods, Spherical tank.

1. Introduction

Normally designing an optimal controller is really a challenge which is used in Non-linear process is very tough compared to the linear processes in the industries. Generally in the process control industries liquids are mostly pumped which is stored in tanks and another time pumped to a different tank. Mostly the chemical treatment is done for liquids in the tank also the level of the fluid should be kept in control. If the level is very high, it causes damage to the process or hazardous material and if the level is too low, it causes disturbances in the sequential operations [1]. So the control of liquid level is a vital need and it is very common in process industries. Spherical tanks are widely used in food processing industries, concrete mixing industries, sewage water treatment industries, petro-chemical industries, paper making etc for many applications. Its shape contributes to improved drainage of solid mixtures, gelatinous liquids at the bottom of the tank. The control of the tank level is a hard task due to frequently varying cross section and non-linearity since these industries need a traditional controlling method to control the tank level. Although there is a development in highly developed process control schemes, PI (Proportional + Integral) controllers are widely used in the industrial control applications due to its simplicity, robust performance and ease in implementation [2].

The PI controller design which is complex, time consuming and model dependant and it is less satisfactory for nonlinear system. Although the PI controller needs only some adjustable parameters, it is hard to be tuned accurately in real time process as tedious plant tests are necessary to attain enhanced controller setting. Because of this reason, finding a simple PI tuning approach with a significant performance improvement has been an significant research issue for process engineers. Consequently, the intention of the paper is to widen a technique which is easier with better performance in closed-loop model. Therefore it is an essential thing to employ numerical method which is also utilized in the nonlinear process. The PI controllers are better than the PID controllers due to the occurrence of noise in the processes.

Newton's Forward interpolation based PI tuning is preferred because of the potential to get controller parameters in an optimized manner with memory size, least computation time, accuracy when compared with other methods. The real time model designed which is also to control the fluid level in the tank. The process model is interfaced in real time with MATLAB by easy cost effective VMAT-01 module. The operating points in the spherical tank process is chosen as 40cm, 60cm and 72cm and corresponding mathematical model is done using the black box modeling and worst case model parameter criterion is utilized to obtain the desired FOPTD model. On behalf of this developed model, PI values are computed by means of the Newton's forward interpolation. The performance of the NF tuned PI controller is better when compared with further recently developed numerical methods and the results are presented.

2.Real Time Working Model Setup

The laboratory set up includes a spherical tank, pump, a water reservoir, rotameter , an electro pneumatic converter (I/P converter) a differential pressure transmitter, pneumatic control valve, a Personal Computer (PC) and an interfacing VMAT-01 module. The output of the differential pressure transmitter is interfaced with the PC by VMAT-01 module - RS-232 port. Using the Simulink blockset within MATLAB the model is developed and related by means of VMAT-1module.

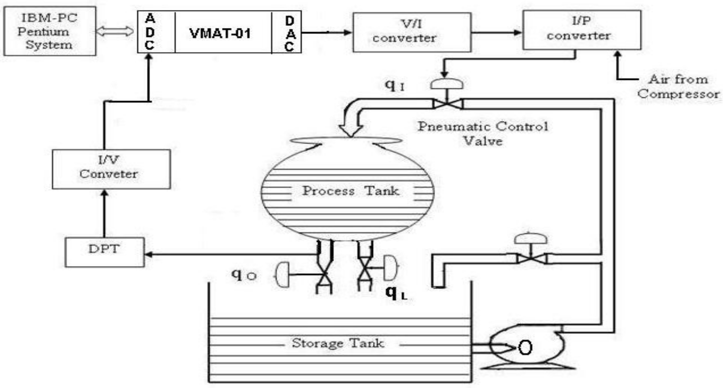

Fig.1. Working Model Setup

The real time setup of spherical tank is revealed in figure 1 &2. The water flow in the Spherical tank from the reservoir is controlled by the pneumatic control valve. The water level in the tank is supervised by means of the differential pressure transmitter in which its output differ from (4-20) mA and the similar is fed to the interfaced VMAT-01 module. Further the output is sent to I/P converter in the form of current signal then differs from (4-20) mA, additionally I/P converter converts the current signal to pressure signal which controls the actuation of the Pneumatic valve in applying the water in a controlled way to the tank which is happening for the continuous closed loop process .The air for actuation of the valve is fed via air compressor constantly maintained at 6kg pressure.

Fig.2. Real time experimental setup of a Spherical tank

3. System Identification

Different types of System Identification persevere to get the model of the process, from which Mathematical modeling and Black Box modeling are considered to be reliable [3].

3.1 Real Time Modeling

The Spherical tank is primarily filled with water from (0-45cm), among the fixed rate of water inflow & outflow. For all sample time, its output (4-20mA) from DPT (Differential Pressure Transmitter) is noted down and is fed to the system through serial port RS-232 with VMAT-01 interfacing module.

In this manner the data is scaled up in terms of level (in cm). By means of the open loop method, for the given change in input variable; the output response of the system is recorded. The process parameters Kp, τp, θ can be estimated by presenting a single step test in process input. The process gain is calculated by change in process output to change in process input. The process gain is calculated by,

$$Kp = \frac{\Delta}{\delta} \tag{1}$$

$$Kp = \frac{Change\ \ in\ process\ \ output}{Change\ \ in\ process\ \ input} \tag{2}$$

Simth assessed the parameters of FOPTD transfer function model by allowing the response of the actual system and that of the model to meet at two points 28.3% & 63.2% which depicts the two parameters time constant θ and time delay τ i.e., the time span between the input change occurrence in accord with significant output response is recorded [3]. Two point method is deployed here in estimating the process parameters and time taken for the process output to reach 28.3% and 63.2% respectively. The time constant and time delay is obtained from the equation (3) and (4)

$$\tau_p = 1.5\ (t_{63.2\%} - t_{28.3\%}) \tag{3}$$

$$\theta = (t_{63.2\%} - \tau_p) \tag{4}$$

The purpose of this method is to get the model of the process which reaches the steady state at fixed inflow & outflow rate, for this the open loop response of the spherical tank process is recorded between the span of the operating points(region) from 30% to 45%. A step size increment of 5% is given in input flow rate & corresponding readings of the output are collected until a new steady state is reached. The same procedure is carried out at all operating regions and the model parameter values such as process gain (Kp),time constant (τ), and time delay (t_d) are noted. The experimental data are approximated to be the FOPDT model and the model parameters are tabulated in table (1).

Table 1. Black box model of a non-linear spherical tank for different operating points

S.No	Operating region	Kp	T	t_d
1	30 – 35	2.94	3450	250
2	35 – 40	3.788	4500	300
3	40 – 45	5.294	5850	150

From this table, the worst case model parameter criterion is utilized to obtain the larger process gain (Kp), smaller time constant (τ) and larger delay (t_d). From this the FOPTD model of a spherical tank process is given by,

$$G(S) = \frac{5.294}{3450} e^{-300s} \tag{5}$$

From this model a Stable PI controller is designed based on numerical techniques.

4. Controller Design

4.1. Gregory Newton's Forward Interpolation Formula

In mathematics it is clear that it is the art of studying in between rows of the table. It is the practice of calculating the intermediary values in a function within the particular set of tabular values which is utilized in finding the least value among many values from the table. Here Z = f(x) represents the function including the values corresponding to $x_0, x_1, .. x_n$ respectively of x[4],[5]. The set ofK_p,K_i values are obtained from the stability boundary locus approach. Getting the values for Z from K_p&$K_i$and x values from ω intervals to produce the difference table. Choosing the x value as the least positive one for getting maximum accuracy and it varies according to the ω intervals. For K_p x value is taken as the centre value in the beginning of the table. For K_i x value is taken as the second top most value in the beginning of the table. If $x_0, x_1, \dots x_n$ are equidistant that is,$X_i - X_{i-1} = h$, for i =1,2,3...n

It can be written as $x_i = x_0 + h$

Let $m_n(x)$ be a polynomial of the nth degree in x such that $y_i = f(x_i) = m_n(x_i), i = 0,1,2 \dots n$.

$$m_n(x) = a_0 + a_1(x - x_0)^{(1)} + a_2(x - x_0)^{(2)} + \cdots + a_r(x - x_0)^{(r)} + \cdots + a_n(x - x_0)^{(n)} \tag{6}$$

$$u = \frac{x - x_0}{h}$$

Where, h is ω interval, x_0 is the initial value[ ]

$$= y_0 + \frac{u^{(1)}}{1!}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \cdots + \frac{u^{(r)}}{r!}\Delta^r y_0 + \ \ldots + \frac{u^{(n)}}{n!}\Delta^n y_0 \quad (7)$$

It is known as Gregory – Newton forward interpolation formula.

4.2.Weighted Geometric Centre Method

The weighted geometric centre is to find the mean ("average") position of all the points in the shape. This definition stretches to the object in n-dimensional space in which its centroid is the mean position of every point in all the coordinate directions [6]. These points are closely placed at least ω values. Through the rising values of ω, it becomes distant and then tightens about thepeak point of stability boundary locus and unwraps then it reaches towards the stability boundary locus to the real root boundary. These provide several considerable advantages such as it can also be applied to the state delay systems and also the time delay systems by means of parametric uncertainties. Those parameters forever guarantee the closed loop stability also provides good time domain performance with small control signal. The method gives an easy PI tuning rule which provides a good transient performance for the time delay systems.

$$k_{pw} = \frac{1}{n}\sum_{j=1}^{n} kpj,$$

$$k_{iw} = \frac{1}{2.n}\sum_{j=1}^{n} kij,$$

4.3. Dimensional analysis method

Dimensional analysis is a mathematical tool regularly applied in engineering and physics to simplify a problem by minimizing the count of variables to minimum of essential ones [7]. In simple, dimensional analysis is a method for eliminating off the point information from a relation between quantities [8]. Based on Buckingham's pi-theorem, the PI parameters are derived from the parameters of the model through finding the second and third dimensionless numbers. Initially, the best values of controller parameters are derived using genetic algorithms. The resultant formulae can also be applied to first order plus long dead time systems. Then optimal values are mapped using curve-fitting techniques. It is used to curtail IAE and also used to assure a minimum Gain margin and Phase margin.

$$KK_c = 0.4849\frac{T}{\tau d} + 0.3047$$

$$\frac{Ti}{T} = 0.4262\,\frac{\tau d}{T} + 0.9581$$

5. Experimental Results

5.1 Real Time Experimental Analysis of Numerical Based PI Controller

Using the real time model set up the servo response of the spherical tank level process is mapped for shift in operating point of 60% with raise and drop off in set point at various time interval +/-5% & +/-10% as revealed in figure 3 to 6.The NF based PI controller shows best results in correlation to WGC and DA based PI controllers with minimal overshoot,faster rise time & minimal settling time. This is evident from the performance index value as tabulated in table 2 to 5 where IAE, ISE values & time domain specifications are on the minimum side. The NF PI controller provides highly stable output at various operating points of the process across the complete span of the tank.

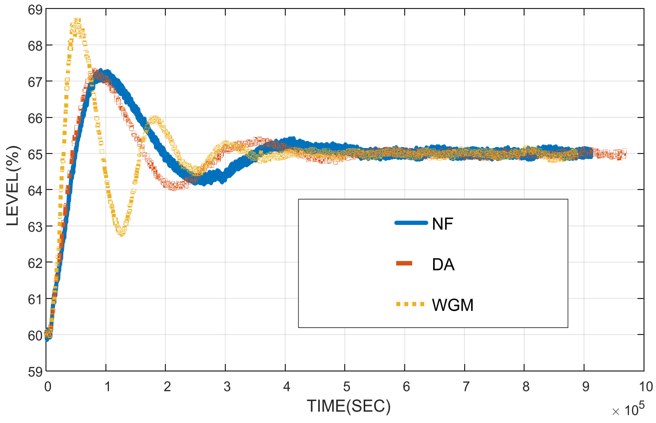

Fig.3. Servo Response of spherical tank level process for 5% Increase inSet point from nominal operating point of 60% using Numerical techniques

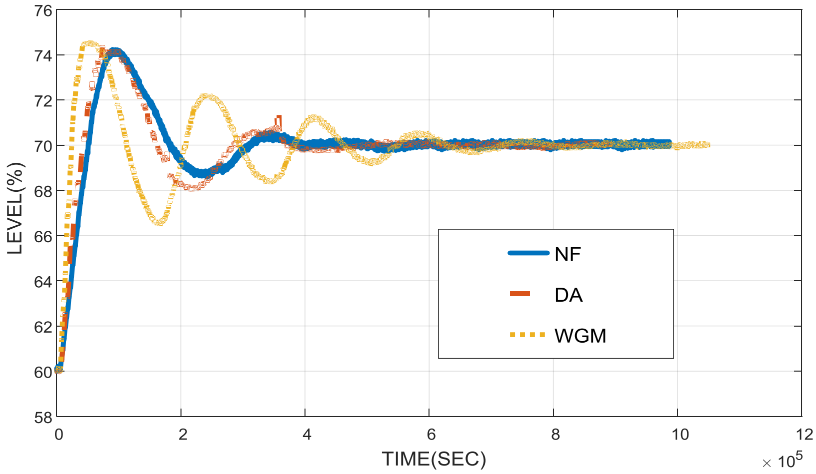

Fig.4. Servo Response of spherical tank level process for 10% Increase inSet point from nominal operating point of 60% using Numerical techniques

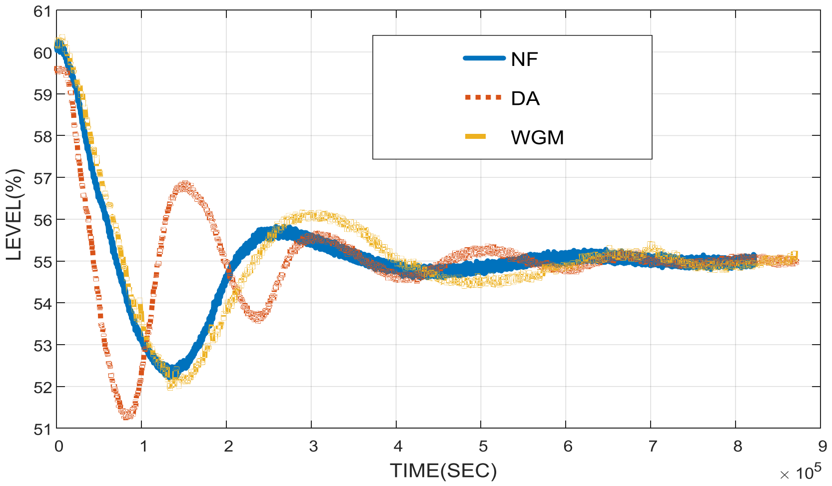

Fig.5. Servo Response of spherical tank level process for 5% decrease inSet point from nominal operating point of 60% using Numerical techniques

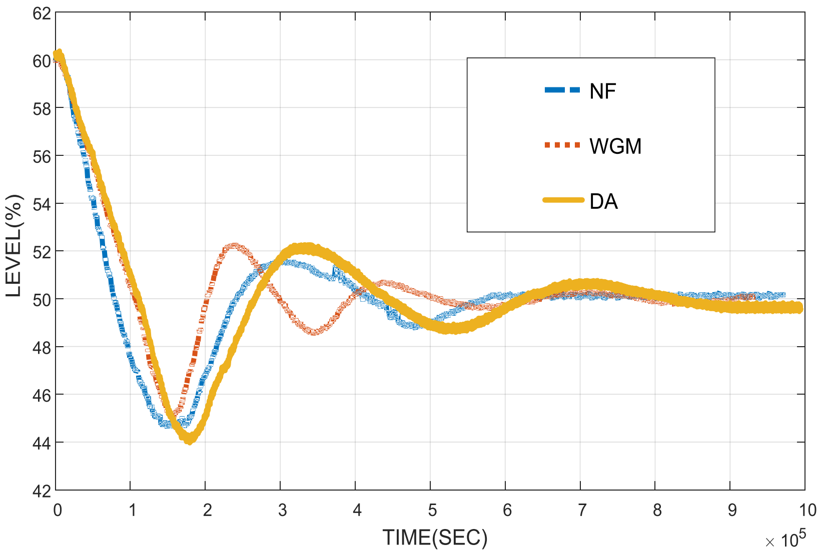

Fig.6. Servo Response of spherical tank level process for 10% decrease inSet point from nominal operating point of 60% using Numerical techniques

Table 2. Performance Index of spherical tank level process for 5% Increase inSet point from nominal operating point of 60% using Numerical techniques

S.No	METHOD	ISE	IAE	t_r(sec)	%M_p	t_s(sec)
1	NF	7.9894e^{+04}	4.9068e^{+04}	$0.5x10^5$	3.076	$4x10^5$
2	WGM	1.0782e^{+05}	5.4459e^{+04}	$0.3x10^5$	5.846	$5.5x10^5$
3	DA	9.2905e^{+04}	5.1585e^{+04}	$0.5x10^5$	3.076	$5x10^5$

Table 3. Performance Index of spherical tank level process for 10% Increase inSet point from nominal operating point of 60% using Numerical techniques

S.No	METHOD	ISE	IAE	t_r(sec)	%M_p	t_s(sec)
1	NF	2.9742e^{+05}	8.0399e^{+04}	$0.4x10^5$	5.71	$4x10^5$
2	WGM	3.2535e^{+05}	1.1014e^{+05}	$0.2x10^5$	6.071	$7x10^5$
3	DA	3.2107e^{+05}	9.1969e^{+04}	$0.3x10^5$	5.71	$5x10^5$

Table 4. Performance Index of spherical tank level process for 5% decrease inSet point from nominal operating point of 60% using Numerical techniques

S.No	METHOD	ISE	IAE	t_r(sec)	%M_p	t_s(sec)
1	NF	1.2976e^{+05}	6.4890e^{+04}	$0.6x10^5$	4.8	$5.5x10^5$

2	WGM	$1.8570e^{+05}$	$8.2875e^{+04}$	$0.45x10^{5}$	6.9	7
3	DA	$1.3425e^{+05}$	$7.1901e^{+04}$	$0.6x10^{5}$	5.45	7.5

Table 5. Performance Index of spherical tank level process for 10% decrease inSet point from nominal operating point of 60% using Numerical techniques

S.No	METHOD	ISE	IAE	t_r(sec)	%M_p	t_s(sec)
1	NF	$5.0711e^{+05}$	$1.1130e^{+05}$	$0.7x10^{5}$	10	$5.5x10^{5}$
2	WGM	$4.9570e^{+05}$	$1.0372e^{+05}$	$1.2x10^{5}$	9.6	$6.5x10^{5}$
3	DA	$5.3425e^{+05}$	$1.3871e^{+05}$	$1.2x10^{5}$	12	$8.5x10^{5}$

The regulatory response of the spherical tank level process at nominal operating point of 60% when subjected to 50% sudden opening of the tank output valve is shown in figure 7 to 9. The NF PI controller yields better performance measures and provides a prompt response to the disturbance caused in correlation with other conventional controllers as given in table 6.This proves the fact that NF PI controller acts upon effectively in level control of the tank even in uncertain conditions without causing any disturbance to the process involved.

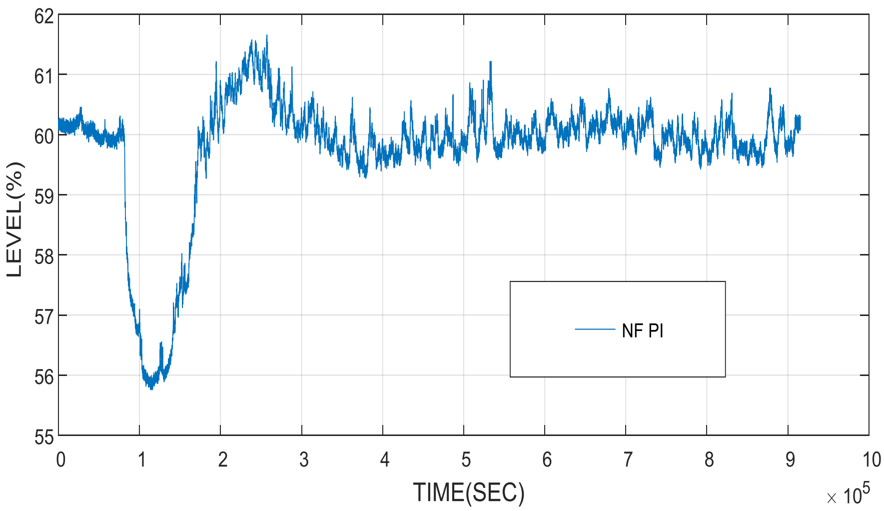

Fig.7. Regulatory Response of spherical tank level process for 50% Output valve opening at nominal operating point of 60% using NF PI controller

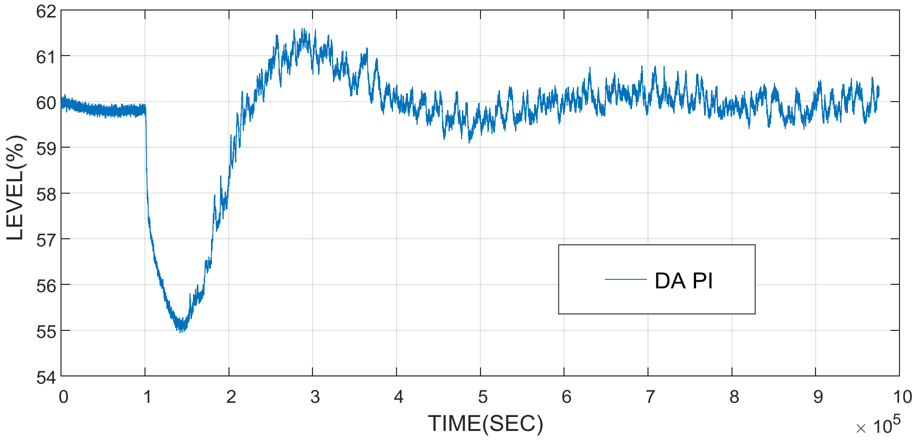

Fig.8. Regulatory Response of spherical tank level process for 50% Output valve opening at nominal operating point of 60% using DA PI controller

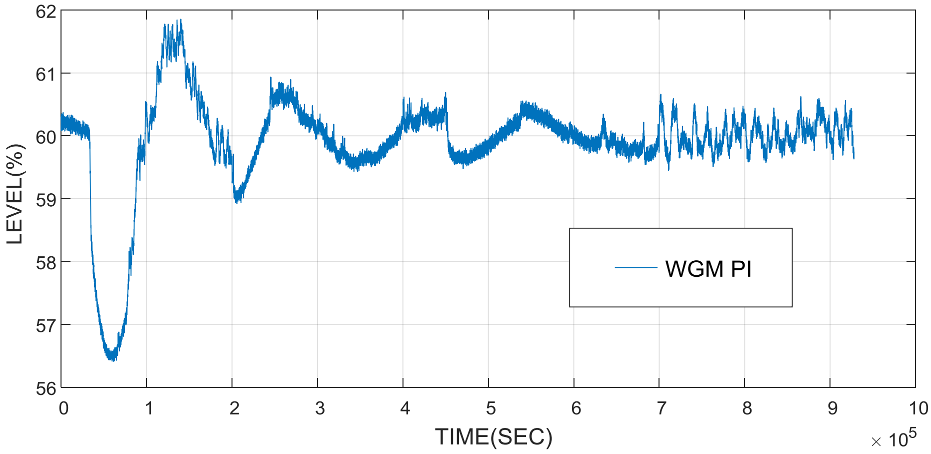

Fig.9. Regulatory Response of spherical tank level process for 50% Output valve opening at nominal operating point of 60% using WGM PI controller

Table 6. Performance Index of spherical tank level process for 50% Output valve openingAt nominal operating point of 60% using Numerical techniques

S.No	METHOD	ISE	IAE	%M_p	t_s(sec)
1	NF	6.3985e^{+04}	4.1240e^{+04}	1.5	3.5
2	DA	1.1348Z^{+05}	5.5822e^{+04}	1.5	5
3	WGM	1.0018e^{+05}	6.4709e^{+04}	1.9	6.5

6.Conclusion

In the present work Spherical tank is taken for the real time experimental study and an FOPTD model is obtained using the worst case model parameter criteria for level control at various operating regions. The design and implementation of real time NF based PI controller is done & compared with WGC and DA based PI controller. Based on the real time results it is accomplished that, the NF based PI controller outperforms the other conventional controllers which is clear from the servo & regulatory responses which reflects its effectiveness in terms of performance measures & time domain specifications.

References

[1] N. Tan, and I. Kaya, "Computation of stabilizing PI controllers for interval systems" *presented at the 11th Mediterranean Conference on Control and Automation, Rhodes, Greece*, 2003.

[2] N. Tan, I. Kaya, C. Yeroglu, and D. P. Atherton, "Computation of stabilizing PI and PID controllers using the stability boundary locus" *Energy Conversion and Management*, vol. 47, no. 18-19, pp. 3045-3058,2006.

[3] G.Sakthivel, T.S. Anandhi and S.P. Natarajan, "Modelling and Real time Implementation of Digital PI Controller for a Non Linear Process" *Journal of Innovative Research in Engineering and Sciences,*pp.274-290,2011.

[4] Kandasamy. P, Thilakavathy. K and Gunavathy. K Numerical Methods. *S.Chand and Co. New Delhi* 1999.

[5]Burden.R.L, and Faries. T.D., "Numerical Analysis", *Seventh Edition, Thomson Asia Pvt. Ltd, Singapore* 2002.

[6] Cem Onat, Department of Mechanical Engg., Inonu University, Turkey, "A New concept on PI Design for Time Delay Systems: Weighted Geometrical Centre", *International Journal Conference of InnovativeComputing and Control,* Vol:9, April 2003.

[7] E. S. Taylor. Dimensional Analysis for Engineers, *Clarendon Press, Oxford,* (1974).

[8]M. Zlokarnik. Dimensional Analysis and Scale-up in Chemical Engineering, *Springer-Verlag, Berlin,* (1991).